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英文版

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数字系统设计与VHDL

Digital Systems Design with VHDL

[美] Charles H. Roth, Jr.
Lizy Kurian John 著

梁松海 改编

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数字系统设计与VHDL 英文版

Digital Systems Design with VHDL

本书对原著进行了结构调整，使之更适合作为本科双语教学教材。第1章首先回顾了逻辑设计基本原理，第2章和第3章分别讲解了VHDL基本知识和高级主题，第4章为简单设计实例，第5章讨论状态机，第6章讨论浮点数运算，第7章讨论硬件测试和可测试性设计，第8章给出了一些高级设计实例。

本书是作者多年来在得克萨斯大学奥斯汀分校讲授数字系统设计课程的经验积累。书中总结了VHDL语言的常规和高级文法特点，介绍了数字系统中运算部件从简单到复杂的设计方法，基于SM图实现复杂控制逻辑的VHDL设计方法，以及运算部件和控制逻辑的系统整合方法。书中提供了大量的设计实例可供参考，适用于本科高年级同学和研究生对数字系统进行VHDL设计与实现。

作者简介

Charles H. Roth, Jr. 美国斯坦福大学博士，1961年就职于得克萨斯大学奥斯汀分校，目前是电气与计算机工程系教授。他的授课和研究领域涵盖了数字系统理论和设计、微计算机系统和VHDL应用，出版了4本著作。

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（英文版）

Digital Systems Design with VHDL

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内容简介

本书对原著进行了结构调整，使之更适合作为本科双语教学教材。第1章首先回顾了逻辑设计基本原理，第2章和第3章分别讲解了VHDL基本知识和高级主题，第4章为简单设计实例，第5章讨论状态机，第6章讨论浮点数运算，第7章讨论硬件测试和可测试性设计，第8章给出了一些高级设计实例。全书将工业标准硬件描述语言VHDL和数字系统设计融为一体，较好地实现了控制逻辑和运算部件的整合设计，并给出了多个设计实例，便于学生在实践中得到提高。

本书适合作为高等院校电子、电气和计算机专业本科生数字系统设计类课程的双语教学教材，也适合作为相关工程技术人员的参考书。

Charles H. Roth, Jr. & Lizy Kurian John
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序

2001年7月间，电子工业出版社的领导同志邀请各高校十几位通信领域方面的老师，商量引进国外教材问题。与会同志对出版社提出的计划十分赞同，大家认为，这对我国通信事业、特别是对高等院校通信学科的教学工作会很有好处。

教材建设是高校教学建设的主要内容之一。编写、出版一本好的教材，意味着开设了一门好的课程，甚至可能预示着一个崭新学科的诞生。20世纪40年代MIT林肯实验室出版的一套28本雷达丛书，对近代电子学科、特别是对雷达技术的推动作用，就是一个很好的例子。

我国领导部门对教材建设一直非常重视。20世纪80年代，在原教委教材编审委员会的领导下，汇集了高等院校几百位富有教学经验的专家，编写、出版了一大批教材；很多院校还根据学校的特点和需要，陆续编写了大量的讲义和参考书。这些教材对高校的教学工作发挥了极好的作用。近年来，随着教学改革不断深入和科学技术的飞速进步，有的教材内容已比较陈旧、落后，难以适应教学的要求，特别是在电子学和通信技术发展神速、可以讲是日新月异的今天，如何适应这种情况，更是一个必须认真考虑的问题。解决这个问题，除了依靠高校的老师和专家撰写新的符合要求的教科书外，引进和出版一些国外优秀电子与通信教材，尤其是有选择地引进一批英文原版教材，是会有好处的。

一年多来，电子工业出版社为此做了很多工作。他们成立了一个“国外电子与通信教材系列”项目组，选派了富有经验的业务骨干负责有关工作，收集了230余种通信教材和参考书的详细资料，调来了100余种原版教材样书，依靠由20余位专家组成的出版委员会，从中精选了40多种，内容丰富，覆盖了电路理论与应用、信号与系统、数字信号处理、微电子、通信系统、电磁场与微波等方面，既可作为通信专业本科生和研究生的教学用书，也可作为有关专业人员的参考材料。此外，这批教材，有的翻译为中文，还有部分教材直接影印出版，以供教师用英语直接授课。希望这些教材的引进和出版对高校通信教学和教材改革能起一定作用。

在这里，我还要感谢参加工作的各位教授、专家、老师与参加翻译、编辑和出版的同志们。各位专家认真负责、严谨细致、不辞辛劳、不怕琐碎和精益求精的态度，充分体现了中国教育工作者和出版工作者的良好美德。

随着我国经济建设的发展和科学技术的不断进步，对高校教学工作会不断提出新的要求和希望。我想，无论如何，要做好引进国外教材的工作，一定要联系我国的实际。教材和学术专著不同，既要注意科学性、学术性，也要重视可读性，要深入浅出，便于读者自学；引进的教材要适应高校教学改革的需要，针对目前一些教材内容较为陈旧的问题，有目的地引进一些先进的和正在发展中的交叉学科的参考书；要与国内出版的教材相配套，安排好出版英文原版教材和翻译教材的比例。我们努力使这套教材能尽量满足上述要求，希望它们能放在学生们的课桌上，发挥一定的作用。

最后，预祝“国外电子与通信教材系列”项目取得成功，为我国电子与通信教学和通信产业的发展培土施肥。也恳切希望读者能对这些书籍的不足之处、特别是翻译中存在的问题，提出意见和建议，以便再版时更正。

吴佑寿

中国工程院院士、清华大学教授

“国外电子与通信教材系列”出版委员会主任

出 版 说 明

进入21世纪以来，我国信息产业在生产和科研方面都大大加快了发展速度，并已成为国民经济发展的支柱产业之一。但是，与世界上其他信息产业发达的国家相比，我国在技术开发、教育培训等方面都还存在着较大的差距。特别是在加入WTO后的今天，我国信息产业面临着国外竞争对手的严峻挑战。

作为我国信息产业的专业科技出版社，我们始终关注着全球电子信息技术的发展方向，始终把引进国外优秀电子与通信信息技术教材和专业书籍放在我们工作的重要位置上。在2000年至2001年间，我社先后从世界著名出版公司引进出版了40余种教材，形成了一套“国外计算机科学教材系列”，在全国高校以及科研部门中受到了欢迎和好评，得到了计算机领域的广大教师与科研工作者的充分肯定。

引进和出版一些国外优秀电子与通信教材，尤其是有选择地引进一批英文原版教材，将有助于我国信息产业培养具有国际竞争能力的技术人才，也将有助于我国国内在电子与通信教学工作中掌握和跟踪国际发展水平。根据国内信息产业的现状、教育部《关于“十五”期间普通高等教育教材建设与改革的意见》的指示精神以及高等院校老师们反映的各种意见，我们决定引进“国外电子与通信教材系列”，并随后开展了大量准备工作。此次引进的国外电子与通信教材均来自国际著名出版商，其中影印教材约占一半。教材内容涉及的学科方向包括电路理论与应用、信号与系统、数字信号处理、微电子、通信系统、电磁场与微波等，其中既有本科专业课程教材，也有研究生课程教材，以适应不同院系、不同专业、不同层次的师生对教材的需求，广大师生可自由选择和自由组合使用。我们还将与国外出版商一起，陆续推出一些教材的教学支持资料，为授课教师提供帮助。

此外，“国外电子与通信教材系列”的引进和出版工作得到了教育部高等教育司的大力支持和帮助，其中的部分引进教材已通过“教育部高等学校电子信息科学与工程类专业教学指导委员会”的审核，并得到教育部高等教育司的批准，纳入了“教育部高等教育司推荐——国外优秀信息科学与技术系列教学用书”。

为做好该系列教材的翻译工作，我们聘请了清华大学、北京大学、北京邮电大学、南京邮电大学、东南大学、西安交通大学、天津大学、西安电子科技大学、电子科技大学、中山大学、哈尔滨工业大学、西南交通大学等著名高校的教授和骨干教师参与教材的翻译和审校工作。许多教授在国内电子与通信专业领域享有较高的声望，具有丰富的教学经验，他们的渊博学识从根本上保证了教材的翻译质量和专业学术方面的严格与准确。我们在此对他们的辛勤工作与贡献表示衷心的感谢。此外，对于编辑的选择，我们达到了专业对口；对于从英文原书中发现的错误，我们通过与作者联络、从网上下载勘误表等方式，逐一进行了修订；同时，我们对审校、排版、印制质量进行了严格把关。

今后，我们将进一步加强同各高校教师的密切关系，努力引进更多的国外优秀教材和教学参考书，为我国电子与通信教材达到世界先进水平而努力。由于我们对国内外电子与通信教育的发展仍存在一些认识上的不足，在选题、翻译、出版等方面的工作中还有许多需要改进的地方，恳请广大师生和读者提出批评及建议。

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改编者序

本书是在*Digital Systems Design Using VHDL*，*Second Edition*的基础上改编的英文版教材（改后英文书名为*Digital Systems Design with VHDL*），适用于本科高年级学生和研究生针对数字系统进行 VHDL 设计与实现的英语或双语教学使用。

本书作者Charles H. Roth, Jr.是美国斯坦福大学博士，1961年就职于得克萨斯大学奥斯汀分校，目前是电气与计算机工程系教授。他的授课和研究领域涵盖了数字系统理论和设计、微计算机系统和 VHDL 应用，出版了 4 本著作。作者在手工电路设计和基于硬件描述语言的电路系统设计领域积累了深厚的工作经验和教学经历，以电路系统设计为核心来开展VHDL教学是本书区别于一般专注于 VHDL 语言文法讲解教材的显著特点，受到了使用者的高度评价。

Digital Systems Design Using VHDL，*Second Edition*的内容体系与国内高校数字系统设计相关课程的教学内容和课程安排存在一些不匹配之处，其中可编程逻辑器件简介和FPGA 设计部分的内容，已有专门的课程讲授，微程序设计和RISC微处理器设计部分，需要计算机体系结构设计相关课程作为先导，并有专门的课程讲授（对这些内容感兴趣的读者可参阅电子工业出版社所出中文版《数字系统设计与 VHDL（第二版）》，ISBN: 978-7-121-06728-0）。为了更好地突出本书在数字系统设计方面的特色，我们将这两部分的内容进行了删除，同时对原著进行了结构调整，使之更适合作为本科双语教学教材。其中第 1 章首先回顾了逻辑设计基本原理，第 2 章和第 3 章分别讲解了 VHDL 基本知识和高级主题，第 4 章为简单设计实例，第 5 章讨论状态机，第 6 章讨论浮点数运算，第 7 章讨论硬件测试和可测试性设计，第 8 章给出了一些高级设计实例。

改编后的版本虽经反复推敲，但由于改编者水平有限，书中仍难免有不妥之处，恳请广大同行和读者给予指正（敬请致函 liangsh@szu.edu.cn），以便进一步提高本书的质量，更好地推动双语教学。

Preface

This textbook is intended for a senior-level course in digital systems design. The book covers both basic principles of digital system design and the use of a hardware description language, VHDL, in the design process. After basic principles have been covered, design is best taught by using examples. For this reason, many digital system design examples, ranging in complexity from a simple binary adder to a microprocessor, are included in the text.

Students using this textbook should have completed a course in the fundamentals of logic design, including both combinational and sequential circuits. Although no previous knowledge of VHDL is assumed, students should have programming experience using a modern high-level language such as C.

Because students typically take their first course in logic design two years before this course, most students need a review of the basics. For this reason, Chapter 1 includes a review of logic design fundamentals. Most students can review this material on their own, so it is unnecessary to devote much lecture time to this chapter. However, a good understanding of timing in sequential circuits and the principles of synchronous design is essential to the digital system design process.

Chapter 2 starts with an overview of modern design flow. It also summarizes various technologies for implementation of digital designs. Then, it introduces the basics of VHDL, and this hardware description language is used throughout the rest of the book. Additional features of VHDL are introduced on an as-needed basis, and more advanced features are covered in Chapter 3. From the start, we relate the constructs of VHDL to the corresponding hardware. Some textbooks teach VHDL as a programming language and devote many pages to teaching the language syntax. Instead, our emphasis is on how to use VHDL in the digital design process. The language is very complex, so we do not attempt to cover all its features. We emphasize the basic features that are necessary for digital design and omit some of the less-used features. Use of standard IEEE VHDL libraries is introduced in this chapter and only IEEE standard libraries are used throughout the text.

VHDL is very useful in teaching top-down design. We can design a system at a high level and express the algorithms in VHDL. We can then simulate and debug the designs at this level before proceeding with the detailed logic design. However, no design is complete until it has actually been implemented in hardware and the hardware has been tested. For this reason, we recommend that the course include some lab exercises in which designs are implemented in hardware.

By the time students reach Chapter 3, they should be thoroughly familiar with the basics of VHDL. At this point we introduce some of the more advanced features of VHDL and illustrate their use. The use of multi-valued logic, including the IEEE-1164 standard logic, is one of the important topics covered. A memory model with tri-state output busses is presented to illustrate the use of the multi-valued logic.

Chapter 4 presents a variety of design examples, including both arithmetic and non-arithmetic examples. Simple examples such as a BCD to 7-segment display decoder to more complex examples such as game scoreboards, keypad scanners and binary dividers are presented. The chapter presents common techniques used for computer arithmetic, including carry look-ahead addition, and binary multiplication and division. Use of a state machine for sequencing the operations in a digital system is an important concept presented in this chapter. Synthesizable VHDL code is presented for the various designs. A variety of examples are presented so that instructors can select their favorite designs for teaching.

Use of sequential machine charts (SM charts) as an alternative to state graphs is presented in Chapter 5. We show how to write VHDL code based on SM charts and how to realize hardware to implement the SM charts. Then, the technique of microprogramming is presented. Transformation of SM charts for different types of microprogramming is discussed. Then, we show how the use of linked state machines facilitates the decomposition of complex systems into simpler ones. The design of a dice-game simulator is used to illustrate these techniques.

Basic techniques for floating-point arithmetic are described in Chapter 6. A simple floating-point format with 2's complement numbers is presented and then the IEEE standard floating-point formats are presented. A floating-point multiplier example is presented starting with development of the basic algorithm, then simulating the system using VHDL, and finally synthesizing and implementing the system using an FPGA.

The important topics of hardware testing and design for testability are covered in Chapter 7. This chapter introduces the basic techniques for testing combinational and sequential logic. Then scan design and boundary-scan techniques, which facilitate the testing of digital systems, are described. The chapter concludes with a discussion of built-in self-test (BIST). VHDL code for a boundary-scan example and for a BIST example is included. The topics in this chapter play an important role in digital system design, and we recommend that they be included in any course on this subject. Chapter 7 can be covered any time after the completion of Chapter 3.

Chapter 8 presents three complete design examples that illustrate the use of VHDL synthesis tools. First, a wristwatch design is presented. It shows the progress of a design from a textual description to a state diagram and then a VHDL model. This example illustrates modular design. The test bench for the wristwatch illustrates the use of multiple procedure calls to facilitate the testing. The second example describes the use of VHDL to model RAM memories. The third example, a serial communications receiver-transmitter, should easily be understood by any student who has completed the material through Chapter 3.

This book is the result of many years of teaching a senior course in digital systems design at the University of Texas at Austin. Throughout the years, the technology for hardware implementation of digital systems has kept changing, but many of the same design principles are still applicable. In the early years of the course, we handwired modules consisting of discrete transistors to implement our designs. Then integrated circuits were introduced, and we were able to implement our designs using breadboards and TTL logic. Now we are able to use FPGAs and CPLDs to realize very complex designs. We originally used our own hardware description language together with a simulator running on a mainframe computer. When PCs came along, we wrote an improved hardware description language and implemented a simulator that ran on PCs. When VHDL was adopted as an IEEE standard and became widely used in industry, we switched to VHDL. The widespread availability of high-quality commercial CAD tools now enables us to synthesize complex designs directly from the VHDL code.

All of the VHDL code in this textbook[①] has been tested using the ModelSim simulator. The ModelSim software is available in a student edition, and we recommend its use in conjunction with this text. The CD that accompanies this text provides a link for downloading the ModelSim student edition and an introductory tutorial to help students get started using the software. All of the VHDL code in this textbook is available on the CD. The CD also contains two software packages, LogicAid and SimUaid, which are useful in teaching digital system design. Instruction manuals and examples of using this software are on the CD.

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We would like to thank the many individuals who have contributed their time and effort to the development of this textbook. Over many years we have received valuable feedback from the students in our digital systems design courses. We would especially like to thank the faculty members who reviewed the previous edition and offered many suggestions for its improvement. These faculty include:

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Peter N. Marinos, Duke University

① 书中提及的 VHDL 代码及其他光盘内容均可从华信教育资源网（www.hxedu.com.cn）下载。

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CHAPTER 1

Review of Logic Design Fundamentals

This chapter reviews many of the logic design topics normally taught in a first course in logic design. Some of the review examples that follow are referenced in later chapters of this text. For more details on any of the topics discussed in this chapter, the reader should refer to a standard logic design textbook such as Roth, *Fundamentals of Logic Design*, 5th Edition (Thomson Brooks/Cole, 2004). First, we review combinational logic and then sequential logic. Combinational logic has no memory, so the present output depends only on the present input. Sequential logic has memory, so the present output depends not only on the present input but also on the past sequence of inputs. The sections on sequential circuit timing and synchronous design are particularly important, since a good understanding of timing issues is essential to the successful design of digital systems.

1.1 Combinational Logic

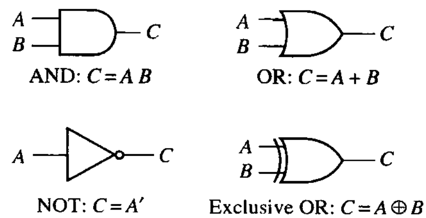
Some of the basic gates used in logic circuits are shown in Figure 1-1. Unless otherwise specified, all the variables that we use to represent logic signals will be two-valued, and the two values will be designated 0 and 1. We will normally use positive logic, for which a low voltage corresponds to a logic 0 and a high voltage corresponds to a logic 1. When negative logic is used, a low voltage corresponds to a logic 1 and a high voltage corresponds to a logic 0.

For the AND gate of Figure 1-1, the output $C = 1$ if and only if the input $A = 1$ *and* the input $B = 1$. We will use a raised dot or simply write the variables side by side to indicate the AND operation; thus $C = A$ AND $B = A \cdot B = AB$. For the OR gate, the output $C = 1$ if and only if the input $A = 1$ *or* the input $B = 1$ (inclusive OR). We will use + to indicate the OR operation; thus $C = A$ OR $B = A + B$. The NOT gate, or inverter, forms the complement of the input; that is, if $A = 1$, $C = 0$, and if $A = 0$, $C = 1$. We will use a prime (′) to indicate the complement (NOT) operation, so $C =$ NOT A $= A'$. The exclusive-OR (XOR) gate has an output $C = 1$ if $A = 1$ and $B = 0$ or if $A = 0$ and $B = 1$. The symbol $\oplus$ represents exclusive OR, so we write

$$C = A \text{ XOR } B = AB' + A'B = A \oplus B \tag{1-1}$$

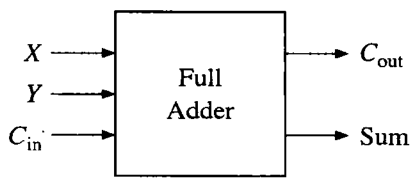
The behavior of a combinational logic circuit can be specified by a truth table that gives the circuit outputs for each combination of input values. As an example,

FIGURE 1-1: Basic Gates



consider the full adder of Figure 1-2, which adds two binary digits (X and Y) and a carry (C_{in}) to give a sum (Sum) and a carry out (C_{out}). The truth table specifies the adder outputs as a function of the adder inputs. For example, when the inputs are $X = 0$, $Y = 0$ and $C_{in} = 1$, adding the three inputs gives $0 + 0 + 1 = 01$, so the sum is 1 and the carry out is 0. When the inputs are 011, $0 + 1 + 1 = 10$, so $Sum = 0$ and $C_{out} = 1$. When the inputs are $X = Y = C_{in} = 1$, $1 + 1 + 1 = 11$, so $Sum = 1$ and $C_{out} = 1$.

FIGURE 1-2: Full Adder



(a) Full adder module

X	Y	C_{in}	C_{out}	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

(b) Truth table

We will derive algebraic expressions for Sum and C_{out} from the truth table. From the table, $Sum = 1$ when $X = 0$, $Y = 0$, and $C_{in} = 1$. The term $X'Y'C_{in}$ equals 1 only for this combination of inputs. The term $X'YC'_{in} = 1$ only when $X = 0$, $Y = 1$, and $C_{in} = 0$. The term $XY'C'_{in}$ is 1 only for the input combination $X = 1$, $Y = 0$, and $C_{in} = 0$. The term XYC_{in} is 1 only when $X = Y = C_{in} = 1$. Therefore, Sum is formed by ORing these four terms together:

$$Sum = X'Y'C_{in} + X'YC'_{in} + XY'C'_{in} + XYC_{in} \tag{1-2}$$

Each of the terms in this sum of products (SOP) expression is 1 for exactly one combination of input values. In a similar manner, C_{out} is formed by ORing four terms together:

$$C_{out} = X'YC_{in} + XY'C_{in} + XYC'_{in} + XYC_{in} \tag{1-3}$$

Each term in Equations (1-2) and (1-3) is referred to as a *minterm*, and these equations are referred to as *minterm expansions*. These minterm expansions can also be written in m-notation or decimal notation as follows:

$$Sum = m_1 + m_2 + m_4 + m_7 = \Sigma m(1, 2, 4, 7)$$
$$C_{out} = m_3 + m_5 + m_6 + m_7 = \Sigma m(3, 5, 6, 7)$$

The decimal numbers designate the rows of the truth table for which the corresponding function is 1. Thus $Sum = 1$ in rows 001, 010, 100, and 111 (rows 1, 2, 4, 7).

A logic function can also be represented in terms of the inputs for which the function value is 0. Referring to the truth table for the full adder, $C_{out} = 0$ when $X = Y = C_{in} = 0$. The term $(X + Y + C_{in})$ is 0 only for this combination of inputs. The term $(X + Y + C'_{in})$ is 0 only when $X = Y = 0$ and $C_{in} = 1$. The term $(X + Y' + C_{in})$ is 0 only when $X = C_{in} = 0$ and $Y = 1$. The term $(X' + Y + C_{in})$ is 0 only when $X = 1$ and $Y = C_{in} = 0$. C_{out} is formed by ANDing these four terms together:

$$C_{out} = (X + Y + C_{in})(X + Y + C'_{in})(X + Y' + C_{in})(X' + Y + C_{in}) \qquad (1\text{-}4)$$

C_{out} is 0 only for the 000, 001, 010, and 100 rows of the truth table and, therefore, must be 1 for the remaining four rows. Each of the terms in the Product of Sums (POS) expression in Equation (1-4) is referred to as a *maxterm*, and (1-4) is called a *maxterm expansion*. This *maxterm expansion* can also be written in decimal notation as

$$C_{out} = M_0 \cdot M_1 \cdot M_2 \cdot M_4 = \Pi M(0, 1, 2, 4)$$

where the decimal numbers correspond to the truth table rows for which $C_{out} = 0$.

1.2 Boolean Algebra and Algebraic Simplification

The basic mathematics used for logic design is Boolean algebra. Table 1-1 summarizes the laws and theorems of Boolean algebra. They are listed in dual pairs; for example, Equation (1-10D) is the dual of (1-10). They can be verified easily for two-valued logic by using truth tables. These laws and theorems can be used to simplify logic functions so they can be realized with a reduced number of components.

A very important law in Boolean algebra is the *DeMorgan's law*. DeMorgan's laws stated in Equations (1-16, 1-16D) can be used to form the complement of an expression on a step-by-step basis. The generalized form of DeMorgan's law in Equation (1-17) can be used to form the complement of a complex expression in one step. Equation (1-17) can be interpreted as follows: To form the complement of a Boolean expression, replace each variable by its complement; also replace 1 with 0, 0 with 1, OR with AND, and AND with OR. Add parentheses as required to assure the proper hierarchy of operations. If AND is performed before OR in F, then parentheses may be required to assure that OR is performed before AND in F'.

Example

Find the complement of F if

$$F = X + E'K\,(C\,(AB + D') \cdot 1 + WZ'\,(G'H + 0))$$
$$F' = X'\,(E + K' + (C' + (A' + B')\,D + 0)\,(W' + Z + (G + H') \cdot 1))$$

Additional parentheses in F' were added when an AND operation in F was replaced with an OR. The dual of an expression is the same as its complement, except that the variables are not complemented.

TABLE 1-1: Laws and Theorems of Boolean Algebra

Operations with 0 and 1:

$X + 0 = X$	(1-5)	$X \cdot 1 = X$	(1-5D)
$X + 1 = 1$	(1-6)	$X \cdot 0 = 0$	(1-6D)

Idempotent laws:

$X + X = X$	(1-7)	$X \cdot X = X$	(1-7D)

Involution law:

$(X')' = X$	(1-8)		

Laws of complementarity:

$X + X' = 1$	(1-9)	$X \cdot X' = 0$	(1-9D)

Commutative laws:

$X + Y = Y + X$	(1-10)	$XY = YX$	(1-10D)

Associative laws:

$(X + Y) + Z = X + (Y + Z) = X + Y + Z$	(1-11)	$(XY)Z = X(YZ) = XYZ$	(1-11D)

Distributive laws:

$X(Y + Z) = XY + XZ$	(1-12)	$X + YZ = (X + Y)(X + Z)$	(1-12D)

Simplification theorems:

$XY + XY' = X$	(1-13)	$(X + Y)(X + Y') = X$	(1-13D)
$X + XY = X$	(1-14)	$X(X + Y) = X$	(1-14D)
$(X + Y')Y = XY$	(1-15)	$XY' + Y = X + Y$	(1-15D)

DeMorgan's laws:

$(X + Y + Z + \cdots)' = X'Y'Z'\cdots$	(1-16)	$(XYZ\ldots)' = X' + Y' + Z' + \cdots$	(1-16D)

$$[f(X_1, X_2, \ldots, X_n, 0, 1, +, \cdot)]' = f(X_1', X_2', \ldots, X_n', 1, 0, \cdot, +) \quad (1\text{-}17)$$

Duality:

$(X + Y + Z + \cdots)^D = XYZ\cdots$	(1-18)	$(XYZ\cdots)^D = X + Y + Z + \cdots$	(1-18D)

$$[f(X_1, X_2, \ldots, X_n, 0, 1, +, \cdot)]^D = f(X_1, X_2, \ldots, X_n, 1, 0, \cdot, +) \quad (1\text{-}19)$$

Theorem for multiplying out and factoring:

$(X + Y)(X' + Z) = XZ + X'Y$	(1-20)	$XY + X'Z = (X + Z)(X' + Y)$	(1-20D)

Consensus theorem:

$XY + YZ + X'Z = XY + X'Z$	(1-21)	$(X + Y)(Y + Z)(X' + Z) = (X + Y)(X' + Z)$	(1-21D)

Four ways of simplifying a logic expression using the theorems in Table 1-1 are as follows:

1. *Combining terms.* Use the theorem $XY + XY' = X$ to combine two terms. For example,

$$ABC'D' + ABCD' = ABD' \; [X = ABD', Y = C]$$

When combining terms by this theorem, the two terms to be combined should contain exactly the same variables, and exactly one of the variables should appear complemented in one term and not in the other. Since $X + X = X$, a given term may be duplicated and combined with two or more other terms. For example, the expression for C_{out} in Equation (1-3) can be simplified by combining the first and fourth terms, the second and fourth terms, and the third and fourth terms:

$$\begin{aligned} C_{out} &= (X'YC_{in} + XYC_{in}) + (XY'C_{in} + XYC_{in}) + (XYC'_{in} + XYC_{in}) \\ &= YC_{in} + XC_{in} + XY \qquad (1\text{-}22) \end{aligned}$$

Note that the fourth term in Equation (1-3) was used three times.

The theorem can still be used, of course, when X and Y are replaced with more complicated expressions. For example,

$$(A + BC)(D + E') + A'(B' + C')(D + E') = D + E'$$
$$[X = D + E', Y = A + BC, Y' = A'(B' + C')]$$

2. *Eliminating terms.* Use the theorem $X + XY = X$ to eliminate redundant terms if possible; then try to apply the consensus theorem ($XY + X'Z + YZ = XY + X'Z$) to eliminate any consensus terms. For example,

$$A'B + A'BC = A'B \; [X = A'B]$$
$$A'BC' + BCD + A'BD = A'BC' + BCD \; [X = C, Y = BD, Z = A'B]$$

3. *Eliminating literals.* Use the theorem $X + X'Y = X + Y$ to eliminate redundant literals. Simple factoring may be necessary before the theorem is applied. For example,

$$\begin{aligned} A'B + A'B'C'D' + ABCD' &= A'(B + B'C'D') + ABCD' && \text{(by (1-12))} \\ &= A'(B + C'D') + ABCD' && \text{(by (1-15D))} \\ &= B(A' + ACD') + A'C'D' && \text{(by (1-10))} \\ &= B(A' + CD') + A'C'D' && \text{(by (1-15D))} \\ &= A'B + BCD' + A'C'D' && \text{(by (1-12))} \end{aligned}$$

The expression obtained after applying 1, 2, and 3 will not necessarily have a minimum number of terms or a minimum number of literals. If it does not and no further simplification can be made using 1, 2, and 3, deliberate introduction of redundant terms may be necessary before further simplification can be made.

4. *Adding redundant terms.* Redundant terms can be introduced in several ways, such as adding XX', multiplying by $(X + X')$, adding YZ to $XY + X'Z$

(consensus theorem), or adding XY to X. When possible, the terms added should be chosen so that they will combine with or eliminate other terms. For example,

$$
\begin{aligned}
& WX + XY + X'Z' + WY'Z' && \text{(Add } WZ' \text{ by the consensus theorem.)} \\
&= WX + XY + X'Z' + WY'Z' + WZ' && \text{(Eliminate } WY'Z'.\text{)} \\
&= WX + XY + X'Z' + WZ' && \text{(Eliminate } WZ'.\text{)} \\
&= WX + XY + X'Z'
\end{aligned}
$$

When multiplying out or factoring an expression, in addition to using the ordinary distributive law (1-12), the second distributive law (1-12D) and theorem (1-20) are particularly useful. The following is an example of multiplying out to convert from a product of sums to a sum of products:

$$
\begin{aligned}
& (A + B + D)(A + B' + C')(A' + B + D')(A' + B + C') \\
&= (A + (B + D)(B' + C'))(A' + B + C'D') && \text{(by (1-12D))} \\
&= (A + BC' + B'D)(A' + B + C'D') && \text{(by (1-20))} \\
&= A(B + C'D') + A'(BC' + B'D) && \text{(by (1-20))} \\
&= AB + AC'D' + A'BC' + A'B'D && \text{(by (1-12))}
\end{aligned}
$$

Note that the second distributive law (1-12D) and theorem (1-20) were applied before the ordinary distributive law. Any Boolean expression can be factored by using the two distributive laws (1-12 and 1-12D) and theorem (1-20). As an example of factoring, read the steps in the preceding example in the reverse order.

The following theorems apply to exclusive-OR:

$$X \oplus 0 = X \tag{1-23}$$

$$X \oplus 1 = X' \tag{1-24}$$

$$X \oplus X = 0 \tag{1-25}$$

$$X \oplus X' = 1 \tag{1-26}$$

$$X \oplus Y = Y \oplus X \qquad \text{(commutative law)} \tag{1-27}$$

$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z \qquad \text{(associative law)} \tag{1-28}$$

$$X(Y \oplus Z) = XY \oplus XZ \qquad \text{(distributive law)} \tag{1-29}$$

$$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y' \tag{1-30}$$

The expression for *Sum* in Equation (1-2) can be rewritten in terms of exclusive-OR by using Equations (1-1) and (1-30):

$$
\begin{aligned}
Sum &= X'(Y'C_{\text{in}} + YC_{\text{in}}') + X(Y'C_{\text{in}}' + YC_{\text{in}}) \\
&= X'(Y \oplus C_{\text{in}}) + X(Y \oplus C_{\text{in}})' = X \oplus Y \oplus C_{\text{in}}
\end{aligned}
\tag{1-31}
$$

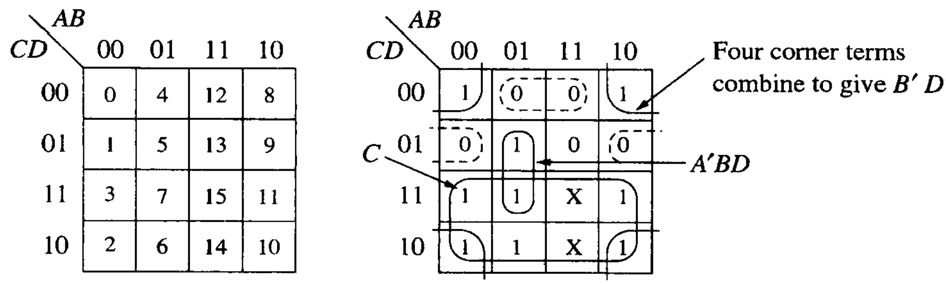
The simplification rules that you studied in this section are important when a circuit has to be optimized to use a smaller number of gates. The existence of equivalent forms also helps when mapping circuits into particular target devices where only certain types of logic (e.g., NAND only or NOR only) are available.

1.3 Karnaugh Maps

Karnaugh maps (K-maps) provide a convenient way to simplify logic functions of three to five variables. Figure 1-3 shows a four-variable Karnaugh map. Each square in the map represents one of the 16 possible minterms of four variables. A 1 in a square indicates that the minterm is present in the function, and a 0 (or blank) indicates that the minterm is absent. An X in a square indicates that we don't care whether the minterm is present or not. *Don't cares* arise under two conditions: (1) The input combination corresponding to the don't care can never occur, and (2) the input combination can occur, but the circuit output is not specified for this input condition.

The variable values along the edge of the map are ordered so that adjacent squares on the map differ in only one variable. The first and last columns and the

FIGURE 1-3: Four-Variable Karnaugh Maps



$F = \Sigma m\,(0, 2, 3, 5, 6, 7, 8, 10, 11) + \Sigma d\,(14, 15)$
$= C + B'D' + A'BD$

(a) Location of minterms (b) Looping terms

top and bottom rows of the map are considered to be adjacent. Two 1's in adjacent squares can be combined by eliminating one variable using $xy + xy' = x$. Figure 1-3 shows a four-variable function with nine minterms and two don't cares. Minterms $A'BC'D$ and $A'BCD$ differ only in the variable C, so they can be combined to form $A'BD$, as indicated by a loop on the map. Four 1's in a symmetrical pattern can be combined to eliminate two variables. The 1's in the four corners of the map can be combined as follows:

$$(A'B'C'D' + AB'C'D') + (A'B'CD' + AB'CD') = B'C'D' + B'CD' = B'D'$$

as indicated by the loop. Similarly, the six 1's and two X's in the bottom half of the map combine to eliminate three variables and form the term C. The resulting simplified function is

$$F = A'BD + B'D' + C$$

The minimum sum-of-products representation of a function consists of a sum of prime implicants. A group of one, two, four, or eight adjacent 1's on a map represents

a prime implicant if it cannot be combined with another group of 1's to eliminate a variable. A prime implicant is essential if it contains a 1 that is not contained in any other prime implicant. When finding a minimum sum of products from a map, essential prime implicants should be looped first, and then a minimum number of prime implicants to cover the remaining 1's should be looped. The Karnaugh map shown in Figure 1-4 has five prime implicants and three essential prime implicants. $A'C'$ is essential because minterm m_1 is not covered by any other prime implicant. Similarly, ACD is essential because of m_{11}, and $A'B'D'$ is essential because of m_2. After looping the essential prime implicants, all 1's are covered except m_7. Since m_7 can be covered by either prime implicant $A'BD$ or BCD, F has two minimum forms:

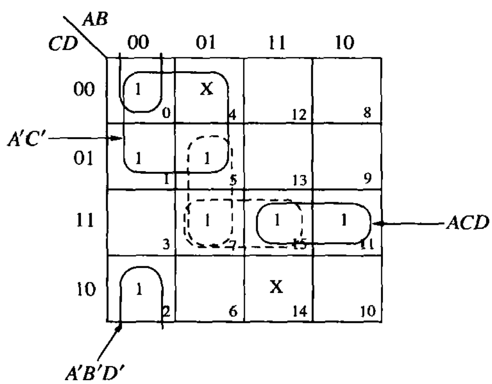
$$F = A'C' + A'B'D' + ACD + A'BD$$

and

$$F = A'C' + A'B'D' + ACD + BCD$$

When don't cares (X's) are present on the map, the don't cares are treated like 1's when forming prime implicants, but the X's are ignored when finding a minimum

FIGURE 1-4: Selection of Prime Implicants



Prime implicants: A'C', ACD, A'B'D', A'BD, BCD

Essential prime implicants: A'C', ACD, A'B'D'

set of prime implicants to cover all the 1's. The following procedure can be used to obtain a minimum sum of products from a Karnaugh map:

1. Choose a minterm (a 1) that has not yet been covered.
2. Find all 1's and X's adjacent to that minterm. (Check the n adjacent squares on an n-variable map.)
3. If a single term covers the minterm and all the adjacent 1's and X's, then that term is an essential prime implicant, so select that term. (Note that don't cares are treated like 1's in steps 2 and 3 but not in step 1.)
4. Repeat steps 1, 2, and 3 until all essential prime implicants have been chosen.
5. Find a minimum set of prime implicants that cover the remaining 1's on the map. (If there is more than one such set, choose a set with a minimum number of literals.)

To find a minimum product of sums from a Karnaugh map, loop the 0's instead of the 1's. Since the 0's of F are the 1's of F', looping the 0's in the proper way gives the minimum sum of products for F', and the complement is the minimum product

of sums for F. For Figure 1-3, we can first loop the essential prime implicants of F' ($BC'D'$ and $B'C'D$, indicated by dashed loops) and then cover the remaining 0 with AB. Thus the minimum sum for F' is

$$F' = BC'D' + B'C'D + AB$$

from which the minimum product of sums for F is

$$F = (B' + C + D)(B + C + D')(A' + B')$$

1.3.1 Simplification Using Map-Entered Variables

Two four-variable Karnaugh maps can be used to simplify functions with five variables. If functions have more than five variables, *map-entered variables* can be used. Consider a truth table as in Table 1-2. There are six input variables (A, B, C, D, E, F) and one output variable (G). Only certain rows of the truth table have been specified. To completely specify the truth table, 64 rows will be required. The input combinations not specified in the truth table result in an output of 0.

TABLE 1-2: Partial Truth Table for a Six-Variable Function

A	B	C	D	E	F	G
0	0	0	0	X	X	1
0	0	0	1	X	X	X
0	0	1	0	X	X	1
0	0	1	1	X	X	1
0	1	0	1	1	X	1
0	1	1	1	1	X	1
1	0	0	1	X	1	1
1	0	1	0	X	X	X
1	0	1	1	X	X	1
1	1	0	1	X	X	X
1	1	1	1	X	X	1

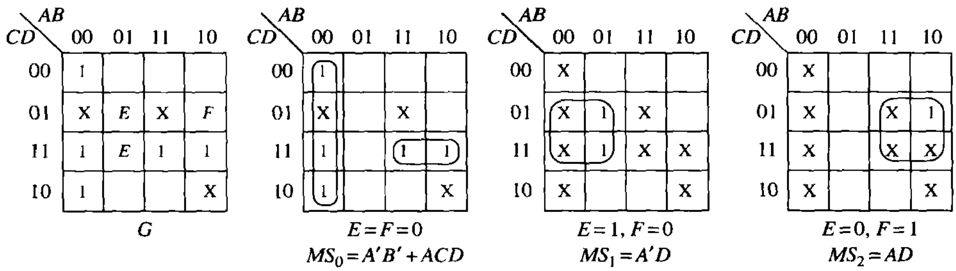
Karnaugh map techniques can be extended to simplify functions such as this using map-entered variables. Since E and F are the input variables with the most number of don't cares (X), a Karnaugh map can be formed with A, B, C, D and the remaining two variables can be entered inside the map. Figure 1-5 shows a four-variable map with variables E and F entered in the squares in the map. When E appears in a square, this means that if $E = 1$, the corresponding minterm is present in the function G, and if $E = 0$, the minterm is absent. The fifth and sixth rows in the truth table result in the E in the box corresponding to minterm 5 and minterm 7. The seventh row results in the F in the box corresponding to minterm 9. Thus, the map represents the six-variable function

$$G(A, B, C, D, E, F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15}$$
$$(+ \text{ don't care terms})$$

where the minterms are minterms of the variables A, B, C, D. Note that m_9 is present in G only when $F = 1$.

Next we will discuss a general method of simplifying functions using map-entered variables. In general, if a variable P_i is placed in square m_j of a map of function F, this means that $F = 1$ when $P_i = 1$ and the variables are chosen so that $m_j = 1$. Given a

FIGURE 1-5: Simplification Using Map-Entered Variables



map with variables $P_1, P_2, \ldots$ entered into some of the squares, the minimum sum-of-products form of F can be found as follows: Find a sum-of-products expression for F of the form

$$F = MS_0 + P_1MS_1 + P_2MS_2 + \cdots \tag{1-32}$$

where

- MS_0 is the minimum sum obtained by setting $P_1 = P_2 = \cdots = 0$.
- MS_1 is the minimum sum obtained by setting $P_1 = 1, P_j = 0\ (j \neq 1)$, and replacing all 1's on the map with don't cares.
- MS_2 is the minimum sum obtained by setting $P_2 = 1, P_j = 0\ (j \neq 2)$, and replacing all 1's on the map with don't cares.

Corresponding minimum sums can be found in a similar way for any remaining map-entered variables.

The resulting expression for F will always be a correct representation of F. This expression will be a minimum sum provided that the values of the map-entered variables can be assigned independently. On the other hand, the expression will not generally be a minimum sum if the variables are not independent (for example, if $P_1 = P_2'$).

For the example of Figure 1-5, maps for finding MS_0, MS_1, and MS_2 are shown, where E corresponds to P_1 and F corresponds to P_2. Note that it is not required to draw a map for $E = 1, F = 1$, because $E = 1$ already covers cases with $E = 1, F = 0$ and $E = 1, F = 1$. The resulting expression is a minimum sum of products for G:

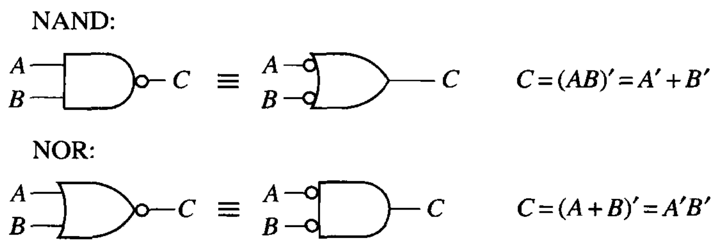
$$G = A'B' + ACD + EA'D + FAD$$

After some practice, it should be possible to write the minimum expression directly from the original map without first plotting individual maps for each of the minimum sums.

1.4 Designing With NAND and NOR Gates

In many technologies, implementation of NAND gates or NOR gates is easier than that of AND and OR gates. Figure 1-6 shows the symbols used for NAND and NOR gates. The *bubble* at a gate input or output indicates a complement. Any logic function can be realized using only NAND gates or only NOR gates.

FIGURE 1-6: NAND and NOR Gates

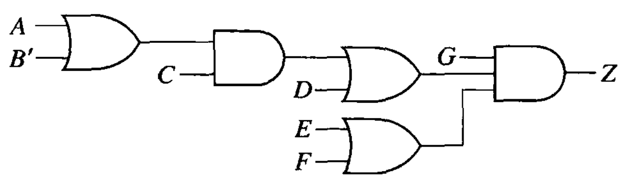


Conversion from circuits of OR and AND gates to circuits of all NOR gates or all NAND gates is straightforward. To design a circuit of NOR gates, start with a product-of-sums representation of the function (circle 0's on the Karnaugh map). Then find a circuit of OR and AND gates that has an AND gate at the output. If an AND gate output does not drive an AND gate input and an OR gate output does not connect to an OR gate input, then conversion is accomplished by replacing all gates with NOR gates and complementing inputs if necessary. Figure 1-7 illustrates the conversion procedure for

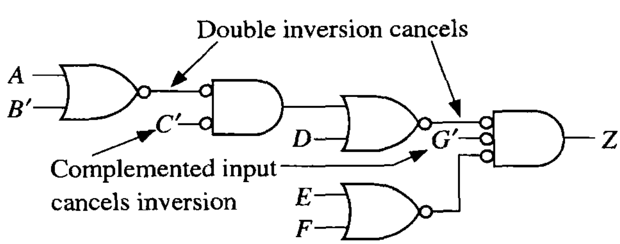
$$Z = G(E + F)(A + B' + D)(C + D) = G(E + F)[(A + B')C + D]$$

Conversion to a circuit of NAND gates is similar, except the starting point should be a sum-of-products form for the function (circle 1's on the map), and the output gate of the AND-OR circuit should be an OR gate.

FIGURE 1-7: Conversion to NOR Gates



(a) AND-OR circuit



(b) Equivalent NOR-gate circuit

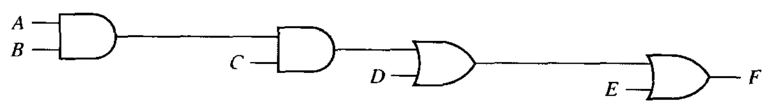
Even if AND and OR gates do not alternate, we can still convert a circuit of AND and OR gates to a NAND or NOR circuit, but it may be necessary to add

extra inverters so that each added inversion is canceled by another inversion. The following procedure may be used to convert to a NAND (or NOR) circuit:

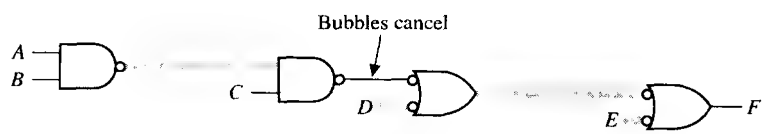
1. Convert all AND gates to NAND gates by adding an inversion bubble at the output. Convert OR gates to NAND gates by adding inversion bubbles at the inputs. (To convert to NOR, add inversion bubbles at all OR gate outputs and all AND gate inputs.)
2. Whenever an inverted output drives an inverted input, no further action is needed, since the two inversions cancel.
3. Whenever a noninverted gate output drives an inverted gate input or vice versa, insert an inverter so that the bubbles will cancel. (Choose an inverter with the bubble at the input or output, as required.)
4. Whenever a variable drives an inverted input, complement the variable (or add an inverter) so the complementation cancels the inversion at the input.

In other words, if we always add bubbles (or inversions) in pairs, the function realized by the circuit will be unchanged. To illustrate the procedure, we will convert Figure 1-8(a) to NANDs. First, we add bubbles to change all gates to NAND gates (Figure 1-8(b)). The highlighted lines indicate four places where we have added only a single inversion. This is corrected in Figure 1-8(c) by adding two inverters and complementing two variables.

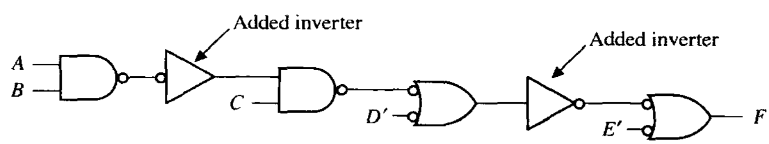
FIGURE 1-8: Conversion of AND-OR Circuit to NAND Gates



(a) AND-OR circuit



(b) First step in NAND conversion



(c) Completed conversion

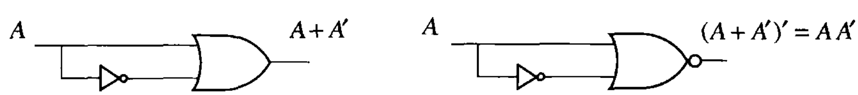
1.5 Hazards in Combinational Circuits

When the input to a combinational circuit changes, unwanted switching transients may appear in the output. These transients occur when different paths from input to output have different propagation delays. If, in response to an input change and for some combination of propagation delays, a circuit output may momentarily go to 0

when it should remain a constant 1, we say that the circuit has a static 1-*hazard*. Similarly, if the output may momentarily go to 1 when it should remain a 0, we say that the circuit has a static 0-*hazard*. If, when the output is supposed to change from 0 to 1 (or 1 to 0), the output may change three or more times, we say that the circuit has a *dynamic hazard*.

Consider the two simple circuits in Figure 1-9. Figure 1-9(a) shows an inverter and an OR gate implementing the function $A + A'$. Logically, the output of this circuit is expected to be a 1 always; however, a delay in the inverter gate can cause static hazards in this circuit. Assume a nonzero delay for the inverter and that the value of A just changed from 1 to 0. There is a short interval of time until the inverter delay has passed when both inputs of the OR gate are 0 and hence the output of the circuit may momentarily go to 0. Similarly, in the circuit in Figure 1-9(b), the expected output is always 0; however, when A changes from 1 to 0, a momentary 1 appears at the output of the inverter because of the delay. This circuit hence has a static 0-hazard. The hazard occurs because both A and A' have the same value for a short duration after A changes.

FIGURE 1-9: Simple Circuits Containing Hazards



(a) Simple circuit with static 1-hazard

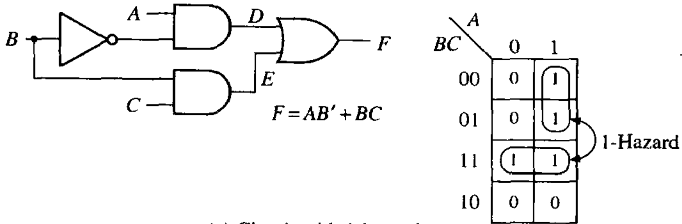
(b) Simple circuit with static 0-hazard

A static 1-hazard occurs in a sum-of-product implementation when two minterms differing by only one input variable are not covered by the same product term. Figure 1-10(a) illustrates another circuit with a static 1-hazard. If $A = C = 1$, the output should remain a constant 1 when B changes from 1 to 0. However, as shown in Figure 1-10(b), if each gate has a propagation delay of 10 ns, E will go to 0 before D goes to 1, resulting in a momentary 0 (a 1-hazard appearing in the output F). As seen on the Karnaugh map, there is no loop that covers both minterm ABC and $AB'C$. So if $A = C = 1$ and B changes from 1 to 0, BC immediately becomes 0, but until an inverter delay passes, AB' does not become a 1. Both terms can momentarily go to 0, resulting in a glitch in F. If we add a loop corresponding to the term AC to the map and add the corresponding gate to the circuit (Figure 1-10(c)), this eliminates the hazard. The term AC remains 1 while B is changing, so no glitch can appear in the output. In general, nonminimal expressions are required to eliminate static hazards.

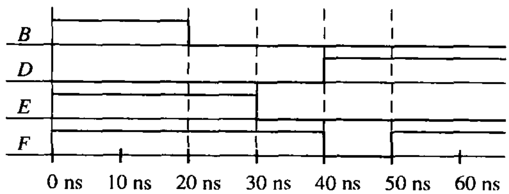
To design a circuit that is free of static and dynamic hazards, the following procedure may be used:

1. Find a sum-of-products expression (F^t) for the output in which every pair of adjacent 1s is covered by a 1-term. (The sum of all prime implicants will always satisfy this condition.) A two-level AND-OR circuit based on this F^t will be free of 1-, 0-, and dynamic hazards.
2. If a different form of circuit is desired, manipulate F^t to the desired form by simple factoring, DeMorgan's laws, and so on. **Treat each x_i and x_i' as independent variables to prevent introduction of hazards.**

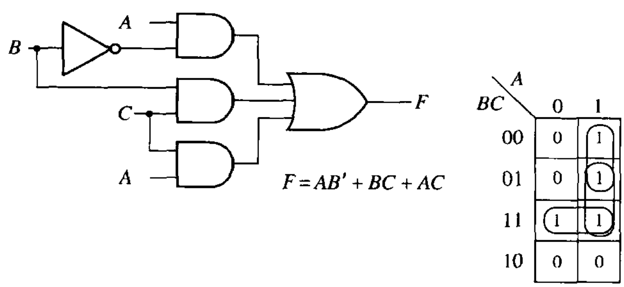
FIGURE 1-10: Elimination of 1-Hazard



(a) Circuit with 1-hazard



(b) Timing chart



(c) Circuit with hazard removed

Alternatively, you can start with a product-of-sums expression in which every pair of adjacent 0s is covered by a 0-term.

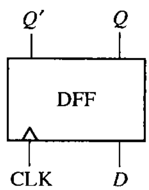
Given a circuit, one can identify the static hazards in it by writing an expression for the output in terms of the inputs exactly as it is implemented in the circuit and manipulating it to a sum-of-products form, treating x_i and x_i' as independent variables. A Karnaugh map can be constructed and all implicants corresponding to each term circled. If any pair of adjacent 1's is not covered by a single term, a static 1-hazard can occur. Similarly, a static 0-hazard can be identified by writing a product-of-sums expression for the circuit.

1.6 Flip-Flops and Latches

Sequential circuits commonly use flip-flops as storage devices. There are several types of flip-flops, such as Delay (D) flip-flops, J-K flip-flops, Toggle (T) flip-flops, and so on. Figure 1-11 shows a clocked D flip-flop. This flip-flop can change state in

response to the rising edge of the clock input. The next state of the flip-flop after the rising edge of the clock is equal to the D input before the rising edge. The *characteristic equation* of the flip-flop is therefore $Q^+ = D$, where Q^+ represents the next state of the Q output after the active edge of the clock and D is the input before the active edge.

FIGURE 1-11: Clocked D Flip-Flop with Rising-Edge Trigger

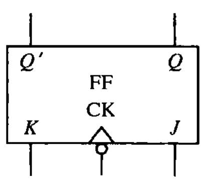


D	Q	Q^+
0	0	0
0	1	0
1	0	1
1	1	1

Figure 1-12 shows a clocked J-K flip-flop and its truth table. Since there is a bubble at the clock input, all state changes occur following the falling edge of the clock input. If $J = K = 0$, no state change occurs. If $J = 1$ and $K = 0$, the flip-flop is set to 1, independent of the present state. If $J = 0$ and $K = 1$, the flip-flop is always reset to 0. If $J = K = 1$, the flip-flop changes state. The characteristic equation, derived from the truth table in Figure 1-12, using a Karnaugh map is

$$Q^+ = JQ' + K'Q \tag{1-33}$$

FIGURE 1-12: Clocked J-K Flip-Flop



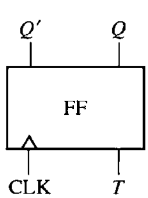
J	K	Q	Q^+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

A clocked T flip-flop (Figure 1-13) changes state following the active edge of the clock if $T = 1$, and no state change occurs if $T = 0$. T flip-flops are particularly useful for designing counters. The characteristic equation for the T flip-flop is

$$Q^+ = QT' + Q'T = Q \oplus T \tag{1-34}$$

A J-K flip-flop is easily converted to a T flip-flop by connecting T to both J and K. Substituting T for J and K in Equation (1-33) yields Equation (1-34).

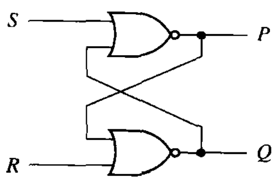
FIGURE 1-13: Clocked T Flip-Flop



T	Q	Q^+
0	0	0
0	1	1
1	0	1
1	1	0

Two NOR gates can be connected to form an unclocked S-R (set-reset) flip-flop, as shown in Figure 1-14. An unclocked flip-flop of this type is often referred to as an S-R latch. If $S = 1$ and $R = 0$, the Q output becomes 1 and $P = Q'$. If $S = 0$ and $R = 1$, Q becomes 0 and $P = Q'$. If $S = R = 0$, no change of state occurs. If $R = S = 1$, $P = Q = 0$, which is not a proper flip-flop state, since the two outputs should always be complements. If $R = S = 1$ and these inputs are simultaneously changed to 0, oscillation may occur. For this reason, S and R are not allowed to be 1 at the same time. For purposes of deriving the characteristic equation, we assume that $S = R = 1$ never occurs, in which case $Q^+ = S + R'Q$. In this case, Q^+ represents the state after any input changes have propagated to the Q output.

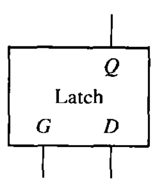
FIGURE 1-14: S-R Latch



S	R	Q	Q^+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	–
1	1	1	–

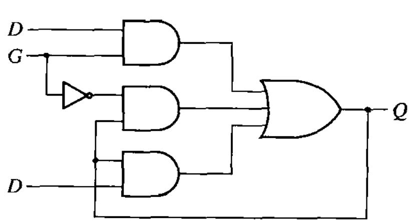
A gated D latch (Figure 1-15), also called a transparent D latch, behaves as follows: If the gate signal $G = 1$, then the Q output follows the D input ($Q^+ = D$). If $G = 0$, then the latch holds the previous value of Q ($Q^+ = Q$). Essentially, the device will not respond to input changes unless G = 1; it simples "latches" the previous input right before G became 0. Some refer to the D latch as a level-sensitive D flip-flop. Essentially, if the gate input G is viewed as a clock, the latch can be considered as a device that operates when the clock level is high and does not respond to the inputs when the clock level is low. The characteristic equation for the D latch is $Q^+ = GD + G'Q$. Figure 1-16 shows an implementation of the D latch using gates. Since the Q^+ equation has a 1-hazard, an extra AND gate has been added to eliminate the hazard.

FIGURE 1-15: Transparent D Latch



G	D	Q	Q^+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

FIGURE 1-16: Implementation of D Latch

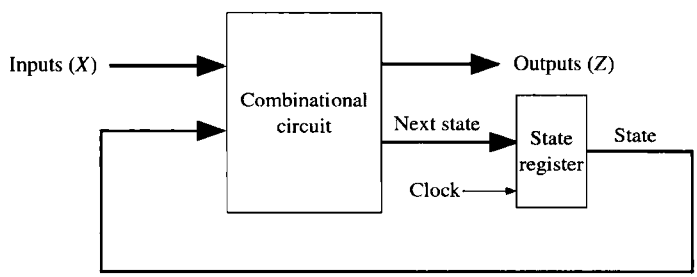


$Q^+ = DG + G'Q + (DQ)$

1.7 Mealy Sequential Circuit Design

There are two basic types of sequential circuits: Mealy and Moore. In a Mealy circuit, the outputs depend on both the present state and the present inputs. In a Moore circuit, the outputs depend only on the present state. A general model of a Mealy sequential circuit consists of a combinational circuit, which generates the outputs and the next state, and a state register, which holds the present state (see Figure 1-17). The state register normally consists of D flip-flops. The normal sequence of events is (1) the X inputs change to a new value; (2) after a delay, the corresponding Z outputs and next state appears at the output of the combinational circuit; and (3) the next state is clocked into the state register and the state changes. The new state feeds back into the combinational circuit and the process is repeated.

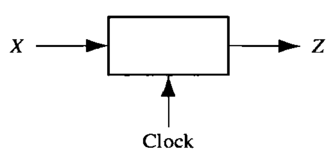
FIGURE 1-17: General Model of Mealy Sequential Machine



1.7.1 Mealy Machine Design Example 1: Sequence Detector

To illustrate the design of a clocked Mealy sequential circuit, let us design a sequence detector. The circuit has the form indicated in the block diagram in Figure 1-18.

FIGURE 1-18: Block Diagram of a Sequence Detector



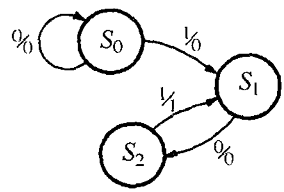
The circuit will examine a string of 0's and 1's applied to the X input and generate an output $Z = 1$ only when the input sequence ends in 1 0 1. The input X can change only between clock pulses. The output $Z = 1$ coincides with the last 1 in 1 0 1. The circuit does not reset when a 1 output occurs. A typical input sequence and the corresponding output sequence are

$$X = 0\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 0$$
$$Z = 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0$$

Let us construct a *state graph* for this sequence detector. We will start in a reset state designated S_0. If a 0 input is received, we can stay in state S_0 as the input

sequence we are looking for does not start with 0. However, if a 1 is received, the circuit should go to a new state. Let us denote that state as S_1. When in S_1, if we receive a 0, the circuit must change to a new state (S_2) to remember that the first two inputs of the desired sequence (1 0) have been received. If a 1 is received in state S_2, the desired input sequence is complete and the output should be a 1. The output will be produced as a Mealy output and will coincide with the last 1 in the detected sequence. Since we are designing a Mealy circuit, we are not going to go to a new state that indicates the sequence 101 has been received. When we receive a 1 in S_2, we cannot go to the start state since the circuit is not supposed to reset with every detected sequence. But the last 1 in a sequence can be the first 1 in another sequence; hence, we can go to state S_1. The partial state graph at this point is indicated in Figure 1-19.

FIGURE 1-19: Partial State Graph of the Sequence Detector



When a 0 is received in state S_2, we have received two 0's in a row and must reset the circuit to state S_0. If a 1 is received when we are in S_1, we can stay in S_1 because the most recent 1 can be the first 1 of a new sequence to be detected. The final state graph is shown in Figure 1-20. State S_0 is the starting state, state S_1 indicates that a sequence ending in 1 has been received, and state S_2 indicates that a sequence ending in 10 has been received. Converting the state graph to a state table yields Table 1-3. In row S_2 of the table, an output of 1 is indicated for input 1.

FIGURE 1-20: Mealy State Graph for Sequence Detector

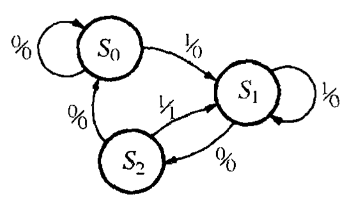


TABLE 1-3: State Table for Sequence Detector

Present State	Next State $X = 0$	Next State $X = 1$	Present Output $X = 0$	Present Output $X = 1$
S_0	S_0	S_1	0	0
S_1	S_2	S_1	0	0
S_2	S_0	S_1	0	1

Next, *state assignment* is performed, whereby specific flip-flop values are associated with specific states. There are two techniques to perform state assignment (1) one-hot state assignment and (2) encoded state assignment. In one-hot state assignment, one flip-flop is used for each state. Hence three flip-flops will be required if this circuit is to be implemented using the one-hot approach. In encoded state assignment, just enough flip-flops to have a unique combination for each state are sufficient. Since we have three states, we need at least two flip-flops to represent all states. We will use encoded state assignment in this design. Let us designate the two flip-flops as A and B. Let the flip-flop states $A = 0$ and $B = 0$ correspond to state S_0; $A = 0$ and $B = 1$ correspond to state S_1; and $A = 1$ and $B = 0$ correspond to state S_2. Now, the transition table of the circuit can be written as in Table 1-4.

TABLE 1-4: Transition Table for Sequence Detector

	A^+B^+		Z	
AB	$X = 0$	$X = 1$	$X = 0$	$X = 1$
00	00	01	0	0
01	10	01	0	0
10	00	01	0	1

From this table, we can plot the K-maps for the next states and the output Z. The next states are typically represented by A^+ and B^+. The three K-maps are shown in Figure 1-21.

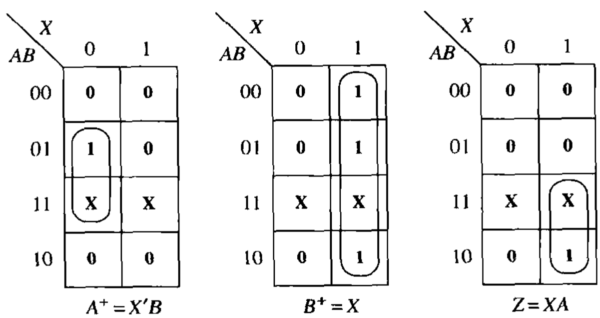


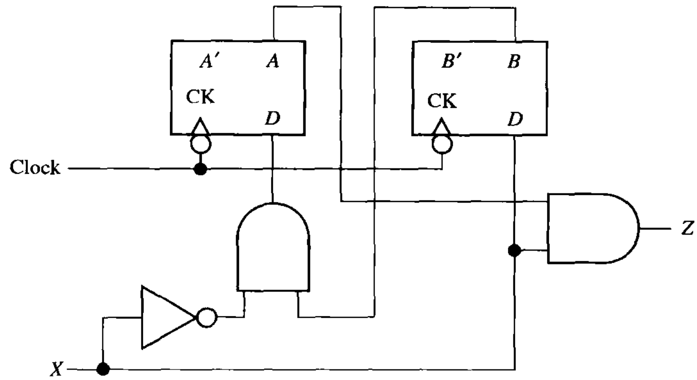
FIGURE 1-21: K-Maps for Next States and Output of Sequence Detector

The next step is deriving the flip-flop inputs to obtain the desired next states. If D flip-flops are used, one simply needs to give the expected next state of the flip-flop to the flip-flop input. So, for flip-flops A and B, $D_A = A^+$ and $D_B = B^+$. The resulting circuit is shown in Figure 1-22.

1.7.2 Mealy Machine Design Example 2: BCD to Excess-3 Code Converter

As an example of a more complex Mealy sequential circuit, we will design a serial code converter that converts an 8-4-2-1 binary-coded-decimal (BCD) digit to an excess-3-coded decimal digit. The input (X) will arrive serially with the least significant bit

FIGURE 1-22: Circuit for Mealy Sequence Detector



(LSB) first. The outputs will be generated serially as well. Table 1-5 lists the desired inputs and outputs at times t_0, t_1, t_2, and t_3. After receiving four inputs, the circuit should reset to its initial state, ready to receive another BCD digit.

TABLE 1-5: Code Converter

X Input (BCD)				*Z* Output (excess-3)			
t_3	t_2	t_1	t_0	t_3	t_2	t_1	t_0
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

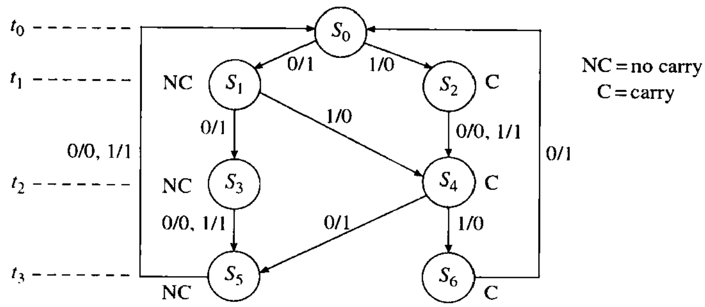
The excess-3 code is formed by adding 0011 to the BCD digit. For example,

$$\begin{array}{r} 0\ 1\ 0\ 0 \\ +\ 0\ 0\ 1\ 1 \\ \hline 0\ 1\ 1\ 1 \end{array} \qquad \begin{array}{r} 0\ 1\ 0\ 1 \\ +\ 0\ 0\ 1\ 1 \\ \hline 1\ 0\ 0\ 0 \end{array}$$

If all of the BCD bits are available simultaneously, this code converter can be implemented as a combinational circuit with four inputs and four outputs. However, here the bits arrive sequentially, one bit at a time. Hence we must implement this code converter sequentially.

Let us now construct a state graph for the code converter (Figure 1-23(a)). Let us designate the start state as S_0. The first bit arrives and we need to add 1 to this bit, as it is the LSB of 0011, the number to be added to the BCD digit to obtain the

FIGURE 1-23: State Graph and Table for Code Converter



(a) Mealy state graph

PS	NS $X=0$	NS $X=1$	Z $X=0$	Z $X=1$
S0	S1	S2	1	0
S1	S3	S4	1	0
S2	S4	S4	0	1
S3	S5	S5	0	1
S4	S5	S6	1	0
S5	S0	S0	0	1
S6	S0	–	1	–

(b) State table

excess-3 code. At t_0, we add 1 to the least significant bit, so if $X = 0, Z = 1$ (no carry), and if $X = 1, Z = 0$ (carry $= 1$). Let us use S_1 to indicate no carry after the first addition, and S_2 to indicate a carry of 1 after the addition to the LSB.

At t_1, we add 1 to the next bit, so if there is no carry from the first addition (state S_1), $X = 0$ gives $Z = 0 + 1 + 0 = 1$ and no carry (state S_3), and $X = 1$ gives $Z = 1 + 1 + 0 = 0$ and a carry (state S_4). If there is a carry from the first addition (state S_2), then $X = 0$ gives $Z = 0 + 1 + 1 = 0$ and a carry (S_4), and $X = 1$ gives $Z = 1 + 1 + 1 = 1$ and a carry (S_4).

At t_2, 0 is added to X, and transitions to S_5 (no carry) and S_6 are determined in a similar manner. At t_3, 0 is again added to X, and the circuit resets to S_0.

Figure 1-23(b) gives the corresponding state table. At this point, we should verify that the table has a minimum number of states before proceeding (see Section 1–9). Then state assignment must be performed. Since this state table has seven states, three flip-flops will be required to realize the table in encoded state assignment. In the one-hot approach, one flip-flop is used for each state. Hence seven flip-flops will be required if this circuit is to be implemented using the one-hot approach. The next step is to make a state assignment that relates the flip-flop states to the states in the table. In the sequence detector example, we simply did a straight binary state assignment. Here we are going to look for an optimal assignment. The best state assignment to use depends on a number of factors. In

many cases, we should try to find an assignment that will reduce the amount of required logic. For some types of programmable logic, a straight binary state assignment will work just as well as any other. For programmable gate arrays, a one-hot assignment may be preferred. In recent years, with the abundance of transistors on silicon chips, the emphasis on optimal state assignment has been reduced.

In order to reduce the amount of logic required, we will make a state assignment using the following guidelines (see Roth, *Fundamentals of Logic Design*, 5th Ed. [Thomson Brooks/Cole, 2004] for details):

I. States that have the same next state (NS) for a given input should be given adjacent assignments (look at the columns of the state table).
II. States that are the next states of the same state should be given adjacent assignments (look at the rows).
III. States that have the same output for a given input should be given adjacent assignments.

Using these guidelines tends to clump 1's together on the Karnaugh maps for the next state and output functions. The guidelines indicate that the following states should be given adjacent assignments:

I. (1, 2), (3, 4), (5, 6) (in the $X = 1$ column, S_1 and S_2 both have NS S_4; in the $X = 0$ column, S_3 and S_4 have NS S_5, and S_5 and S_6 have NS S_0)

II. (1, 2), (3, 4), (5, 6) (S_1 and S_2 are NS of S_0; S_3 and S_4 are NS of S_1; and S_5 and S_6 are NS of S_4)

III. (0, 1, 4, 6), (2, 3, 5)

Figure 1-24(a) gives an assignment map, which satisfies the guidelines, and the corresponding transition table. Since state 001 is not used, the next state and outputs for this state are don't cares. The next state and output equations are derived from this table in Figure 1-25. Figure 1-26 shows the realization of the code converter using NAND gates and D flip-flops.

FIGURE 1-24: State Assignment for BCD to Excess-3 Code Converter

Q_2Q_3 \ Q_1	0	1
00	S0	S1
01		S2
11	S5	S3
10	S6	S4

(a) Assignment map

	$Q_1^+ Q_2^+ Q_3^+$		Z	
$Q_1Q_2Q_3$	$X=0$	$X=1$	$X=0$	$X=1$
000	100	101	1	0
100	111	110	1	0
101	110	110	0	1
111	011	011	0	1
110	011	010	1	0
011	000	000	0	1
010	000	xxx	1	x
001	xxx	xxx	x	x

(b) Transition table

FIGURE 1-25: Karnaugh Maps for Code Converter

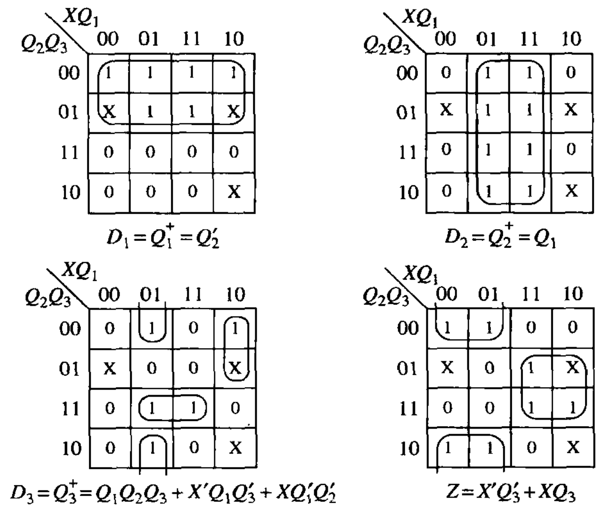
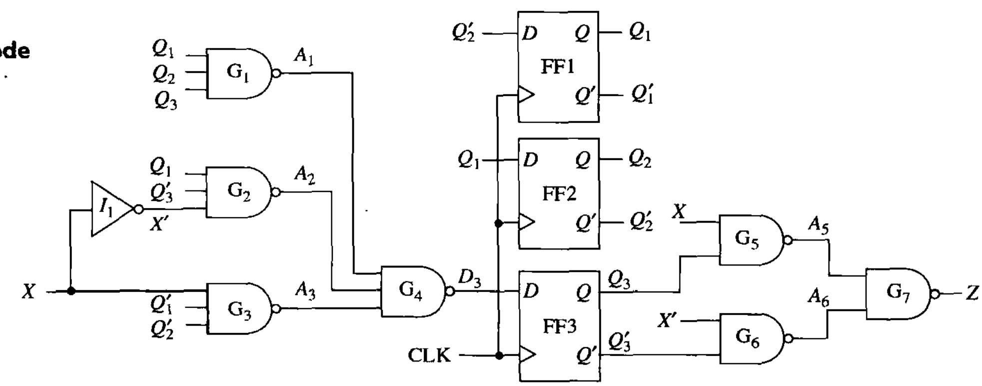


FIGURE 1-26: Realization of Code Converter



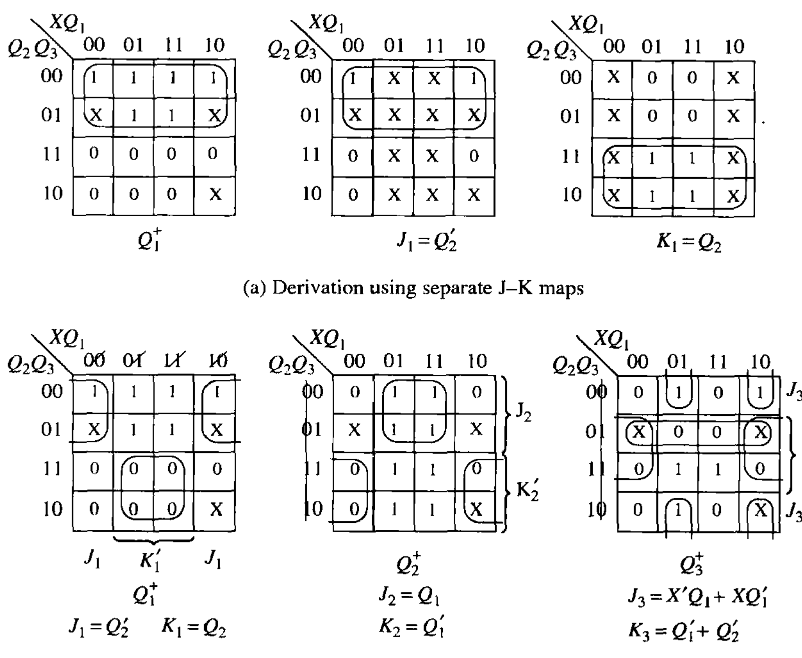
If J-K flip-flops are used instead of D flip-flops, the input equations for the J-K flip-flops can be derived from the next state maps. Given the present state flip-flop (Q) and the desired next state (Q^+), the J and K inputs can be determined from Table 1-6, also known as the excitation table. This table is derived from the truth table in Figure 1-12.

TABLE 1-6: Excitation Table for a J-K Flip-Flop

Q	Q^+	J	K	
0	0	0	X	(No change in Q; J must be 0, K may be 1 to reset Q to 0.)
0	1	1	X	(Change to $Q = 1$; J must be 1 to set or toggle.)
1	0	X	1	(Change to $Q = 0$; K must be 1 to reset or toggle.)
1	1	X	0	(No change in Q; K must be 0, J may be 1 to set Q to 1.)

Figure 1-27 shows derivation of J-K flip-flop input equations for the state table of Figure 1-23 using the state assignment of Figure 1-24. First, we derive the J-K input equations for flip-flop Q_1 using the Q_1^+ map as the starting point. From the preceding table, whenever Q_1 is 0, $J = Q_1^+$ and $K = X$. So, we can fill in the $Q_1 = 0$ half of the J_1 map the same as Q_1^+ and the $Q_1 = 0$ half of the K_1 map as all X's. When Q_1 is 1, $J_1 = X$ and $K_1 = (Q_1^+)'$. So, we can fill in the $Q_1 = 1$ half of the J_1 map with X's and the $Q_1 = 1$ half of the K_1 map with the complement of the Q_1^+. Since half of every J and K map is don't cares, we can avoid drawing separate J and K maps and read the J's and K's directly from the Q^+ maps, as illustrated in Figure 1-27(b). This shortcut method is based on the following: If $Q = 0$, then $J = Q^+$, so loop the 1's on the $Q = 0$ half of the map to get J. If $Q = 1$, then $K = (Q^+)'$, so loop the 0's on the $Q = 1$ half of the map to get K. The J and K equations will be independent of Q, since Q is set to a constant value (0 or 1) when reading J and K. To make reading the J's and K's off the map easier, we cross off the Q values on each map. In effect, using the shortcut method is equivalent to splitting the four-variable Q^+ map into two three-variable maps, one for $Q = 0$ and one for $Q = 1$.

FIGURE 1-27: Derivation of J-K Input Equations



(a) Derivation using separate J–K maps

(b) Derivation using the shortcut method

The following summarizes the steps required to design a sequential circuit:

1. Given the design specifications, determine the required relationship between the input and output sequences. Then find a state graph and state table.
2. Reduce the table to a minimum number of states. First eliminate duplicate rows by row matching; then form an implication table and follow the procedure in Section 1.9.

3. If the reduced table has m states ($2^{n-1} < m \leq 2^n$), n flip-flops are required. Assign a unique combination of flip-flop states to correspond to each state in the reduced table. This is the encoded state assignment technique. Alternately, a one-hot assignment with m flip-flops can be used.
4. Form the transition table by substituting the assigned flip-flop states for each state in the reduced state tables. The resulting transition table specifies the next states of the flip-flops and the output in terms of the present states of the flip-flops and the input.
5. Plot next-state maps and input maps for each flip-flop and derive the flip-flop input equations. Derive the output functions.
6. Realize the flip-flop input equations and the output equations using the available logic gates.
7. Check your design using computer simulation or another method.

Steps 2 through 7 may be carried out using a suitable computer-aided design (CAD) program.

1.8 Moore Sequential Circuit Design

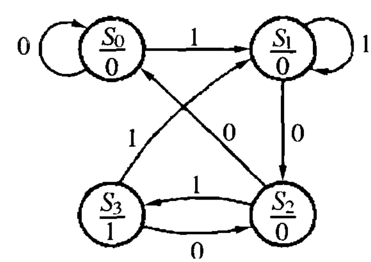
In a Moore circuit, the outputs depend only on the present state. Moore machines are typically easier to design and debug compared to Mealy machines, but they often contain more states than equivalent Mealy machines. In Moore machines, there are no outputs that happen during the transition. The outputs are associated entirely to the state.

1.8.1 Moore Machine Design Example 1: Sequence Detector

As an example, let us design the sequence detector of Section 1.7.1 using the Moore Method. The circuit will examine a string of 0's and 1's applied to the X input and generate an output $Z = 1$ only when the input sequence ends in 101. The input X can change only between clock pulses. The circuit does not reset when a 1 output occurs.

As in the Mealy machine example, we start in a reset state designated S_0 in Figure 1-28. If a 0 input is received, we can stay in state S_0 as the input sequence we are looking for does not start with 0. However, if a 1 is received, the circuit goes to a new state, S_1. When in S_1, if we receive a 0, the circuit must change to a new state (S_2) to remember that the first two inputs of the desired sequence (10) have been

FIGURE 1-28: State Graph of the Moore Sequence Detector



received. If a 1 is received in state S_2, the circuit should go to a new state to indicate that the desired input sequence is complete. Let us designate this new state as S_3. In state S_3, the output must have a value of 1. The outputs in states S_0, S_1 and S_2 must be 0's. The sequence 100 resets the circuit to S_0. A sequence 1010 takes the circuit back to S_2 because another 1 input should cause Z to become 1 again.

The state table corresponding to the circuit is given by Table 1-7. Note that there is a single column for output because the output is determined by the present state and does not depend on X. Note that this sequence detector requires one more state than the Mealy sequence detector in Table 1-3, which detects the same input sequence.

TABLE 1-7: State Table for Sequence Detector

Present State	Next State $X = 0$	Next State $X = 1$	Present Output (Z)
S_0	S_0	S_1	0
S_1	S_2	S_1	0
S_2	S_0	S_3	0
S_3	S_2	S_1	1

Because there are four states, two flip-flops are required to realize the circuit. Using the state assignment $AB = 00$ for S_0, $AB = 01$ for S_1, $AB = 11$ for S_2, and $AB = 10$ for S_3, the transition table shown in Table 1-8 is obtained.

TABLE 1-8: Transition Table for Moore Sequence Detector

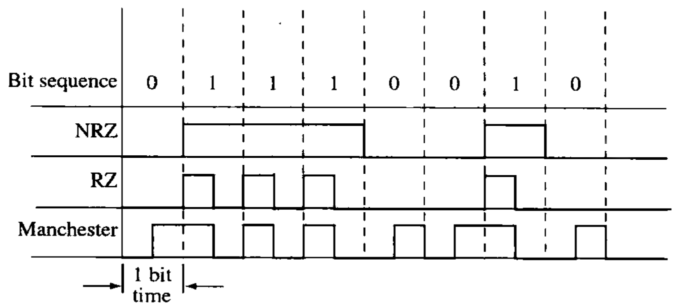
AB	A^+B^+ $X = 0$	A^+B^+ $X = 1$	Z
00	00	01	0
01	11	01	0
11	00	10	0
10	11	01	1

The output function $Z = AB'$. Note that Z depends only on the flip-flop states and is independent of X, while for the corresponding Mealy machine, Z was a function of X. (It was equal to AX in Figure 1-21.) The transition table can be used to write the next state maps and inputs to the flip-flops can be derived.

1.8.2 Moore Machine Design Example 2: NRZ to Manchester Code Converter

As another example of designing a Moore sequential machine, we will design a converter for serial data. Binary data is frequently transmitted between computers as a serial stream of bits. Figure 1-29 shows three different coding schemes for serial data. The example shows transmission of the bit sequence 0, 1, 1, 1, 0, 0, 1, 0. With the NRZ (nonreturn-to-zero) code, each bit is transmitted for one bit time without any change. In contrast, for the RZ (return-to-zero) code, a 0 is transmitted as 0 for one full bit time, but a 1 is transmitted as a 1 for the first half of the bit time, and then

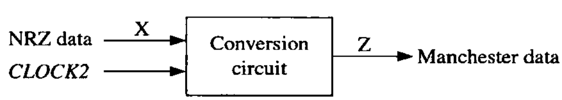
FIGURE 1-29: Coding Schemes for Serial Data Transmission



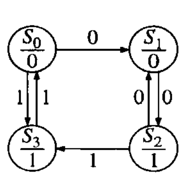
the signal returns to 0 for the second half. For the Manchester code, a 0 is transmitted as 0 for the first half of the bit time and a 1 for the second half, but a 1 is transmitted as a 1 for the first half and a 0 for the second half. Thus, the Manchester encoded bit always changes in the middle of the bit time.

We will design a Moore sequential circuit that converts an NRZ-coded bit stream to a Manchester-coded bit stream (Figure 1-30). In order to do this, we will use a clock (*CLOCK2*) that is twice the frequency of the basic bit clock. If the NRZ bit is 0, it will be 0 for two *CLOCK2* periods, and if it is 1, it will be 1 for two *CLOCK2* periods. Thus, starting in the reset state (S_0), the only two possible input sequences are 00 and 11, and the corresponding output sequences are 01 and 10. When a 0 is received, the circuit goes to S_1 and outputs a 0; when the second 0 is received, it goes to S_2 and outputs a 1. Starting in S_0, if a 1 is received, the circuit goes to S_3 and outputs a 1, and when the second 1 is received, it must go to a state with a 0 output. Going back to S_0 is appropriate since S_0 has a 0 output and the circuit is ready to receive another 00 or 11 sequence. When in S_2, if a 00 sequence is received, the circuit can go to S_1 and then back to S_2. If a 11 sequence is received in S_2, the circuit can go to S_3 and then back to S_0. The corresponding Moore state table has two don't cares, which correspond to input sequences that cannot occur.

FIGURE 1-30: Moore Circuit for NRZ-to-Manchester Conversion



(a) Conversion circuit



(b) State graph

Present State	Next State $X=0$	Next State $X=1$	Present Output (Z)
S_0	S_1	S_3	0
S_1	S_2	—	0
S_2	S_1	S_3	1
S_3	—	S_0	1

(c) State table

Figure 1-31 shows the timing chart for the Moore circuit. Note that the Manchester output is shifted one clock time with respect to the NRZ input. This

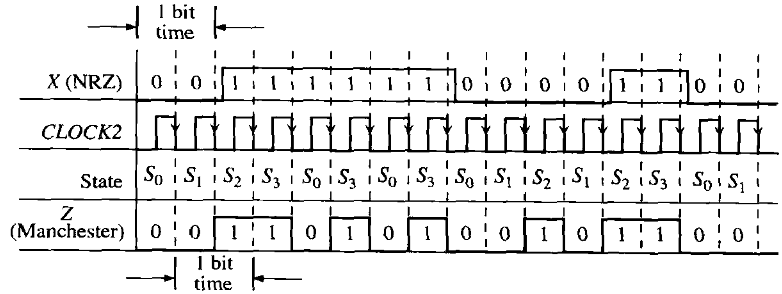


FIGURE 1-31: Timing for Moore Circuit

shift occurs because a Moore circuit cannot respond to an input until the active edge of the clock occurs. This is in contrast to a Mealy circuit, for which the output can change after the input changes and before the next clock.

1.9 Equivalent States and Reduction of State Tables

The concept of equivalent states is important for the design and testing of sequential circuits. It helps to reduce the hardware consumed by circuits. Two states in a sequential circuit are said to be *equivalent* if we cannot tell them apart by observing input and output sequences. Consider two sequential circuits, N_1 and N_2 (see Figure 1-32). N_1 and N_2 could be copies of the same circuit. N_1 is started in state s_i, and N_2 is started in state s_j. We apply the same input sequence, $\underline{X}$, to both circuits and observe the output sequences, $\underline{Z}_1$ and $\underline{Z}_2$. (The underscore notation indicates a sequence.) If $\underline{Z}_1$ and $\underline{Z}_2$ are the same, we reset the circuits to states s_i and s_j, apply a different input sequence, and observe $\underline{Z}_1$ and $\underline{Z}_2$. If the output sequences are the same for all possible input sequences, we say the s_i and s_j are equivalent ($s_i \equiv s_j$). Formally, we can define equivalent states as follows: $s_i \equiv s_j$ if and only if, for every input sequence $\underline{X}$, the output sequences $\underline{Z}_1 = \lambda_1(s_i, \underline{X})$ and $\underline{Z}_2 = \lambda_2(s_j, \underline{X})$ are the same. This is not a very practical way to test for state equivalence since, at least in theory, it requires input sequences of infinite length. In practice, if we have a bound on number of states, then we can limit the length of the test sequences.

A more practical way to determine state equivalence uses the state equivalence theorem: $s_i \equiv s_j$ if and only if for every single input X, the outputs are the same and the

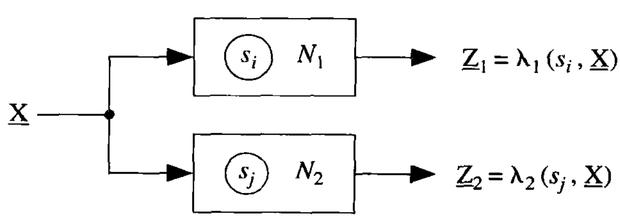


FIGURE 1-32: Sequential Circuits

next states are equivalent. When using the definition of equivalence, we must consider all input sequences, but we do not need any information about the internal state of the system. When using the state equivalence theorem, we must look at both the output and next state, but we need to consider only single inputs rather than input sequences.

The table of Figure 1-33(a) can be reduced by eliminating equivalent states. First, observe that states a and h have the same next states and outputs when $X = 0$ and also when $X = 1$. Therefore, $a \equiv h$ so we can eliminate row h and replace h with a in the table. To determine if any of the remaining states are equivalent, we will use the state equivalence theorem. From the table, since the outputs for states a and b are the same, $a \equiv b$ if and only if $c \equiv d$ and $e \equiv f$. We say that c-d and e-f are implied pairs for a-b. To keep track of the implied pairs, we make an *implication chart*, as shown in Figure 1-33(b). We place c-d and e-f in the square at the intersection of row a and column b to indicate the implication. Since states d and e have different outputs, we place an X in the d-e square to indicate that $d \not\equiv e$. After completing the implication chart in this way, we make another pass through the chart. The e-g square contains c-e and b-g. Since the c-e square has an X, $c \not\equiv e$, which implies $e \not\equiv g$, so we X out the e-g square. Similarly, since $a \not\equiv g$, we X out the f-g square. On the next pass through the chart, we X out all the squares that contain e-g or f-g as implied pairs (shown on the chart with dashed x's). In the next pass, no additional squares are X'ed out, so the process terminates. Since all the squares corresponding to non-equivalent states have been X'ed out, the coordinates of the remaining squares indicate equivalent state pairs. From the first column, $a \equiv b$; from third column, $c \equiv d$; and from the fifth column, $e \equiv f$.

The implication table method of determining state equivalence can be summarized as follows:

1. Construct a chart that contains a square for each pair of states.
2. Compare each pair of rows in the state table. If the outputs associated with states i and j are different, place an X in square i-j to indicate that $i \not\equiv j$. If the outputs are the same, place the implied pairs in square i-j. (If the next states of i and j are m and n for some input x, then m-n is an implied pair.) If the outputs and next states are the same (or if i-j implies only itself), place a check ($\surd$) in square i-j to indicate that $i \equiv j$.
3. Go through the table square by square. If square i-j contains the implied pair m-n, and square m-n contains an X, then $i \not\equiv j$, and an X should be placed in square i–j.
4. If any X's were added in step 3, repeat step 3 until no more X's are added.
5. For each square i-j that does not contain an X, $i \equiv j$.

If desired, row matching can be used to partially reduce the state table before constructing the implication table. Although we have illustrated this procedure for a Mealy table, the same procedure applies to a Moore table.

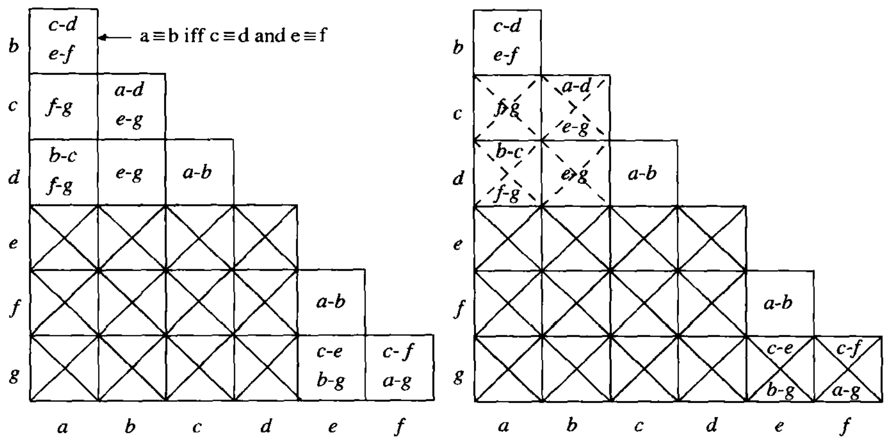
Two sequential circuits are said to be equivalent if every state in the first circuit has an equivalent state in the second circuit, and vice versa.

Optimization techniques such as this are incorporated in CAD tools. The importance of state minimization has slightly diminished in recent years due to the abundance of transistors on chips; however, it is still important to do obvious state minimizations to reduce the circuit's area and power.

FIGURE 1-33: State Table Reduction

Present State	Next State $X=0$	1	Present Output $X=0$	1
a	c	f	0	0
b	d	e	0	0
c	~~h~~a	g	0	0
d	b	g	0	0
e	e	b	0	1
f	f	a	0	1
g	c	g	0	1
~~h~~	~~e~~	~~f~~	~~0~~	~~0~~

(a) State table reduction by row matching



(b) Implication chart (first pass)

(c) After second and third passes

	$X=$ 0	1	$X=$ 0	1
a	c	e	0	0
c	a	g	0	0
e	e	a	0	1
g	c	g	0	1

(d) Final reduced table

1.10 Sequential Circuit Timing

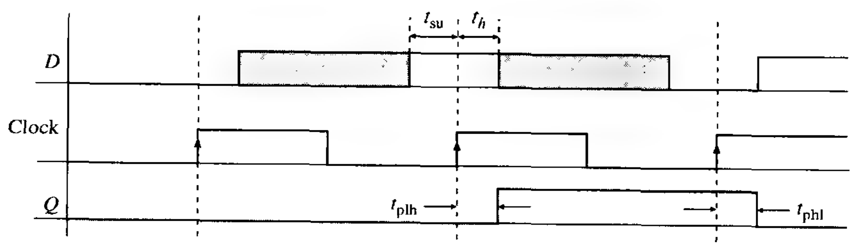
The correct functioning of sequential circuits involves several timing issues. Propagation delays of flip-flops, gates and wires, setup times and hold times of flip-flops, clock synchronization, clock skew, etc become important issues while designing sequential circuits. In this section, we look at various topics related to sequential circuit timing.

1.10.1 Propagation Delays; Setup and Hold Times

There is a certain amount of time, albeit small, that elapses from the time the clock changes to the time the Q output changes. This time, called *propagation delay*, is indicated in Figure 1-34. The propagation delay can depend on whether the output is changing from high to low or vice versa. In the figure, the propagation delay for a low-to-high change in Q is denoted by t_{plh}, and for a high-to-low change it is denoted by t_{phl}.

For an ideal D flip-flop, if the D input changed at exactly the same time as the active edge of the clock, the flip-flop would operate correctly. However, for a real flip-flop, the D input must be stable for a certain amount of time before the active edge of the clock. This interval is called the *setup time* (t_{su}). Furthermore, D must be stable for a certain amount of time after the active edge of the clock. This interval is called the *hold time* (t_h). Figure 1-34 illustrates setup and hold times for a D flip-flop that changes state on the rising edge of the clock. D can change at any time during the shaded region on the diagram, but it must be stable during the time interval t_{su} before the active edge and for t_h after the active edge. If D changes at any time during the forbidden interval, it cannot be determined whether the flip-flop will change state. Even worse, the flip-flop may malfunction and output a short pulse or even go into oscillation. Minimum values for t_{su} and t_h and maximum values for t_{plh} and t_{phl} can be read from manufacturers' data sheets.

FIGURE 1-34: Setup and Hold Times for D Flip-Flop

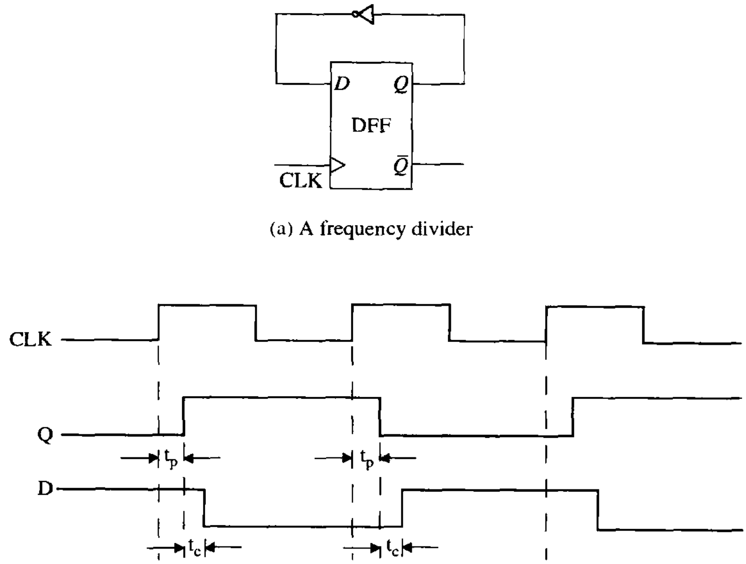


1.10.2 Maximum Clock Frequency of Operation

In a synchronous sequential circuit, state changes occur immediately following the active edge of the clock. The maximum clock frequency for a sequential circuit depends on several factors. The clock period must be long enough so that all flip-flop and register inputs will have time to stabilize before the next active edge of the clock. Propagation delays and setup and hold times create complications in sequential circuit timing.

Consider a simple circuit of the form of Figure 1-35(a). The output of a D flip-flop is fed back to its input through an inverter. Assume a clock as indicated by the waveform CLK in Figure 1-35(b). If the current output of the flip-flop is 1, a value of 0 will appear at the flip-flop's D input after the propagation delay of the inverter. Assuming that the next active edge of the clock arrives after the setup time has elapsed, the output of the flip-flop will change to 0. This process will continue, yielding the output Q

FIGURE 1-35: Simple Frequency Divider



(a) A frequency divider

(b) Frequency divider timing diagram

of the flip-flop to be a waveform with twice the period of the clock. Essentially the circuit behaves as a frequency divider.

If we increase the frequency of the clock slightly, the circuit will still work yielding half of the increased frequency at the output. However, if we increase the frequency to be very high, the output of the inverter may not get enough time to stabilize and meet the setup time requirements. Similarly, if the inverter was very fast and fed the inverted output to the D input extremely quickly, there will be timing problems because the hold time of the flip-flop may not be met. So we can easily see a variety of ways in which timing problems could arise from propagation delays and setup and hold time requirements.

1.10.3 Timing Conditions for Proper Operation

For a circuit of the general form of Figure 1-17, assume that the maximum propagation delay through the combinational circuit is t_{cmax} and the maximum propagation delay from the time the clock changes to the time the flip-flop output changes is t_{pmax}, where t_{pmax} is the maximum of t_{plh} and t_{phl}. There are four conditions this circuit has to meet in order to ensure proper operation.

1. **Clock period should be long enough to satisfy flip-flop setup time.** The clock period should be long enough to allow the flip-flop outputs to change and the combinational circuitry to change while still leaving enough time to satisfy the setup time. Once the clock arrives, it could take a delay of up to t_{pmax} before the flip-flop output changes. Then it could take a delay of up to t_{cmax} before the output of the combinational circuitry changes. Thus the maximum time from the active edge of the clock to the time the change in Q propagates back to the

D flip-flop inputs is $t_{pmax} + t_{cmax}$. In order to ensure proper flip-flop operation, the combinational circuit output must be stable t_{su} before the end of the clock period. If the clock period is t_{ck},

$$t_{ck} \geq t_{pmax} + t_{cmax} + t_{su}$$

The difference between t_{ck} and $(t_{pmax} + t_{cmax} + t_{su})$ is referred to as the setup time margin.

2. **Clock period should be long enough to satisfy flip-flop hold time.** A hold-time violation could occur if the change in Q fed back through the combinational circuit and caused D to change too soon after the clock edge. The hold time is satisfied if

$$t_{pmin} + t_{cmin} \geq t_h$$

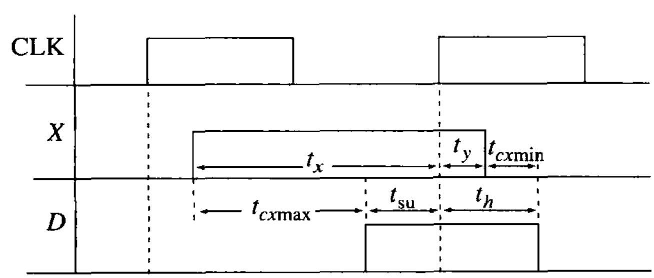
When checking for hold-time violations, the worst case occurs when the timing parameters have their minimum values. Since $t_{pmin} > t_h$ for normal flip-flops, a hold-time violation due to Q changing does not occur.

3. **External input changes to the circuit should satisfy flip-flop setup time.** A setup time violation could occur if the X input to the circuit changes too close to the active edge of the clock. When the X input to a sequential circuit changes, we must make sure that the input change propagates to the flip-flop inputs such that the setup time is satisfied before the active edge of the clock. If X changes t_x time units before the active edge of the clock (see Figure 1-36), then it could take up to the maximum propagation delay of the combinational circuit, before the change in X propagates to the flip-flop input. There should still be a margin of t_{su} left before the edge of the clock. Hence, the setup time is satisfied if

$$t_x \geq t_{cxmax} + t_{su}$$

where t_{cxmax} is the maximum propagation delay from X to the flip-flop input.

FIGURE 1-36: Setup and Hold Timing for Changes in X



4. **External input changes to the circuit should satisfy flip-flop hold times.** In order to satisfy the hold time, we must make sure that X does not change too soon after the clock. If a change in X propagates to the flip-flop input in zero time, X should not change for a duration of t_h after the clock edge. Fortunately, it takes some positive propagation delay for the change in X

to reach the flip-flop. If t_{cxmin} is the minimum propagation delay from X to the flip-flop input, changes in X will not reach the flip-flop input until at least a time of t_{cxmin} has elapsed after the clock edge. So, if X changes t_y time units after the active edge of the clock, then the hold time is satisfied if

$$t_y \geq t_h - t_{cxmin}$$

If t_y is negative, X can change before the active clock edge and still satisfy the hold time.

Given a circuit, we can determine the safe frequency of operation and safe regions for input changes using the above principles. As an example, consider the frequency divider circuit in Figure 1-35(a). If the minimum and maximum delays of the inverter are 1 ns and 3 ns, and if t_{pmin} and t_{pmax} are 5 ns and 8 ns, the maximum frequency at which it can be clocked can be derived using requirement (1) above. Assume that the setup and hold times of the flip-flop are 4 ns and 2 ns. For proper operation, $t_{ck} \geq t_{pmax} + t_{cmax} + t_{su}$. In this example, t_{pmax} for the flip-flops is 8 ns, t_{cmax} is 3 ns, and t_{su} is 4 ns. Hence

$$t_{ck} \geq 8 + 3 + 4 = 15 \text{ ns}$$

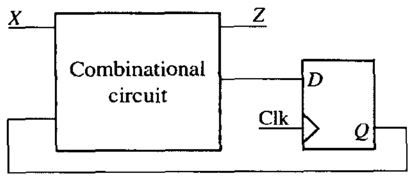
The maximum clock frequency is then $1/t_{ck}$ = 66.67 MHz. We should also make sure that the hold time requirement is satisfied. Hold time requirement means that the D input should not change before 2 ns after the clock edge. This will be satisfied if $t_{pmin} + t_{cmin} \geq 2$ ns. In this circuit, t_{pmin} is 5 ns and t_{cmin} is 1 ns. Thus the Q output is guaranteed to not change until 5 ns after the clock edge, and at least 1 ns more should elapse before the change can propagate through the inverter. Hence the D input will not change until 6 ns after the clock edge, which automatically satisfies the hold time requirements. Since there are no external inputs, these are the only timing constraints that we need to satisfy.

Now consider a circuit as in Figure 1-37(a). Assume that the delay of the combinational circuit is in the range 2 to 4 ns, the flip-flop propagation delays are in the range 5 to10 ns, the setup time is 8 ns, and hold time is 3 ns. In order to satisfy the setup time, the clock period has to be greater than $t_{pmax} + t_{cmax} + t_{su}$. So

$$t_{ck} \geq 10 + 4 + 8 = 22 \text{ ns}$$

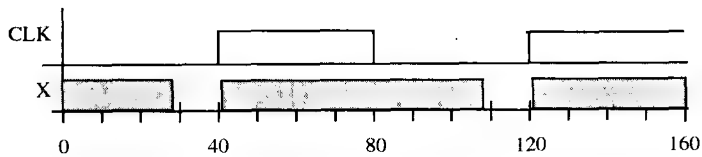
The hold time requirement is satisfied if the output does not change until 3 ns after the clock. Here, the output is not expected to change until $t_{pmin} + t_{cmin}$. Since t_{pmin} is 5 ns and t_{cmin} is 2 ns, the output is not expected to change until 7 ns, which automatically satisfies the hold time requirement. This circuit has external inputs that allow us to identify safe regions where the input X can change using requirements (3) and (4) above. The X input should be stable for a duration of $t_{cxmax} + t_{su}$ (i.e., 4 ns + 8 ns) before the clock edge. Similarly, it should be stable for a duration of $t_h - t_{cxmin}$ (i.e., 3 ns − 2 ns) after the clock edge. Thus, the X input should not change 12 ns before the clock edge and 1 ns after the clock edge. Although the hold time is 3 ns, we see that the input X can change 1 ns after the clock edge, because it takes at least another 2 ns (minimum delay of combinational circuit) before the input change can propagate to the D input of the flip-flop. The shaded regions in

FIGURE 1-37: Safe Regions for Input Changes



Q	Q^+ 0	Q^+ 1	Z 0	Z 1
0	0	1	0	1
1	0	0	1	0

(a) A sequential circuit



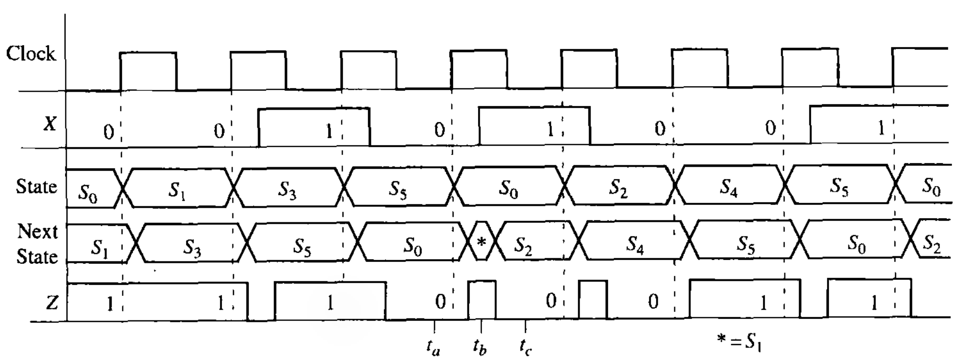
(b) Safe regions for changes in X

the waveform for X indicate safe regions where the input signal X may change without causing erroneous operation in the circuit.

1.10.4 Glitches in Sequential Circuits

Sequential circuits often have external inputs that are asynchronous. Input changes can cause temporary false values called glitches at the outputs and next states. For example, if the state table of Figure 1-23(b) is implemented in the form of Figure 1-17, the timing waveforms are as shown in Figure 1-38. Propagation delays in the flip-flop have been neglected; hence state changes are shown to coincide with clock edges. In this example, the input sequence is 0 0 1 0 1 0 0 1, and X is assumed to change in the middle of the clock pulse. At any given time, the next state and Z output can be read from the next state table. For example, at time t_a, State = S_5 and $X = 0$, so Next State = S_0 and $Z = 0$. At time t_b following the rising edge of the clock, State = S_0 and X is still 0, so Next State = S_1 and $Z = 1$. Then X changes to 1, and at time t_c Next State = S_2 and $Z = 0$. Note that there is a *glitch* (sometimes called a false output) at t_b. The Z output

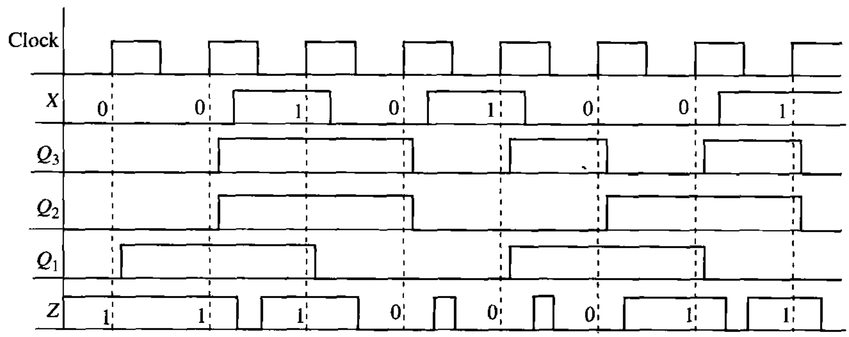
FIGURE 1-38: Timing Diagram for Code Converter



momentarily has an incorrect value at t_b, because the change in X is not exactly synchronized with the active edge of the clock. The correct output sequence, as indicated on the waveform, is 1 1 1 0 0 0 1 1. Several glitches appear between the correct outputs; however, these are of no consequence if Z is read at the right time. The glitch in the next state at t_b (S_1) also does not cause a problem, because the next state has the correct value at the active edge of the clock.

The timing waveforms derived from the circuit of Figure 1-26 are shown in Figure 1-39. They are similar to the general timing waveforms given in Figure 1-38 except that State has been replaced with the states of the three flip-flops, and a propagation delay of 10 ns has been assumed for each gate and flip-flop.

FIGURE 1-39: Timing Diagram for Figure 1-26



1.10.5 Synchronous Design

One of the most commonly used digital design techniques is *synchronous design*. In this type of design, a clock is used to synchronize the operation of all flip-flops, registers, and counters in the system. Synchronous circuits are more reliable compared to asynchronous circuits. In synchronous circuits, events are expected to occur immediately following the active edge of the clock. Outputs from one part have a full clock cycle to propagate to the next part of the circuit. Synchronous design philosophy makes design and debugging easier compared to asynchronous techniques.

Figure 1-40 illustrates a synchronous digital system. Assume that the system is built from several modules or devices. The devices could be flip-flops, registers, counters, adders, multipliers, and so on. All of the sequential devices are synchronized with respect to the same clock in a synchronous system. A traditional way to view a digital system is to consider it as a control section plus a data section. The various devices shown in Figure 1-40 are part of the data section. The control section is a sequential machine that generates control signals to control the operation of the data section. For example, if the data section contains a shift register, the control section may generate signals that determine when the register is to be loaded (Ld) and when it is to be shifted (Sh). A common clock synchronizes the operation of the control and data sections. The data section may generate status signals (not shown in this figure) that affect the control sequence. For example, if a data operation produces an arithmetic overflow, then the data section might generate a condition

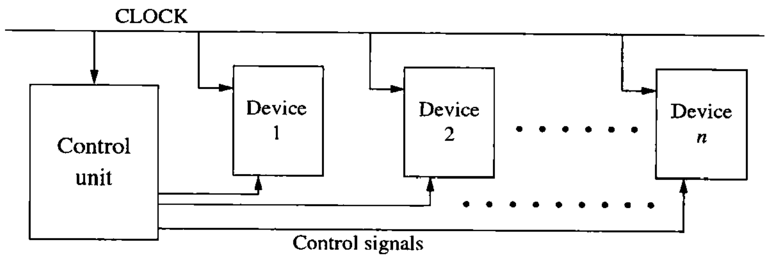


FIGURE 1-40: A Synchronous Digital System

signal V to indicate an overflow. The control section is also called *controller* and the data section is often called *architecture* or *data path*.

In a synchronous digital system, we desire to see all changes happen immediately at the active edge of the clock, but that might not happen in a practical circuit. Modern integrated circuits (ICs) are fabricated at feature sizes such as or smaller than 0.1 microns. Modern microprocessors are clocked at several gigahertz. In these chips, wire delays are significant compared to the clock period. Even if two flip-flops are connected to the same clock, the clock edge might arrive at the two flip-flops at different times due to unequal wire delays. If unequal amounts of combinational circuitry (e.g., buffers or inverters) are used in the clock path to different devices, that also could result in unequal delays, making the clock reach different devices at slightly different times. This problem is called **clock skew**.

There are also problems that occur due to glitches in control signals. Consider Figure 1-41, which illustrates the operation of a digital system that uses devices that change state on the falling edge of the clock. Several flip-flops may change state in response to this falling edge. The time at which each flip-flop changes state is determined by the propagation delay for that flip-flop. The changes in flip-flop states in the control section will propagate through the combinational circuit that generates the control signals, and some of the control signals may change as a result. The exact times at which the control signals change depend on the propagation delays in the gate circuits that generate the signals as well as the flip-flop delays. Thus, after

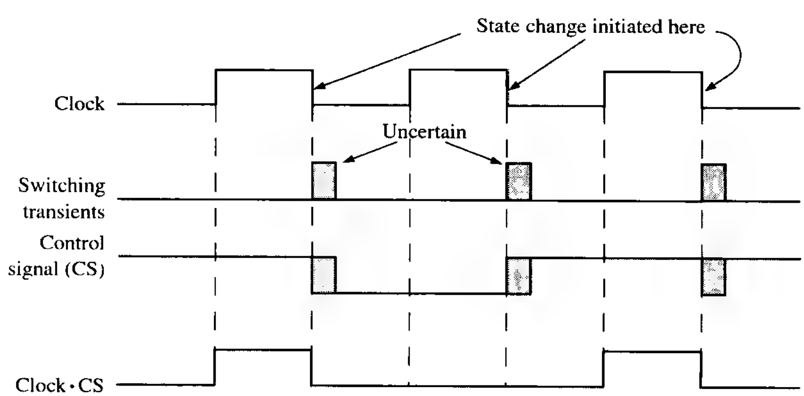


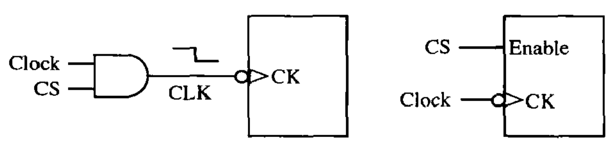
FIGURE 1-41: Timing Chart for System with Falling-Edge Devices

the falling edge of the clock, there is a period of uncertainty during which control signals may change. Glitches and spikes may occur in the control signals due to hazards. Furthermore, when signals are changing in one part of the circuit, noise may be induced in another part of the circuit. As indicated by the shading in Figure 1-41, there is a time interval after each falling edge of the clock in which there may be noise in a control signal (CS), and the exact time at which the control signal changes is not known.

If we want a device in the data section to change state on the falling edge of the clock only if the control signal $CS = 1$, we can AND the clock with CS, as shown in Figure 1-42(a). This technique is called clock gating. The transitions will occur in synchronization with the clock CLK except for a small delay in the AND gate. The gated CLK signal is clean because the clock is 0 during the time interval in which the switching transients occur in CS.

Gating the clock with the control signal, as illustrated in Figure 1-42(a), can solve some synchronization problems. However, clock gating can also lead to clock skew and additional timing problems in high-speed circuits. Instead of gating the clock with the control signal, it is more desirable to use devices with clock enable (CE) pins and feed the control signal to the enable pin, as illustrated in Figure 1-42(b). Many registers, counters, and other devices used in synchronous systems have an enable input. When enable = 1, the device changes state in response to the clock, and when enable = 0, no state change occurs. Use of the enable input eliminates the need for a gate on the clock input, and associated timing problems are avoided.

FIGURE 1-42: Techniques to Synchronize Control Signals

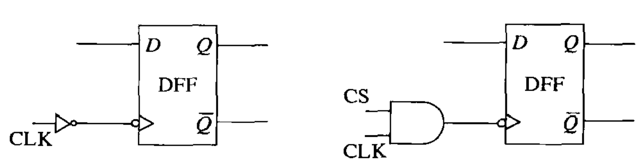


(a) Control signal gating

(b) Use of a clock enable (CE) input to synchronize

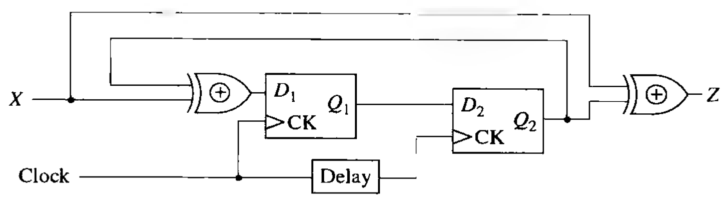
We discourage designers from gating clocks or feeding the output of combinational circuits to clock inputs. While clock skew from wire delays is unavoidable to some extent, clock skew due to combinational circuitry in the clock path can easily be avoided. Circuits as in Figure 1-43 should be avoided as much as possible to minimize timing problems.

FIGURE 1-43: Examples of Circuits to Avoid



Due to wire delays or other unforeseen problems, at times we end up with circuits where the clock edge reaches different flip-flops at different times. Consider the circuit in Figure 1-44, where the clock reaches the two flip-flops at slightly different times. Proper synchronous operation means that both flip-flops operate as if they receive the same clock. Despite the delay in the clock to the second flip-flop, its state change must be triggered before the new value of Q_1 reaches D_2. The maximum clock frequency for synchronous operation should be decided considering the delay between the clocks as well.

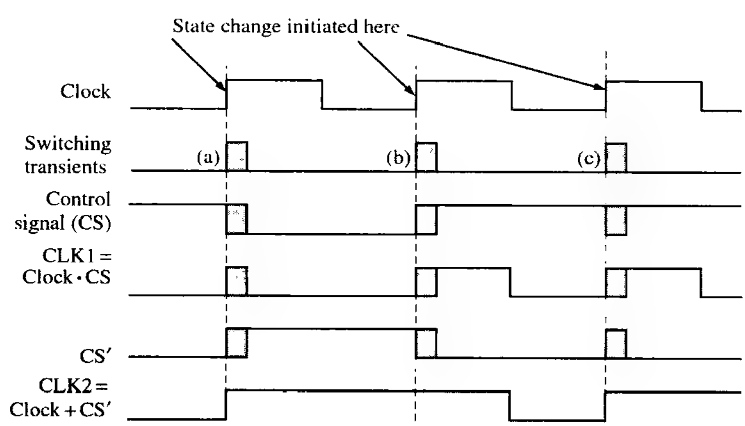
FIGURE 1-44: A Circuit with Clock Skew



If devices do not have enables and synchronous operation cannot be obtained without clock gating, we should pay attention to gate the clocks correctly. A device with negative edge triggering can be made to function correctly by ANDing the clock signal with the control signal, as in Figure 1-42(a). In the following paragraphs, we describe issues associated with control signal gating for positive edge triggered devices.

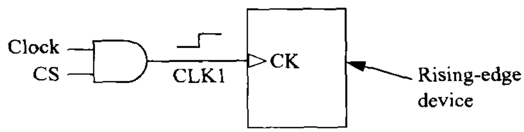
Figure 1-45 illustrates the operation of a digital system that uses devices that change state on the rising edge of the clock. In this case, the switching transients that result in noise and uncertainty will occur following the rising edge of the clock. The shading indicates the time interval in which the control signal CS may be noisy. If we want a device to change state on the rising edge of the clock when $CS = 1$, transition is expected at (a) and (c), but no change is expected at (b) since $CS = 0$ when the clock edge arrives. In order to create a gated control signal, it is tempting to

FIGURE 1-45: Timing Chart for System with Rising-Edge Devices

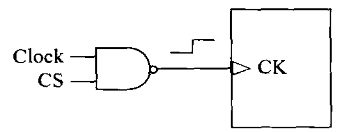


AND the clock with CS, as shown in Figure 1-46(a). The resulting signal, which goes to the CK input of the device, may be noisy and timed incorrectly. In particular, the $CLK1$ pulse at (a) will be short and noisy. It may be too short to trigger the device, or it may be noisy and trigger the device more than once. In general, it will be out of synchronization with the clock, because the control signal does not change until after some of the flip-flops in the control circuit have changed state. The rising edge of the pulse at (b) again will be out of synch with the clock, and it may be noisy. But even worse, the device will trigger near point (b) when it should not trigger there at all. Since $CS = 0$ at the time of the rising edge of the clock, triggering should not occur until the next rising edge, when $CS = 1$.

FIGURE 1-46: Incorrect Clock Gating for Rising-Edge Devices



(a) With AND gate



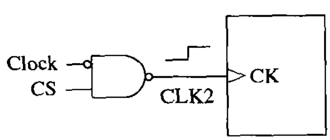
(b) With NAND gate

For a rising-edge device, if we changed the AND gate in Figure 1-42 to NAND gate as in Figure 1-46(b), it would be incorrect because the synchronization will happen at the wrong edge. The correct way to gate the control signal will be as in Figure 1-47, which will result in the CK input to the device having a positive edge only when the control signal is positive and clock is going to have a positive edge. The CK input is then

$$CLK2 = (CS \cdot clock')' = CS' + clock$$

The last waveform in Figure 1-45 illustrates this gated control signal. While this circuit can solve the synchronization problem, we encourage designers to refrain from gating clocks at all if possible.

FIGURE 1-47: Correct Control Signal Gating for Rising-Edge Device



In summary, synchronous design is based on the following principles:

- Method: All clock inputs to flip-flops, registers, counters, and so on are driven directly from the system clock.
- Result: All state changes occur immediately following the active edge of the clock signal.
- Advantage: All switching transients, switching noise, and so on occur between clock pulses and have no effect on system performance.

Asynchronous design is generally more difficult than synchronous design. Since there is no clock to synchronize the state changes, problems may arise when several state variables must change at the same time. A race occurs if the final

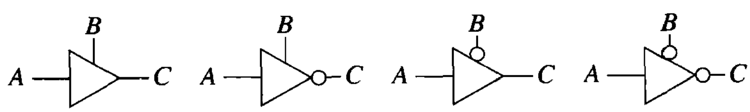
state depends on the order in which the variables change. Asynchronous design requires special techniques to eliminate problems with races and hazards. On the other hand, synchronous design has several disadvantages: In high-speed circuits where the propagation delay in the wiring is significant, the clock signal must be carefully routed so that it reaches all the clock inputs at essentially the same time (i.e., to minimize clock skew). The maximum clock rate is determined by the worst-case delay of the longest path. The system inputs may not be synchronized with the clock, so use of synchronizers may be required. Synchronous systems also consume more power than asynchronous systems. The clock distribution circuitry in synchronous chips often consumes a significant fraction of the chip power.

1.11 Tristate Logic and Busses

Normally, if we connect the outputs of two gates or flip-flops together, the circuit will not operate properly. It can also cause damage to the circuit. Hence, when we need to connect multiple gate outputs to the same wire or channel, one way to do that is by using tristate buffers. Tristate buffers are gates with a high impedance state (hi-Z) in addition to high and low logic states. The high impedance state is equivalent to an open circuit. In digital systems, transferring data back and forth between several system components is often necessary. Tristate busses can be used to facilitate data transfers between registers. When several gates are connected onto a wire, what we expect is that at any one point, one of the gates is going to actually drive the wire, and the other gates should behave as if they are not connected to the wire. The high impedance state achieves this.

Tristate buffers can be inverting or non-inverting. The control input can be active high or active low. Figure 1-48 shows four kinds of tristate buffers. B is the control input used to enable or disable the buffer output. When a buffer is enabled, the output (C) is equal to the input (A) or its complement. However, we can connect two tristate buffer outputs, provided that only one output is enabled at a time.

FIGURE 1-48: Four Kinds of Tristate Buffers



(a)

B	A	C
0	0	Hi-Z
0	1	Hi-Z
1	0	0
1	1	1

(b)

B	A	C
0	0	Hi-Z
0	1	Hi-Z
1	0	1
1	1	0

(c)

B	A	C
0	0	0
0	1	1
1	0	Hi-Z
1	1	Hi-Z

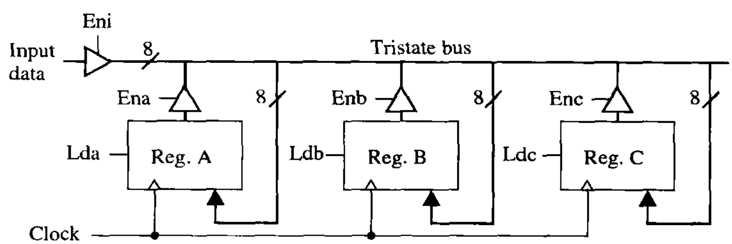
(d)

B	A	C
0	0	1
0	1	0
1	0	Hi-Z
1	1	Hi-Z

Figure 1-49 shows a system with three registers connected to a tristate bus. Each register is 8 bits wide, and the bus consists of 8 wires connected in parallel. Each tristate buffer symbol in the figure represents 8 buffers operating in parallel

with a common enable input. Only one group of buffers is enabled at a time. For example, if $Enb = 1$, the register B output is driven onto the bus. The data on the bus is routed to the inputs of register A, register B, and register C. However, data is loaded into a register only when its load input is 1 and the register is clocked. Thus, if $Enb = Ldc = 1$, the data in register B will be copied into register C when the active edge of the clock occurs. If $Eni = Lda = Ldb = 1$, the input data will be loaded in registers A and B when the registers are clocked.

FIGURE 1-49: Data Transfer Using Tristate Bus



1.12 Problems

1.1 Write out the truth table for the following equation.

$$F = (A \oplus B) \cdot C + A' \cdot (B' \oplus C)$$

1.2 A full subtracter computes the difference of three inputs X, Y, and B_{in}, where $Diff = X - Y - B_{in}$. When $X < (Y + B_{in})$, the borrow output B_{out} is set. Fill in the truth table for the subtracter and derive the sum-of-products and product-of-sums equations for $Diff$ and B_{out}.

1.3 Simplify Z using a four-variable map with map-entered variables. $ABCD$ represents the state of a control circuit. Assume that the circuit can never be in state 0100, 0001, or 1001.

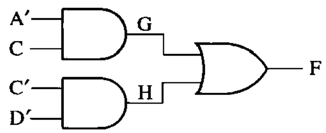
$$Z = BC'DE + ACDF' + ABCD'F' + ABC'D'G + B'CD + ABC'D'H'$$

1.4 For the following functions, find the minimum sum of products using four-variable maps with map-entered variables. In (a) and (b), m_i represents a minterm of variables A, B, C, and D.

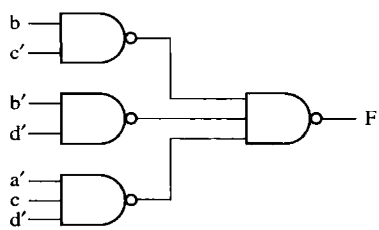
(a) $F(A, B, C, D, E) = \Sigma m(0, 4, 6, 13, 14) + \Sigma d(2, 9) + E(m_1 + m_{12})$
(b) $Z(A, B, C, D, E, F, G) = \Sigma m(2, 5, 6, 9) + \Sigma d(1, 3, 4, 13, 14) + E(m_{11} + m_{12}) + F(m_{10}) + G(m_0)$
(c) $H = A'B'CDF' + A'CD + A'B'CD'E + BCDF'$
(d) $G = C'E'F + DEF + AD'E'F' + BC'E'F + AD'EF'$

Hint: Which variables should be used for the map sides and which variables should be entered into the map?

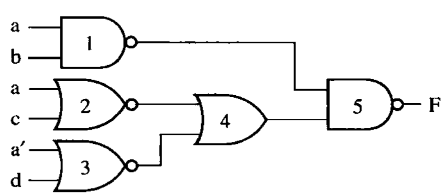
1.5 Identify the static 1-hazards in the following circuit. State the condition under which each hazard can occur. Draw a timing diagram (similar to Figure 1–10(b)) that shows the sequence of events when a hazard occurs.



1.6 Find all of the 1-hazards in the given circuit. Indicate what changes are necessary to eliminate the hazards.



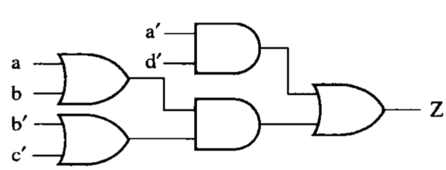
1.7 **(a)** Find all the static hazards in the following circuit. For each hazard, specify the values of the input variables and which variable is changing when the hazard occurs. For one of the hazards, specify the order in which the gate outputs must change.



(b) Design a NAND-gate circuit that is free of static hazards to realize the same function.

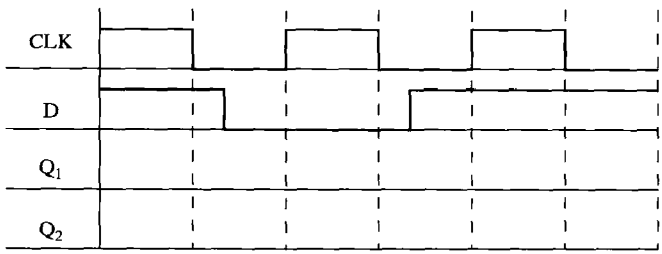
1.8 **(a)** Find all the static hazards in the following circuit. State the condition under which each hazard can occur.

(b) Redesign the circuit so that it is free of static hazards. Use gates with at most three inputs.



1.9 **(a)** Show how you can construct a T flip-flop using a J-K flip-flop.
(b) Show how you can construct a J-K flip-flop using a D flip-flop and gates.

1.10 Construct a clocked D flip-flop, triggered on the rising edge of CLK, using two transparent D latches and any necessary gates. Complete the following timing diagram, where Q_1 and Q_2 are latch outputs. Verify that the flip-flop output changes to D after the rising edge of the clock.



1.11 A synchronous sequential circuit has one input and one output. If the input sequence 0101 or 0110 occurs, an output of two successive 1's will occur. The first of these 1's should occur coincident with the last input of the 0101 or 0110 sequence. The circuit should reset when the second 1 output occurs. For example,

input sequence: $X = 0\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1 \ldots$
output sequence: $Z = 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1 \ldots$

(a) Derive a Mealy state graph and table with a minimum number of states (six states).
(b) Try to choose a good state assignment. Realize the circuit using J-K flip-flops and NAND gates. Repeat using NOR gates. (Work this part by hand.)
(c) Check your answer to (b) using the *LogicAid* program. Also use the program to find the NAND solution for two other state assignments.

1.12 A sequential circuit has one input (X) and two outputs (Z_1 and Z_2). An output $Z_1 = 1$ occurs every time the input sequence 010 is completed provided that the sequence 100 has never occurred. An output $Z_2 = 1$ occurs every time the input sequence 100 is completed. Note that once a $Z_2 = 1$ output has occurred, $Z_1 = 1$ can never occur, but *not* vice versa.

(a) Derive a Mealy state graph and table with a minimum number of states (eight states).
(b) Try to choose a good state assignment. Realize the circuit using J-K flip-flops and NAND gates. Repeat using NOR gates. (Work this part by hand.)
(c) Check your answer to (b) using the *LogicAid* program. Also use the program to find the NAND solution for two other state assignments.

1.13 A sequential circuit has one input (X) and two outputs (S and V). X represents a 4-bit binary number N, which is input least significant bit first. S represents a 4-bit binary number equal to $N + 2$, which is output least significant bit first. At the time the fourth input occurs, $V = 1$ if $N + 2$ is too large to be represented by 4 bits; otherwise, $V = 0$. The value of S should be the proper value, not a don't care, in both cases. The circuit always resets after the fourth bit of X is received.

(a) Derive a Mealy state graph and table with a minimum number of states (six states).
(b) Try to choose a good state assignment. Realize the circuit using D flip-flops and NAND gates. Repeat using NOR gates. (Work this part by hand.)
(c) Check your answer to (b) using the *LogicAid* program. Also use the program to find the NAND solution for two other state assignments.

1.14 A sequential circuit has one input (X) and two outputs (D and B). X represents a 4-bit binary number N, which is input least significant bit first. D represents a 4-bit binary number equal to $N - 2$, which is output least significant bit first. At the time the fourth input occurs, $B = 1$ if $N - 2$ is negative; otherwise, $B = 0$. The circuit always resets after the fourth bit of X is received.

(a) Derive a Mealy state graph and table with a minimum number of states (six states).
(b) Try to choose a good state assignment. Realize the circuit using J-K flip-flops and NAND gates. Repeat using NOR gates. (Work this part by hand.)
(c) Check your answer to (b) using the *LogicAid* program. Also use the program to find the NAND solution for two other state assignments.

1.15 A Moore sequential circuit has one input and one output. The output goes to 1 when the input sequence 111 has occurred and the output goes to 0 if the input sequence 000 occurs. At all other times, the output holds its value.
Example:

$$X = 0\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0$$
$$Z = 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0$$

Derive a Moore state graph and table for the circuit.

1.16 Derive the state transition table and flip-flop input equations for a modulo-6 counter that counts 000 through 101 and then repeats. Use J-K flip-flops.

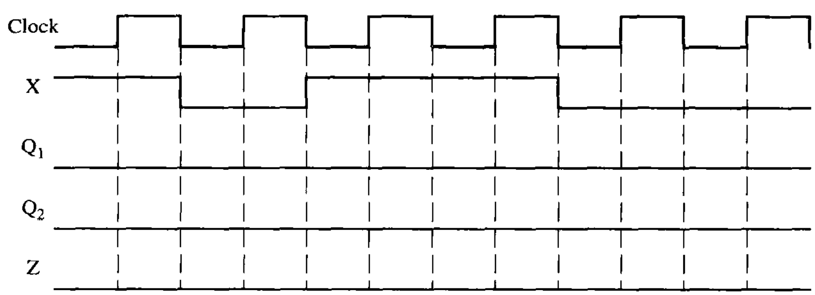
1.17 Derive the state transition table and D flip-flop input equations for a counter that counts from 1 to 6 and then repeats.

1.18 Reduce the following state table to a minimum number of states.

Present State	Next State $X=0$	Next State $X=1$	Output $X=0$	Output $X=1$
A	B	G	0	1
B	A	D	1	1
C	F	G	0	1
D	H	A	0	0
E	G	C	0	0
F	C	D	1	1
G	G	E	0	0
H	G	D	0	0

1.19 A Mealy sequential circuit is implemented using the circuit shown in Figure 1-44. Assume that if the input X changes, it changes at the same time as the falling edge of the clock.

(a) Complete the timing diagram below. Indicate the proper times to read the output (Z). Assume that "delay" is 0 ns and that the propagation delay for the flip-flop and XOR gate has a nominal value of 10 ns. The clock period is 100 ns.

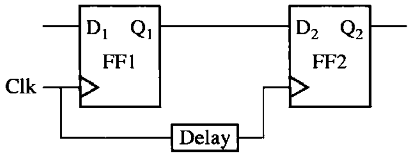


(b) Assume the following delays: XOR gate—10 to 20 ns, flip-flop propagation delay—5 to 10 ns, setup time—5 ns, and hold time—2 ns. Also assume that the "delay" is 0 ns. Determine the maximum clock rate for proper synchronous operation. Consider both the feedback path that includes the flip-flop propagation delay and the path starting when X changes.

(c) Assume a clock period of 100 ns. Also assume the same timing parameters as in (b). What is the maximum value that "delay" can have and still achieve proper synchronous operation? That is, the state sequence must be the same as for no delay.

1.20 Two flip-flops are connected as shown below. The delay represents wiring delay between the two clock inputs, which results in clock skew. This can cause possible loss of synchronization. The flip-flop propagation delay from clock to Q is 10 ns $< t_p <$ 15 ns; the setup and hold times are 4 ns and 2 ns, respectively.

(a) What is the maximum value that the delay can have and still achieve proper synchronous operation? Draw a timing diagram to justify your answer.



(b) Assuming that the delay is < 3 ns, what is the minimum allowable clock period?

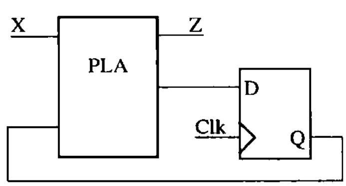
1.21 A D flip-flop has a propagation delay from clock to Q of 7 ns. The setup time of the flip-flop is 10 ns and the hold time is 5 ns. A clock with a period of 50 ns (low until 25 ns, high from 25 to 50 ns, and so on) is fed to the clock input of the flip-flop. Assume a two-level AND-OR circuitry between the external input signals and the flip-flop inputs. Assume gate delays are between 2 and 4 ns. The flip-flop is positive edge triggered.

(a) Assume the D input equals 0 from $t = 0$ until $t = 10$ ns, 1 from 10 until 35, 0 from 35 to 70, and 1 thereafter. Draw timing diagrams illustrating the clock, D, and Q until 100 ns. If outputs cannot be determined (because of not satisfying setup and hold times), indicate this by XX in the region.

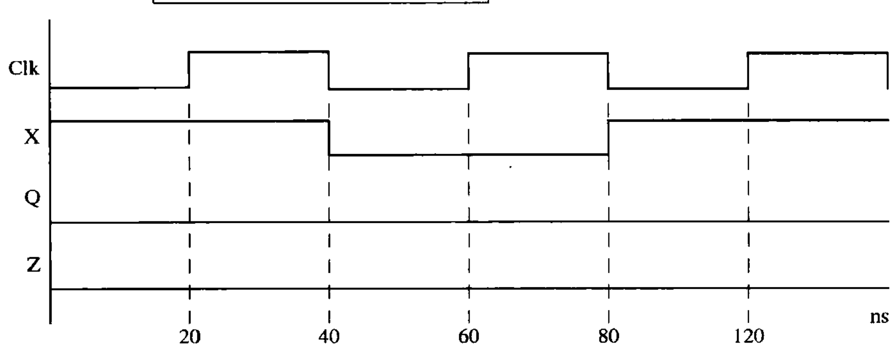
(b) The D input of the flip-flop should not change between __ ns before the clock edge and __ ns after the clock edge.

(c) External inputs should not change between __ ns before the clock edge and __ ns after the clock edge.

1.22 A sequential circuit consists of a PLA and a D flip-flop, as shown.



Q	0	1	0	1
0	0	1	0	1
1	0	0	1	0
	Q^+		Z	



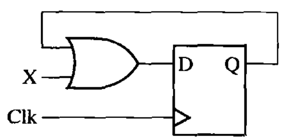
(a) Complete the timing diagram, assuming that the propagation delay for the PLA is in the range 5 to 10 ns, and the propagation delay from clock to output of the D flip-flop is 5 to 10 ns. Use cross-hatching on your timing diagram to indicate the intervals in which Q and Z can change, taking the range of propagation delays into account.

(b) Assuming that X always changes at the same time as the falling edge of the clock, what is the maximum setup and hold time specification that the flip-flop can have and still maintain proper operation of the circuit?

1.23 A D flip-flop has a propagation delay from clock to Q of 15 ns. The setup time of the flip-flop is 10 ns and the hold time is 2 ns. A clock with a period of 50 ns (low until 25 ns, high from 25 to 50 ns, and so on) is fed to the clock input of the flip-flop. The flip-flop is positive edge triggered. D goes up at 20, down at 40, up at 60, down at 80, and so on. Draw timing diagrams illustrating the clock, D, and Q until 100 ns. If outputs cannot be determined (because of not satisfying setup and hold times), indicate it by placing XX in that region.

1.24 A D flip-flop has a setup time of 5 ns, a hold time of 3 ns, and a propagation delay from the rising edge of the clock to the change in flip-flop output in the range of 6 to 12 ns. An OR gate delay is in the range of 1 to 4 ns.

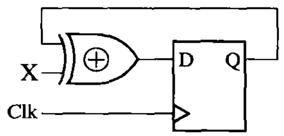
(a) What is the minimum clock period for proper operation of the following circuit?



(b) What is the earliest time after the rising clock edge that X is allowed to change?

(c) Show how you can construct a T flip-flop using a J-K flip-flop using a block diagram. Circuits inside the flip-flops are NOT to be shown.

1.25 In the following circuit, the XOR gate has a delay in the range of 2 to 16 ns. The D flip-flop has a propagation delay from clock to Q in the range 12 to 24 ns. The setup time is 8 ns, and the hold time is 4 ns.



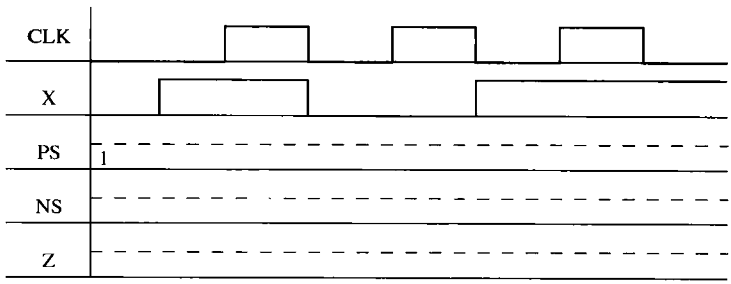
(a) What is the minimum clock period for proper operation of the circuit?

(b) What are the earliest and latest times after the rising clock edge that X is allowed to change and still have proper synchronous operation? (Assume minimum clock period from (a).)

1.26 A Mealy sequential machine has the following state table:

	NS		Z	
PS	$X = 0$	$X = 1$	$X = 0$	$X = 1$
1	2	3	0	1
2	3	1	1	0
3	2	2	1	0

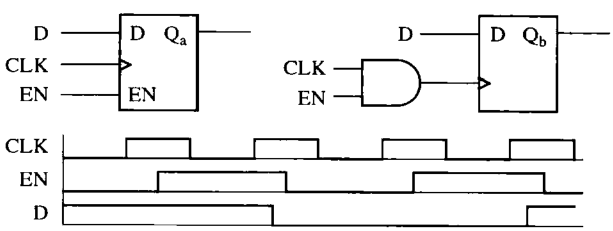
Complete the following timing diagram. Clearly mark on the diagram the times at which you should read the values of Z. All state changes occur after the rising edge of the clock.



1.27 **(a)** Do the following two circuits have essentially the same timing?

(b) Draw the timing for Q_a and Q_b given the timing diagram.

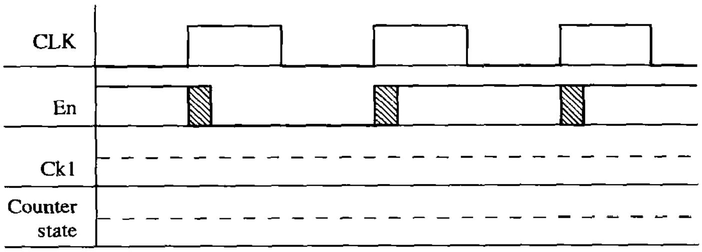
(c) If your answer to (a) is no, show what change(s) should be made in the second circuit so that the two circuits have essentially the same timing (do not change the flip-flop).



1.28 A simple binary counter has only a clock input ($CK1$). The counter increments on the rising edge of $CK1$.

(a) Show the proper connections for a signal En and the system clock (CLK), so that when $En = 1$, the counter increments on the rising edge of CLK and when $En = 0$, the counter does not change state.

(b) Complete the following timing diagram. Explain, in terms of your diagram, why the switching transients that occur on *En* after the rising edge of *CLK* do not affect the proper operation of the counter.



1.29 Referring to Figure 1-49, specify the values of *Eni*, *Ena*, *Enb*, *Enc*, *Lda*, *Ldb*, and *Ldc* so that the data stored in Reg. C will be copied into Reg. A and Reg. B when the circuit is clocked.

CHAPTER 2

Introduction to VHDL

As integrated circuit technology has improved to allow more and more components on a chip, digital systems have continued to grow in complexity. While putting a few transistors on an integrated circuit (IC) was a miracle when it happened, technology improvements have advanced the **VLSI** (very large scale integration) field continually. The early integrated circuits belonged to **SSI** (small scale integration), **MSI** (medium scale integration), or **LSI** (large scale integration) categories depending on the density of integration. SSI referred to ICs with 1 to 20 gates, MSI referred to ICs with 20 to 200 gates, and LSI referred to devices with 200 to a few thousand gates. Many popular building blocks, such as adders, multiplexers, decoders, registers, and counters, are available as MSI standard parts. When the term *VLSI* was coined, devices with 10,000 gates were called VLSI chips. The boundaries between the different categories are fuzzy today. Many modern microprocessors contain more than 100 million transistors. Compared to what was referred to as VLSI in its initial days, modern integration capability could be described as ULSI (ultra large scale integration). Despite the changes in integration ability and the fuzzy definition, the term *VLSI* remains popular, while terms like *LSI* are not practically used any more.

As digital systems have become more complex, detailed design of the systems at the gate and flip-flop level has become very tedious and time-consuming. Two or three decades ago, digital systems were created using hand-drawn schematics, bread-boards, and wires that were connected to the bread-board. Now, hardware design often involves no hands-on tasks with bread-boards and wires.

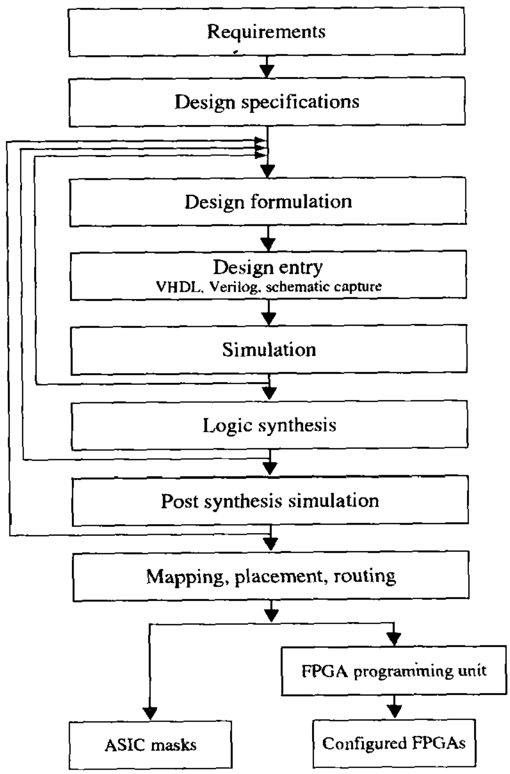
In this chapter, first we present an introduction to computer-aided design. Then we present an introduction to hardware description languages. Basic features of VHDL are presented and examples are presented to illustrate how digital hardware is described, simulated, and synthesized using VHDL. Advanced features of VHDL are presented in Chapter 8.

2.1 Computer-Aided Design

Computer-aided design (CAD) tools have advanced significantly in the past decade, and nowadays, digital design is performed using a variety of software tools. Prototypes or even final designs can be created without discrete components and interconnection wires.

Figure 2-1 illustrates the steps in modern digital system design. Like any engineering design, the first step in the design flow is formulating the problem, stating the **design requirements** and arriving at the **design specification.** The next step is to **formulate the design** at a conceptual level, either at a block diagram level or at an algorithmic level.

FIGURE 2-1: Design Flow in Modern Digital System Design



Design entry is the next step in the design flow. In olden days, this would have been a hand-drawn schematic or blueprint. Now with CAD tools, the design conceptualized in the previous step needs to be entered into the CAD system in an appropriate manner. Designs can be entered in multiple forms. A few years ago, CAD tools used to provide a graphical method to enter designs. This was called **schematic capture.** The schematic editors typically were supplemented with a library of standard digital building blocks like gates, flip-flops, multiplexers, decoders, counters, registers, and so on. ORCAD (a company that produced design automation tools) provided a very popular schematic editor. Nowadays, **hardware description languages** (HDLs) are used to enter designs. Two popular HDLs are VHDL and Verilog. The acronym VHDL stands for **VHSIC hardware description language**, and **VHSIC** in turn stands for **very high speed integrated circuit**.

A hardware description language allows a digital system to be designed and debugged at a higher level of abstraction than schematic capture with gates, flip-flops, and standard MSI building blocks. The details of the gates and flip-flops do not need to be handled during early phases of design. A design can be entered in what is called a **behavioral description** of the design. In a behavioral HDL description, one only specifies the general working of the design at a flow-chart or algorithmic level without associating to any specific physical parts, components, or implementations. Another method to enter a design in VHDL and Verilog is the **structural description** entry. In structural design, specific components or specific implementations of components are associated with the design. A structural VHDL or Verilog model of a design can be considered as a textual description of a schematic diagram that you would have drawn interconnecting specific gates and flip-flops.

Once the design has been entered, it is important to simulate it to confirm that the conceptualized design does function correctly. Initially, one should perform the **simulation** at the high-level behavioral model. This early simulation unveils problems in the initial design. If problems are discovered, the designer goes back and alters the design to meet the requirements.

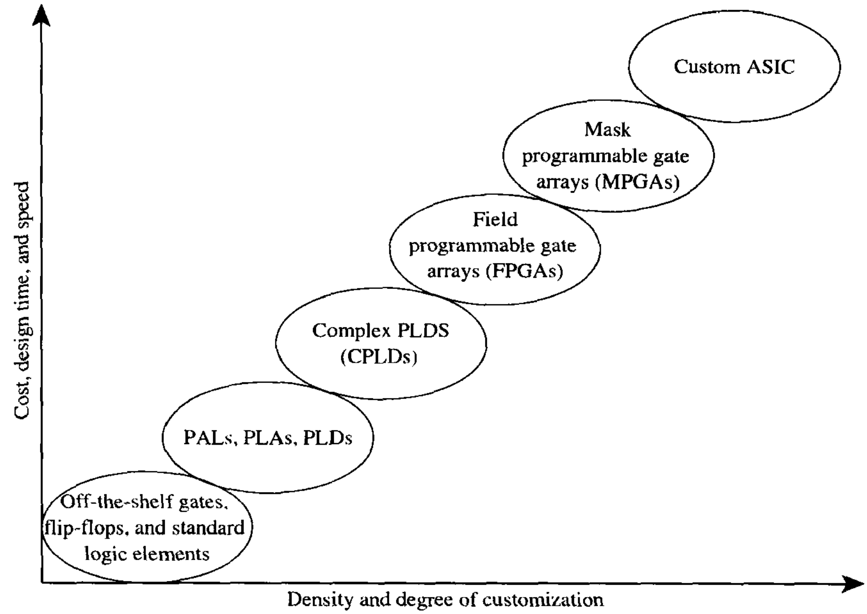
Once the functionality of the design has been verified through simulation, the next step is **synthesis**. *Synthesis* means "conversion of the higher-level abstract description of the design to actual components at the gate and flip-flop level." Use of computer-aided design tools to do this conversion (a.k.a. synthesis) is becoming widespread. The output of the synthesis tool, consisting of a list of gates and a list of interconnections specifying how to interconnect them, is often referred to as a **netlist.** Synthesis is analogous to writing software programs in a high-level language such as C and then using a compiler to convert the programs to machine language. Just like a C compiler can generate optimized or unoptimized machine code, a synthesis tool can generate optimized or unoptimized hardware. The synthesis software generates different hardware implementations depending on algorithms embedded in the software to perform the translation and optimization techniques incorporated into the tool. A synthesis tool is nothing but a compiler to convert design descriptions to hardware, and it is not unusual to name synthesis packages with phrases similar to design compiler, silicon compiler, and so on.

The next step in the design flow is **post-synthesis simulation.** The earlier simulation at a higher level of abstraction does not take into account specific implementations of the hardware components that the design is using. If post-synthesis simulation unveils problems, one should go back and modify the design to meet timing requirements. Arriving at a proper design implementation is an iterative process.

Next, a designer moves into specific realizations of the design. A design can be implemented in several different target technologies. It could be a completely custom IC or it could be implemented in a standard part that is easily available from a vendor. The target technologies that are commonly available now are illustrated in Figure 2-2.

At the lowest level of sophistication and density is an old-fashioned printed circuit board with off-the-shelf gates, flip-flops, and other standard logic building blocks. Slightly higher in density are programmable logic arrays (PLAs), programmable array logic (PAL), and simple programmable logic devices (SPLDs). PLDs

FIGURE 2-2: Spectrum of Design Technologies



with higher density and gate count are called complex programmable logic devices (CPLDs). Then there are the popular field programmable gate arrays (FPGAs) and mask programmable gate arrays (MPGAs), or simply gate arrays. The highest level of density and performance is a fully custom application-specific integrated circuit (ASIC).

Two most common target technologies nowadays are FPGAs and ASICs. The initial steps in the design flow are largely the same for either realization. Toward the final stages in the design flow, different operations are performed depending on the target technology. This is indicated in Figure 2-1. The design is **mapped** into specific target technology and **placed** into specific parts in the target ASIC or FPGA. The paths taken by the connections between components are decided during the **routing**. If an ASIC is being designed, the routed design is used to generate a photomask that will be used in the IC manufacturing process. If a design is to be implemented in an FPGA, the design is translated to a format specifying what is to be done to various programmable points in the FPGA. In modern FPGAs, programming simply involves writing a sequence of 0's and 1's into the programmable cells in the FPGA, and no specific programming unit other than a personal computer (PC) is required.

2.2 Hardware Description Languages

Hardware description languages (HDLs) are a popular mode of design entry. As mentioned previously, two popular HDLs are VHDL and Verilog. This book uses VHDL for illustrating principles of modern digital system design.

VHDL is a hardware description language used to describe the behavior and structure of digital systems. VHDL is a general-purpose HDL that can be used to describe and simulate the operation of a wide variety of digital systems, ranging in complexity from a few gates to an interconnection of many complex integrated circuits. VHDL was originally developed under funding from the Department of Defense (DoD) to allow a uniform method for specifying digital systems. When VHDL was developed, the main purpose was to have a mechanism to describe and document hardware unambiguously. Synthesizing hardware from high-level descriptions was not one of the original purposes. The VHDL language has since become an IEEE (Institute of Electronic and Electrical Engineers) standard, and it is widely used in industry. IEEE created a VHDL standard in 1987 (VHDL-87) and later modified the standard in 1993 (VHDL-93). Further revisions were done to the standard in 2000 and 2002.

VHDL can describe a digital system at several different levels—**behavioral, data flow**, and **structural**. For example, a binary adder could be described at the behavioral level in terms of its function of adding two binary numbers without giving any implementation details. The same adder could be described at the data flow level by giving the logic equations for the adder. Finally, the adder could be described at the structural level by specifying the gates and the interconnections between the gates that comprise the adder.

VHDL leads naturally to a top-down design methodology, in which the system is first specified at a high level and tested using a simulator. After the system is debugged at this level, the design can gradually be refined, eventually leading to a structural description closely related to the actual hardware implementation. VHDL was designed to be technology independent. If a design is described in VHDL and implemented in today's technology, the same VHDL description could be used as a starting point for a design in some future technology. Although initially conceived as a hardware documentation language, most of VHDL can now be used for simulation and logic synthesis.

Verilog is another popular HDL. It was developed by the industry at about the same time the U.S. DoD was funding the creation of VHDL. Verilog was introduced by Gateway Design Automation in 1984 as a proprietary HDL. Synopsis created synthesis tools for Verilog around 1988. Verilog became an IEEE standard in 1995.

VHDL has its syntactic roots in ADA while Verilog has its syntactic roots in C. ADA was a general-purpose programming language, also sponsored by the Department of Defense. Due to the similarity with C, some find Verilog easier or less intimidating to learn. Many find VHDL to be excellent for supporting design and documentation of large systems. VHDL and Verilog enjoy approximately 50/50 market share. Both languages can accomplish most requirements for digital design rather easily. Often design companies continue to use what they are used to, and hence, Verilog users continue to use Verilog and VHDL users continue to use VHDL. If you know one of these languages, it is not difficult to transition to the other.

More recently, there also have been efforts in system design languages such as **System C, Handel-C**, and **System Verilog**. System C is created as an extension to C++, and hence some who are very comfortable with general-purpose software

development find it less intimidating. These languages are primarily targeted at describing large digital systems at a high level of abstraction. They are primarily used for verification and validation. When different parts of a large system are designed by different teams, one team can use a system level behavioral description of the block being designed by the other team during initial design. Problems that might otherwise become obvious only during system integration may become evident in early stages reducing the design cycle for large systems. System-level simulation languages are used during design of large systems.

2.2.1 Learning a Language

There are several challenges when you learn a new language, whether it be a language for common communication (English, Spanish, French, etc.), a computer language like C, or a special-purpose language such as VHDL. If it is not your first language, you typically have a tendency to compare it to a language you know. In the case of VHDL, if you already know another hardware description language, it is good to compare it with VHDL, but you should be careful when comparing it with languages like C. VHDL and Verilog have a very different purpose than languages like C, and a comparison with C is not a meaningful activity. We will be describing the language assuming it is your first HDL; however, we will assume basic knowledge of computer languages like C and the basic compilation and execution flow.

When one learns a new language, one needs to study the alphabet of the new language, its vocabulary, grammar, syntax rules, and semantics of language descriptions. The process of learning VHDL is not much different. One needs to learn the alphabet, vocabulary or lexical elements of the language, syntax (grammar and rules), and semantics (meaning of descriptions). VHDL-87 uses the ASCII character set while VHDL-93 allows use of the full ISO character set. The ISO character set includes the ASCII characters and additionally includes accented characters. The ASCII character set only includes the first 128 characters of the ISO character set. The lexical elements of the language include various **identifiers, reserved words**, special symbols, and literals. We have listed these in Appendix A. The syntax or grammar determines what combinations of lexical elements can be combined to make valid VHDL descriptions. These are the rules that govern the use of different VHDL constructs. Then one needs to understand the semantics or meaning of VHDL descriptions. It is here that one understands what descriptions represent combinational hardware versus sequential hardware. And just like fluency in a natural language comes by speaking, reading, and writing the language, mastery of VHDL comes by repeated use of the language to create models for various digital systems.

Since VHDL is a hardware description language, it differs from an ordinary programming language in several ways. Most importantly, VHDL has statements that execute concurrently since they must model real hardware in which the components are all in operation at the same time. VHDL is popularly used for the purposes of describing, documenting, simulating, and automatically generating hardware. Hence, its constructs are tailored for these purposes. We will present the various methods to model different kinds of digital hardware using examples in the following sections.

Common Abbreviations

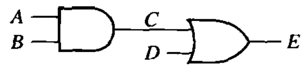
VHDL:	VHSIC hardware description language
VHSIC:	Very high speed integrated circuit
HDL:	Hardware description language
CAD:	Computer-aided design
EDA:	Electronic design automation
LSI:	Large scale integration
MSI:	Medium scale integration
SSI:	Small scale integration
VLSI:	Very large scale integration
ULSI:	Ultra large scale integration
ASCII:	American standard code for information exchange
ISO:	International Standards Organization
ASIC:	Application-specific integrated circuit
FPGA:	Field programmable gate array
PLA:	Programmable logic array
PAL:	Programmable array logic
PLD:	Programmable logic device
CPLD:	Complex programmable logic device

2.3 VHDL Description of Combinational Circuits

The biggest difficulty in modeling hardware using a general-purpose computer language is representing concurrently operating hardware. Computer programs that you are normally accustomed to are sequences of instructions with a well-defined order. At any point of time during execution, the program is at a specific point in its flow and it encounters and executes different parts of the program sequentially. In order to model combinational circuits, which have several gates (all of which are working simultaneously), one needs to be able to "simulate" the execution of several parts of the circuit at the same time.

VHDL models combinational circuits by what are called **concurrent statements**. Concurrent statements are statements which are always ready to execute. These are statements which get evaluated any time and every time a signal on the right side of the statement changes.

We will start by describing a simple gate circuit in VHDL. If each gate in the circuit of Figure 2-3 has a 5-ns propagation delay, the circuit can be described by two VHDL statements as shown, where *A*, *B*, *C*, *D*, and *E* are signals. A signal in VHDL usually corresponds to a signal in a physical system. The symbol "<=" is the signal




```
C <= A and B after 5 ns;
E <= C or D after 5 ns;
```

FIGURE 2-3: A Simple Gate Circuit

assignment operator, which indicates that the value computed on the right side is assigned to the signal on the left side. When the statements in Figure 2-3 are simulated, the first statement will be evaluated anytime A or B changes, and the second statement will be evaluated anytime C or D changes. Suppose that initially $A = 1$ and $B = C = D = E = 0$. If B changes to 1 at time 0, C will change to 1 at time = 5 ns. Then E will change to 1 at time = 10 ns.

VHDL signal assignment statements, like the ones in the preceding example, are examples of concurrent statements. The VHDL simulator monitors the right side of each concurrent statement, and anytime a signal changes, the expression on the right side is immediately re-evaluated. The new value is assigned to the signal on the left side after an appropriate delay. This is exactly the way the hardware works. Anytime a gate input changes, the gate output is recomputed by the hardware, and the output changes after the gate delay. The location of the concurrent statement in the program is not important.

When we initially describe a circuit, we may not be concerned about propagation delays. If we write

```
C <= A and B;
E <= C or D;
```

this implies that the propagation delays are 0 ns. In this case, the simulator will assume an infinitesimal delay referred to as Δ (delta). Assume that initially $A = 1$ and $B = C = D = E = 0$. If B is changed to 1 at time = 1 ns, then C will change at time $1 + \Delta$ and E will change at time $1 + 2\Delta$.

Unlike a sequential program, the order of the preceding concurrent statements is unimportant. If we write

```
E <= C or D;
C <= A and B;
```

the simulation results would be exactly the same as before.

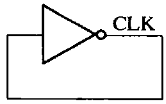
In general, a signal assignment statement has the form

```
signal_name <= expression [after delay];
```

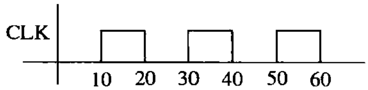
The expression is evaluated when the statement is executed, and the signal on the left side is scheduled to change after `delay`. The square brackets indicate that **`after`** `delay` is optional; they are not part of the statement. If **`after`** `delay` is omitted, then the signal is scheduled to be updated after a delta delay. Note that the time at which the statement executes and the time at which the signal is updated are not the same.

Even if a VHDL program has no explicit loops, concurrent statements may execute repeatedly as if they were in a loop. Figure 2-4 shows an inverter with the output connected back to the input. If the output is '0', then this '0' feeds back to the input and the inverter output changes to '1' after the inverter delay, assumed

FIGURE 2-4: Inverter with Feedback




```
CLK <= not CLK after 10 ns;
```



to be 10 ns. Then the '1' feeds back to the input and the output changes to '0' after the inverter delay. The signal *CLK* will continue to oscillate between '0' and '1' as shown in the waveform. The corresponding concurrent VHDL statement will produce the same result. If *CLK* is initialized to '0', the statement executes and *CLK* changes to '1' after 10 ns. Since *CLK* has changed, the statement executes again, and *CLK* will change back to '0' after another 10 ns. This process will continue indefinitely.

The statement in Figure 2-4 generates a clock waveform with a half period of 10 ns. On the other hand, the concurrent statement

```
CLK <= not CLK;
```

will cause a run-time error during simulation. Since there is 0 delay, the value of CLK will change at times $0 + \Delta$, $0 + 2\Delta$, $0 + 3\Delta$, and so on. Since Δ is an infinitesimal time, time will never advance to 1 ns.

In general, **VHDL is not case sensitive**; that is, uppercase and lowercase letters are treated the same by the compiler and by the simulator. Thus, the statements

```
Clk <= NOT clk After 10 ns;
```

and

```
CLK <= not CLK after 10 ns;
```

would be treated exactly the same. Signal names and other **VHDL identifiers** may contain letters, numbers, and the underscore character (_). An identifier must start with a letter, and it cannot end with an underscore. Thus C123 and ab_23 are legal identifiers, but 1ABC and ABC_ are not. Every VHDL statement must be terminated with a semicolon. Spaces, tabs, and carriage returns are treated in the same way. This means that a VHDL statement can be continued over several lines, or several statements can be placed on one line. In a line of VHDL code, anything following a double dash (--) is treated as a comment. Words such as **and, or,** and **after** are reserved words (or keywords) which have a special meaning to the VHDL compiler. In this text, we will put all reserved words in boldface type.

Figure 2-5 shows three gates that have the signal *A* as a common input and the corresponding VHDL code. The three concurrent statements execute simultaneously whenever *A* changes, just as the three gates start processing the signal change at the same time. However, if the gates have different delays, the gate outputs can change at different times. If the gates have delays of 2 ns, 1 ns, and 3 ns, respectively, and *A* changes at time 5 ns, then the gate outputs *D, E,* and *F* can change at times 7 ns, 6 ns,

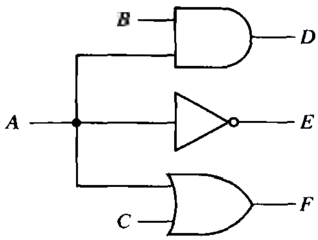


FIGURE 2-5: Three Gates with a Common Input and Different Delays

```
-- when A changes, these concurrent
-- statements all execute at the
-- same time
  D <= A and B after 2 ns;
  E <= not A after 1 ns;
  F <= A or C after 3 ns;
```

and 8 ns, respectively. The VHDL statements work in the same way. Even though the statements execute simultaneously, the signals D, E, and F are updated at times 7 ns, 6 ns, and 8 ns. However, if no delays were specified, then D, E, and F would all be updated at time $5 + \Delta$.

In the preceding examples, every signal is of type bit, which means it can have a value of '0' or '1'. (Bit values in VHDL are enclosed in single quotes to distinguish them from integer values.)

In digital design, we often need to perform the same operation on a group of signals. A one-dimensional array of bit signals is referred to as a **bit-vector**. If a 4-bit vector named B has an index range 0 through 3, then the four elements of the bit-vector are designated $B(0)$, $B(1)$, $B(2)$, and $B(3)$. One can declare a bit-vector using a statement such as:

```
B: in bit_vector(3 downto 0);
```

The statement `B <= "1100"` assigns '1' to $B(3)$, '1' to $B(2)$, '0' to $B(1)$, and '0' to $B(0)$.

Figure 2-6 shows an array of four AND gates. The inputs are represented by bit-vectors A and B, and the output by bit-vector C. Although we can write four VHDL statements to represent the four gates, it is much more efficient to write a single VHDL statement that performs the **and** operation on the bit-vectors A and B. When applied to bit-vectors, the **and** operator performs the **and** operation on corresponding pairs of elements.

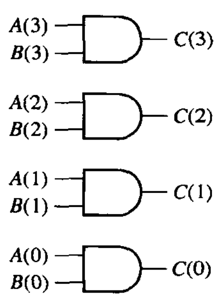


FIGURE 2-6: Array of AND Gates

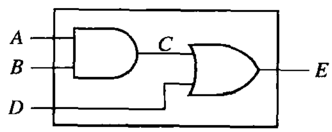
```
-- the hard way
C(3) <= A(3) and B(3);
C(2) <= A(2) and B(2);
C(1) <= A(1) and B(1);
C(0) <= A(0) and B(0);
```

```
-- the easy way assuming C, A and
-- B are 4-bit bit-vectors

C <= A and B;
```

2.4 VHDL Modules

The general structure of a VHDL module is an **entity** description and an **architecture** description. The **entity** description declares the input and output signals, and the **architecture** description specifies the internal operation of the module. As an example, consider Figure 2-7. The **entity** declaration gives the name two_gates to the module. The **port** declaration specifies the inputs and outputs to the module. *A*, *B*, and *D* are input signals of type bit, and *E* is an output signal of type bit. The architecture is named gates. The signal *C* is declared within the architecture since it is an internal signal. The two concurrent statements that describe the gates are placed between the keywords **begin** and **end**.

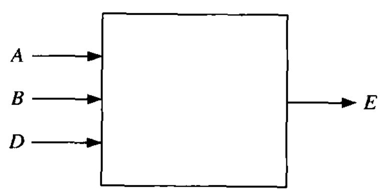
FIGURE 2-7: VHDL Module with Two Gates




```
entity two_gates is
  port(A, B, D: in bit; E: out bit);
end two_gates;

architecture gates of two_gates is
signal C: bit;
begin
  C <= A and B; -- concurrent
  E <= C or D;  -- statements
end gates;
```

The **entity** description can be considered as the black box picture of the module being designed and its external interface (i.e., it represents the interconnections from this module to the external world, as in Figure 2-8).

FIGURE 2-8: Black Box View of the Two-Gate Module



Just as in the preceding simple example, when we describe a system in VHDL, we must specify an entity and architecture at the top level and also specify an entity and architecture for each of the component modules that are part of the system (see Figure 2-9). Each entity declaration includes a list of interface signals that can be used to connect to other modules or to the outside world. We will use entity declarations of the form

```
entity entity-name is
  [port(interface-signal-declaration);]
end [entity] [entity-name];
```

The items enclosed in square brackets are optional. The `interface-signal-declaration` normally has the following form:

```
list-of-interface-signals: mode type [:= initial-value]
{; list-of-interface-signals: mode type [:= initial-value]};
```

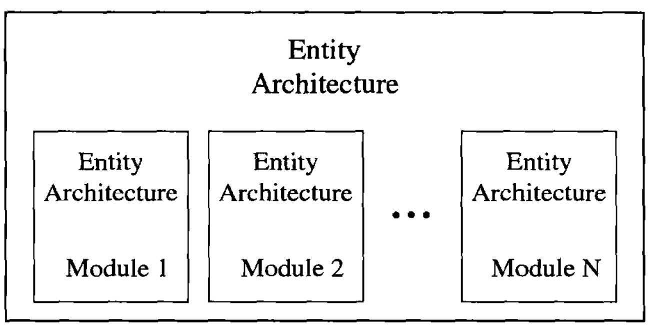
The curly brackets indicate zero or more repetitions of the enclosed clause. **Mode** indicates the direction of information; whether information is flowing into the port or out of it. Input port signals are of mode **in**, output port signals are of mode **out**, and bidirectional signals are of mode **inout**. **Type** specifies the data type or kind of information that can be communicated. So far, we have only used type bit and bit-vector; other types are described in Section 2.10. The optional `initial-value` is used to initialize the signals on the associated list; otherwise, the default initial value is used for the specified type. For example, the port declaration

```
port(A, B: in integer := 2; C, D: out bit);
```

indicates that A and B are input signals of type integer that are initially set to 2, and C and D are output signals of type bit that are initialized by default to '0'. These initial values are significant only for simulation and not for synthesis.

In addition to **in, out** and **inout** modes, there are two other modes: **buffer** and **linkage**. The **buffer** mode is similar to **inout** mode, in that it can be read and written into in the entity. The **buffer** mode is useful if a signal is truly an output, but we would like to read the ports internally as well. A linkage port is useful when VHDL entities are connected to non-VHDL entities. Both of these modes involve several restrictions and we generally restrict ourselves to **in**, **out** and **inout** modes.

FIGURE 2-9: VHDL Program Structure



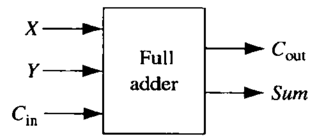
Associated with each **entity** is one or more **architecture** declarations of the form

```
architecture architecture-name of entity-name is
  [declarations]
begin
  architecture body
end [architecture] [architecture-name];
```

In the `declarations` section, we can declare signals and components that are used within the architecture. The architecture body contains statements that describe the operation of the module.

Next, we will write the entity and architecture for a full adder module. A full adder adds 2 bits and a carry input to generate a sum bit and a carry output bit. The entity specifies the inputs and outputs of the adder module as shown in Figure 2-10. The port declaration specifies that X, Y, and C_{in} are input signals of type bit, and that C_{out} and Sum are output signals of type bit.

FIGURE 2-10: Entity Declaration for a Full Adder Module




```
entity FullAdder is
  port(X, Y, Cin: in bit;        --Inputs
       Cout, Sum: out bit);      --Outputs
end FullAdder;
```

The operation of the full adder is specified by an architecture declaration:

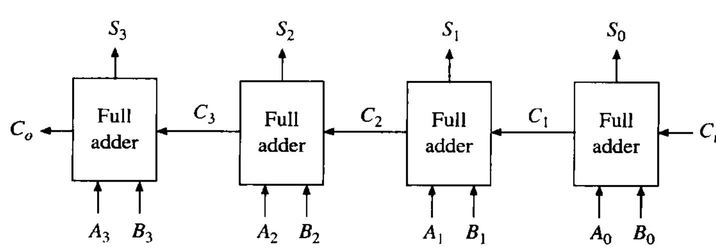
```
architecture Equations of FullAdder is
begin        -- concurrent assignment statements
  Sum  <= X xor Y xor Cin after 10 ns;
  Cout <= (X and Y) or (X and Cin) or (Y and Cin) after 10 ns;
end Equations;
```

In this example, the architecture name (`Equations`) is arbitrary, but the entity name (`FullAdder`) must match the name used in the associated entity declaration. The VHDL assignment statements for *Sum* and C_{out} represent the logic equations for the full adder. Several other architectural descriptions, such as a truth table or an interconnection of gates, could have been used instead. In the C_{out} equation, parentheses are required around (X **and** Y) since VHDL does not specify an order of precedence for the logic operators except the NOT operator.

2.4.1 Four-Bit Full Adder

Next, we will show how to use the `FullAdder` module defined above as a **component** in a system, which consists of four full adders connected to form a 4-bit binary adder (see Figure 2-11). We first declare the 4-bit adder as an entity (see Figure 2-12). Since the inputs and the sum output are 4 bits wide, we declare them as bit-vectors which are dimensioned 3 **downto** 0. (We could have used a range 1 **to** 4 instead).

FIGURE 2-11: Four-Bit Binary Adder



Next, we specify the FullAdder as a component within the architecture of Adder4 (Figure 2-12). The **component** specification is very similar to the **entity** declaration for the full adder, and the input and output port signals correspond to those declared for the full adder. Anytime a module created in one part of the code has to be used in another part, a component declaration needs to be used. The component declaration does not need to be in the same file where you are using the component. It can be where the component entity and architecture are defined. It is typical to create libraries of components for reuse in code, and typically the component declarations are placed in the library file.

Following the component statement, we declare a 3-bit internal carry signal C. In the body of the architecture, we create several instances of the FullAdder component. (In CAD jargon, we "instantiate" four copies of the FullAdder.) Each copy of FullAdder has a name (such as FA0) and a port map. The signal names following the port map correspond one-to-one with the signals in the component port. Thus, $A(0)$, $B(0)$, and C_i correspond to the inputs X, Y, and C_{in}, respectively. $C(1)$ and $S(0)$ correspond to the C_{out} and Sum outputs. Note that the order of the signals in the port map must be the same as the order of the signals in the port of the component declaration.

FIGURE 2-12: Structural Description of a 4-Bit Adder

```
entity Adder4 is
  port(A, B: in bit_vector(3 downto 0); Ci: in bit; -- Inputs
       S: out bit_vector(3 downto 0); Co: out bit); -- Outputs
end Adder4;
architecture Structure of Adder4 is
component FullAdder
  port (X, Y, Cin: in bit;        -- Inputs
        Cout, Sum: out bit);      -- Outputs
end component;
signal C: bit_vector(3 downto 1); -- C is an internal signal
begin     --instantiate four copies of the FullAdder
  FA0: FullAdder port map (A(0), B(0), Ci, C(1), S(0));
  FA1: FullAdder port map (A(1), B(1), C(1), C(2), S(1));
  FA2: FullAdder port map (A(2), B(2), C(2), C(3), S(2));
  FA3: FullAdder port map (A(3), B(3), C(3), Co, S(3));
end Structure;
```

In preparation for simulation, we can place the entity and architecture for the FullAdder and for Adder4 together in one file and compile. Alternatively, we could compile the FullAdder separately and place the resulting code in a library which is linked in when we compile Adder4.

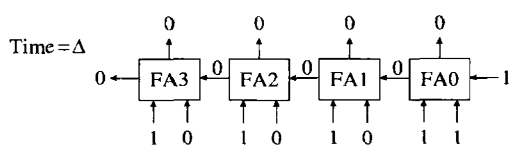
All of the simulation examples in this text use the **ModelSim VHDL simulator** from Mentor Graphics. Most other VHDL simulators use similar command files and can produce output in a similar format. We will use the following simulator commands to test Adder4:

```
add list A B Co C Ci S  -- put these signals on the output list
force A 1111            -- set the A inputs to 1111
force B 0001            -- set the B inputs to 0001
force Ci 1              -- set Ci to 1
run 50 ns               -- run the simulation for 50 ns
force Ci 0
force A 0101
force B 1110
run 50 ns
```

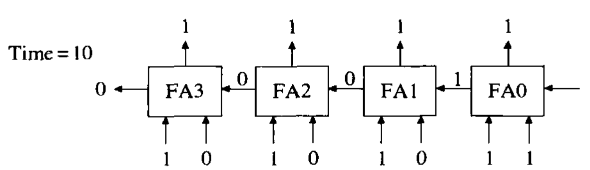
We have chosen to run the simulation for 50 ns since this is more than enough time for the carry to propagate through all of the full adders. The simulation results for the preceding command list are as follows:

ns	delta	a	b	co	c	ci	s
0	+0	0000	0000	0	000	0	0000
0	+1	1111	0001	0	000	1	0000
10	+0	1111	0001	0	001	1	1111
20	+0	1111	0001	0	011	1	1101
30	+0	1111	0001	0	111	1	1001
40	+0	1111	0001	1	111	1	0001
50	+0	0101	1110	1	111	0	0001
60	+0	0101	1110	1	110	0	0101
70	+0	0101	1110	1	100	0	0111
80	+0	0101	1110	1	100	0	0011

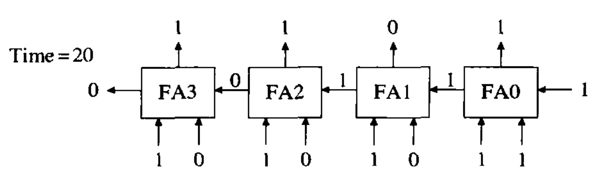
The listing shows how the carry propagates one position every 10 ns. The full adder inputs change at time = Δ:



The sum and carry are computed by each FA and appear at the FA outputs 10 ns later:



Since the inputs to FA1 have changed, the outputs change 10 ns later:



The final simulation results are

1111 + 0001 + 1 = 0001 with a carry of 1 (at time = 40 ns) and

0101 + 1110 + 0 = 0011 with a carry of 1 (at time = 80 ns)

The simulation stops at 80 ns since no further changes occur after that time.

In this section we have shown how to construct a VHDL module using an entity-architecture pair. The 4-bit adder module demonstrates the use of VHDL components to write structural VHDL code. Components used within the architecture are declared at the start of the architecture using a component declaration of the form

```
component component-name
  port(list-of-interface-signals-and-their-types);
end component;
```

The port clause used in the component declaration has the same form as the port clause used in an entity declaration. The connections to each component used in a circuit are specified using a component instantiation statement of the form

```
label: component-name port map (list-of-actual-signals);
```

The list of actual signals must correspond one-to-one to the list of interface signals specified in the component declaration.

2.4.2 Use of "Buffer" Mode

Let us consider the example in Figure 2-13. Assume that all variables are 0 at 0 ns, but *A* changes to 1 at 10 ns.

FIGURE 2-13: VHDL Code Which Will Not Compile

```
entity gates is
  port(A, B, C: in bit; D, E: out bit);
end gates;

architecture example of gates is
begin
  D <= A or B after 5 ns;   -- statement 1
  E <= C or D after 5 ns;   -- statement 2
end example;
```

The code in Figure 2-13 will not actually compile, simulate, or synthesize in most tools because *D* is declared only as an output. Statement 2 uses *D* on the right side of the assignment. Hence, *D* should be either **inout** or **buffer** mode as in Figure 2-14. Use of inout mode results in the synthesis tools creating a truly bidirectional signal. In actuality, *D* is not an external input to the circuit, and hence the mode **buffer** is more appropriate. The mode **buffer** indicates a signal that is an output to the external world; however, its value can also be read inside the entity's architecture. The following code uses buffer mode for signal *D* instead of out mode.

FIGURE 2-14: VHDL Code Illustrating Use of Mode Buffer

```
entity gates is
  port(A, B, C: in bit; D: buffer bit; E: out bit);
end gates;

architecture example of gates is
begin
  D <= A or B after 5 ns;   -- statement 1
  E <= C or D after 5 ns;   -- statement 2
end example;
```

All signals remain at '0' until time 10 ns. The change in A at 10 ns results in statement 1 reevaluating. The value of D becomes '1' at time equal to 15 ns. The change in D at time 15 ns results in statement 2 reevaluating. Signal E changes to '1' at time 20 ns. The description represents two gates, each with a delay of 5 ns.

2.5 Sequential Statements and VHDL Processes

The concurrent statements from the previous section are useful in modeling combinational logic. Combinational logic constantly reacts to input changes. In contrast, synchronous sequential logic responds to changes dependent on the clock. Many input changes might be ignored since output and state changes occur only at valid conditions of the clock. Modeling sequential logic requires primitives to model selective activity conditional on clock, edge-triggered devices, sequence of operations, and so on. In this unit, we will learn VHDL processes which help to model sequential logic.

A VHDL process has the following basic form:

```
process(sensitivity-list)
begin
  sequential-statements
end process;
```

When a process is used, the statements between the **begin** and the **end** are executed sequentially. The expression in parentheses after the word **process** is called a sensitivity list, and the process executes whenever any signal in the sensitivity list changes. For example, if the process begins with `process(A, B, C)`, then the process executes whenever any one of A, B, or C changes. Whenever one of the signals in the sensitivity list changes, the sequential statements in the process body are executed in sequence one time. When a process finishes executing, it goes back to the beginning and waits for a signal on the sensitivity list to change again.

When the concurrent statements

```
C <= A and B;   -- concurrent
E <= C or D;    -- statements
```

are used in a process, they become sequential statements executed in the order in which they appear in the process. Remember that when they were concurrent statements outside a process, their sequence did not matter. But, if they are in a process, the sequence determines the order of execution.

```
process(A, B, C, D)
begin
  C <= A and B;  -- sequential
  E <= C or D;   -- statements
end process;
```

The process executes once when any of the signals *A*, *B*, *C*, or *D* changes. If *C* changes when the process executes, then the process will execute a second time because *C* is on the sensitivity list.

VHDL processes can be used for modeling combinational logic and sequential logic; however, processes are not necessary for modeling combinational logic. They are, however, required for modeling sequential logic. One should be very careful when using processes to represent combinational logic. Consider the code in Figure 2-15, where a process is used. One may write this code thinking of two cascaded gates; however, it does not actually represent such a circuit.

FIGURE 2-15: VHDL Code with a Process

```
entity nogates is
  port(A, B, C: in bit;
       D: buffer bit;
       E: out bit);
end nogates;

architecture behave of nogates is
begin
  process(A, B, C)
  begin
    D <= A or B after 5 ns;  -- statement 1
    E <= C or D after 5 ns;  -- statement 2
  end process;
end behave;
```

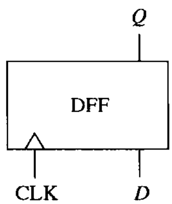
The sensitivity list of the process only includes *A*, *B*, and *C*, the only external inputs to the circuit. Let us assume that all variables are '0' at 0 ns. Then *A* changes to '1' at 10 ns. That causes the process to execute. Both statements inside the process execute once sequentially, but the change in *D* does not happen right at execution. Hence, execution of statement 2 is with the value of *D* at the beginning of the process. *D* becomes '1' at 15 ns, but *E* stays at '0'. Since the change in *D* does not propagate to signal *E*, this VHDL model is not equivalent to two gates. If *D* was included in the sensitivity list of the process, the process would execute again making *E* change at 20 ns. This would result in simulation outputs matching a circuitry with cascaded gates, but it is preferable to realize gates using concurrent statements.

Understanding sequential statements and operation of processes will take several more examples. In the next section, we explain how simple flip-flops can be modeled using processes, and then we explain the basics of the VHDL simulation process. After that, we present more examples illustrating the working of processes and the simulation process.

2.6 Modeling Flip-Flops Using VHDL Processes

A flip-flop can change state either on the rising or on the falling edge of the clock input. This type of behavior is modeled in VHDL by a process. For a simple D flip-flop with a Q output that changes on the rising edge of CLK, the corresponding process is given in Figure 2-16.

FIGURE 2-16: VHDL Code for a Simple D Flip-Flop




```
process(CLK)
begin
  if CLK'event and CLK = '1'  -- rising edge of CLK
    then Q <= D;
  end if;
end process;
```

In Figure 2-16, whenever *CLK* changes, the process executes once through and then waits at the start of the process until *CLK* changes again. The `if` statement tests for a rising edge of the clock, and Q is set equal to D when a rising edge occurs. The expression `CLK'event` is used to accomplish the functionality of an edge-triggered device. The expression `'event` is a predefined attribute for any signal. There are two types of signal attributes in VHDL, those that return values and those that return signals. The `'event` attribute returns a value. The expression `CLK'event` (read as "clock tick event") is TRUE whenever the signal *CLK* changes. If `CLK = '1'` is also TRUE, this means that the change was from '0' to '1', which is a rising edge.

If VHDL is used only for simulation purposes, one might use a statement such as

```
if CLK = '1'
        ...
```

and obtain action corresponding to rising edge. However, when VHDL code is used to synthesize hardware, this statement will result in latches, whereas the expression `CLK'event` results in edge-triggered devices.

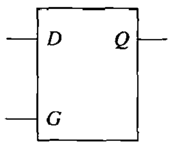
If the flip-flop has a delay of 5 ns between the rising edge of the clock and the change in the Q output, we would replace the statement `Q <= D;` with `Q <= D after 5 ns;` in the preceding process.

The statements between **begin** and **end** in a process operate as sequential statements. In the preceding process, `Q <= D;` is a sequential statement that only executes

following the rising edge of *CLK*. In contrast, the concurrent statement `Q <= D;` executes whenever *D* changes. If we synthesize the above process, the synthesizer infers that *Q* must be a flip-flop since it only changes on the rising edge of *CLK*. If we synthesize the concurrent statement `Q <= D;`, the synthesizer will simply connect *D* to *Q* with a wire or a buffer.

In Figure 2-16, note that *D* is not on the sensitivity list because changing *D* will not cause the flip-flop to change state. Figure 2-17 shows a transparent latch and its VHDL representation. Both *G* and *D* are on the sensitivity list since if *G* = '1', a change in *D* causes *Q* to change. If *G* changes to '0', the process executes, but *Q* does not change.

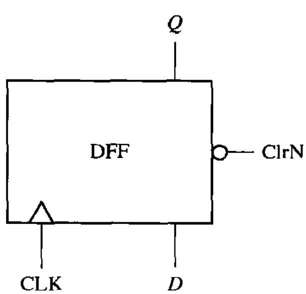
FIGURE 2-17: VHDL Code for a Transparent Latch




```
process(G, D)
begin
  if G = '1' then Q <= D; end if;
end process;
```

If a flip-flop has an active-low asynchronous clear input (*ClrN*) that resets the flip-flop independently of the clock, then we must modify the process of Figure 2-16 so that it executes when either *CLK* or *ClrN* changes. To do this, we add *ClrN* to the sensitivity list. The VHDL code for a D flip-flop with asynchronous clear is given in Figure 2-18. Since the asynchronous *ClrN* signal overrides *CLK*, *ClrN* is tested first and the flip-flop is cleared if *ClrN* is '0'. Otherwise, *CLK* is tested, and *Q* is updated if a rising edge has occurred.

FIGURE 2-18: VHDL Code for a D Flip-Flop with Asynchronous Clear




```
process(CLK, ClrN)
begin
  if CLRn = '0' then Q <= '0';
  else if CLK'event and CLK = '1'
    then Q <= D;
  end if;
  end if;
end process;
```

In the preceding examples, we have used two types of sequential statements—signal assignment statements and `if` statements. The basic `if` statement has the form

```
if condition then
  sequential statements1
else sequential statements2
end if;
```

The condition is a Boolean expression which evaluates to TRUE or FALSE. If it is TRUE, `sequential statements1` are executed; otherwise, `sequential statements2` are executed.

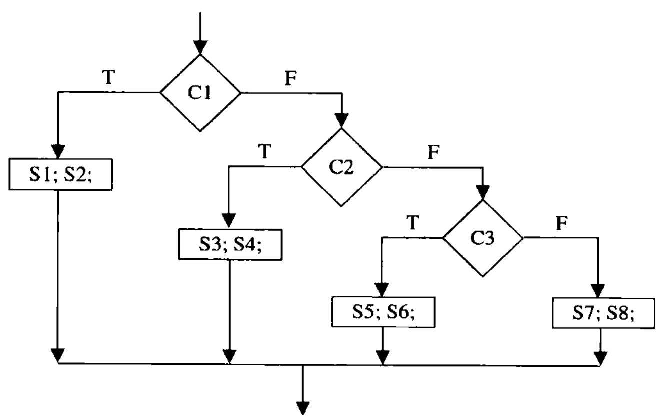
VHDL **if** statements are sequential statements that can be used within a process, but they cannot be used as concurrent statements outside of a process.

The most general form of the **if** statement is

```
if condition then
  sequential statements
{elsif condition then
  sequential statements}
  -- 0 or more elsif clauses may be included
[else sequential statements]
end if;
```

The curly brackets indicate that any number of **elsif** clauses may be included, and the square brackets indicate that the **else** clause is optional. The example of Figure 2-19 shows how a flow chart can be represented using nested **if**s or the equivalent using **elsif**s. In this example, C1, C2, and C3 represent conditions that can be true or false, and S1, S2, . . . , S8 represent sequential statements. Each **if** requires a corresponding **end if**, but **elsif**s do not.

FIGURE 2-19: Equivalent Representations of a Flow Chart Using Nested Ifs and Elsifs

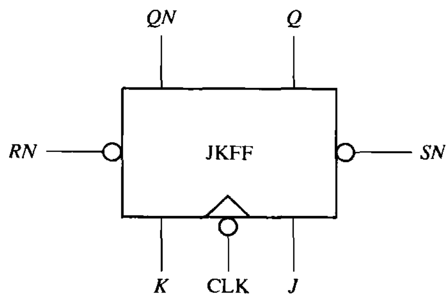



```
if (C1) then S1; S2;
   else if (C2) then S3; S4;
        else if (C3) then S5; S6;
              else S7; S8;
              end if;
        end if;
end if;
```

```
if (C1) then S1; S2;
   elsif (C2) then S3; S4;
   elsif (C3) then S5; S6;
   else S7; S8;
end if;
```

Next, we will write a VHDL module for a J-K flip-flop (Figure 2-20). This flip-flop has active-low asynchronous preset (SN) and clear (RN) inputs. State changes related to J and K occur on the falling edge of the clock. In this chapter, we use a suffix N to indicate an active-low (negative-logic) signal. For simplicity, we will assume that the condition $SN = RN = 0$ does not occur.

FIGURE 2-20: J-K Flip-Flop



The VHDL code for the J-K flip-flop is given in Figure 2-21. The port declaration in the entity defines the input and output signals. Within the architecture we define a signal Q_{int} that represents the state of the flip-flop internal to the module. The two concurrent statements after **begin** transmit this internal signal to the Q and QN outputs of the flip-flop. We do it this way because an output signal in a port cannot appear on the right side of an assignment statement within the architecture. This is another solution to the problem presented in Figure 2-13. The flip-flop can change state in response to changes in SN, RN, and CLK, so these three signals are in the sensitivity list of the process. Since RN and SN reset and set the flip-flop independently of the clock, they are tested first. If RN and SN are both '1', then we test for the falling edge of the clock. The condition (`CLK'event and CLK = '0'`) is TRUE only if CLK has just changed from '1' to '0'. The next state of the flip-flop is determined by its characteristic equation:

$$Q^+ = JQ' + K'Q$$

FIGURE 2-21: J-K Flip-Flop Model

```
entity JKFF is
  port(SN, RN, J, K, CLK: in bit; -- inputs
       Q, QN: out bit);
end JKFF;

architecture JKFF1 of JKFF is
signal Qint: bit;                  -- Qint can be used as input or output
begin
  Q <= Qint;                       -- output Q and QN to port
  QN <= not Qint;                  -- combinational output
                                   -- outside process
```

```
  process(SN, RN, CLK)
  begin
    if RN = '0' then Qint <= '0' after 8 ns; -- RN = '0' will clear the FF
    elsif SN = '0' then Qint <= '1' after 8 ns; -- SN='0' will set the FF
    elsif CLK'event and CLK = '0' then          -- falling edge of CLK
      Qint <= (J and not Qint) or (not K and Qint) after 10 ns;
    end if;
  end process;
end JKFF1;
```

The 8-ns delay represents the time it takes to set or clear the flip-flop output after *SN* or *RN* changes to '0'. The 10-ns delay represents the time it takes for *Q* to change after the falling edge of the clock.

2.7 Processes Using Wait Statements

An alternative form for a process uses wait statements instead of a sensitivity list. A process cannot have both wait statements and a sensitivity list. A process with wait statements may have the form

```
process
begin
  sequential-statements
  wait-statement
  sequential-statements
  wait-statement
  . . .
end process;
```

This process will execute the `sequential-statements` until a wait statement is encountered. Then it will wait until the specified wait condition is satisfied. It will then execute the next set of `sequential-statements` until another wait is encountered. It will continue in this manner until the end of the process is reached. Then it will start over again at the beginning of the process.

Wait statements can be of three different forms:

```
wait on sensitivity-list;
wait for time-expression;
wait until Boolean-expression;
```

The first form waits until one of the signals on the sensitivity-list changes. For example, `wait on A, B, C;` waits until *A*, *B*, or *C* changes and then execution proceeds. The second form waits until the time specified by `time-expression` has lapsed. If `wait for 5 ns` is used, the process waits for 5 ns before continuing. If `wait for 0 ns` is used, the wait is for one delta time. Wait statements of the form `wait for xxx ns` are useful for writing VHDL code for simulation; however, they

should not be used when writing VHDL code for synthesis since they are not synthesizable. For the third form of wait statement, the `Boolean-expression` is evaluated whenever one of the signals in the expression changes, and the process continues execution when the expression evaluates to TRUE. For example,

```
wait until A = B;
```

will wait until either *A* or *B* changes. Then `A = B` is evaluated and if the result is TRUE, the process will continue; otherwise, the process will continue to wait until *A* or *B* changes again and `A = B` is TRUE.

A process cannot have both wait statements and a sensitivity list. It is not acceptable to have some of the signals to be in a sensitivity list and others in wait statements.

After a VHDL simulator is initialized, it executes each process with a sensitivity list one time through, and then waits at the beginning of the process for a change in one of the signals on the sensitivity list. If a process has a wait statement, it will initially execute until a wait statement is encountered. The following two processes are equivalent:

```
process(A, B, C, D)
begin
  C <= A and B after 5 ns;
  E <= C or D after 5 ns;
end process;
```

```
process
begin
  C <= A and B after 5 ns;
  E <= C or D after 5 ns;
  wait on A, B, C, D;
end process;
```

The wait statement at the end of the process replaces the sensitivity list at the beginning. In this way, both processes will initially execute the sequential statements one time and then wait until *A*, *B*, *C*, or *D* changes.

The order in which sequential statements execute in a process is not necessarily the order in which the signals are updated. Consider the following example:

```
process
begin
  wait until clk'event and clk = '1';
  A <= E after 10 ns;        -- (1)
  B <= F after 5 ns;         -- (2)
  C <= G;                    -- (3)
  D <= H after 5 ns;         -- (4)
end process;
```

This process waits for a rising clock edge. Suppose the clock rises at time = 20 ns. Statements (1), (2), (3), (4) immediately execute in sequence. *A* is scheduled to change to *E* at time = 30 ns; *B* is scheduled to change to *F* at time = 25 ns; *C* is scheduled to change to *G* at time = 20 + delta; and *D* is scheduled to change to *H* at time 25 ns. As the simulated time advances, first *G* changes. Then *F* and *D* change at time = 25 ns, and finally *E* changes at time 30 ns. When *clk* changes to '0', the wait statement is reevaluated, but it keeps waiting until *clk* changes to '1', and then the remaining statements execute again.

If several VHDL statements in a process update the same signal at a given time, the last value overrides. For example,

```
process(CLK)
begin
  if CLK'event and CLK = '0' then
    Q <= A; Q <= B; Q <= C;
  end if;
end process;
```

Every time *CLK* changes from '1' to '0', after delta time, *Q* will change to *C*.

A process must have either a sensitivity list or wait statements. The VHDL code in Figure 2-22 will not simulate because there is no sensitivity list or wait statement.

FIGURE 2-22: Example of VHDL Code That Will Not Simulate

```
entity gates is
  port(A, B, C: in bit; D, E: out bit);
end gates;

architecture exam of gates is
begin
  process
  begin
    D <= A or B after 2 ns;
    E <= not C and A;
  end process;
end exam;
```

In this section, we have introduced processes with sensitivity lists and processes with wait statements. The statements within a process are called sequential statements because they execute in sequence, in contrast with concurrent statements that execute only when a signal on the right-hand-side changes. Signal assignment statements can be either concurrent or sequential. However, **if** statements are always sequential.

2.8 Two Types of VHDL Delays: Transport and Inertial Delays

In one of the initial examples in this chapter, we used the statement

```
C <= A and B after 5 ns;
```

to model an AND gate with a propagation delay of 5 ns. The preceding statement will model the AND gate's delay; however, it also introduces some complication, which many readers will not normally expect. If you simulate this AND gate with

inputs that change very often in comparison to the gate delay (e.g., at 1 ns, 2 ns, 3 ns, etc.), the simulation output will not show the changes. This is due to how VHDL delays work.

VHDL provides two types of delays—transport delays and inertial delays. The default delay is inertial delay; hence, the after clause in the preceding statement represents an **inertial delay**. Inertial delays are slightly different from simple delays that readers normally assume.

Inertial delay is intended to model gates and other devices that do not propagate short pulses from the input to the output. If a gate has an ideal inertial delay T, in addition to delaying the input signals by time T, any pulse with a width less than T is rejected. For example, if a gate has an inertial delay of 5 ns, a pulse of width 5 ns would pass through, but a pulse of width 4.999 ns would be rejected. Real devices do not behave in this way. Perhaps they would reject very narrow spurious pulses, but it might be unreasonable to assume that all pulses narrower than the delay duration will be rejected. VHDL does allow one to model devices which reject only very narrow pulses. Rejection of pulses of any arbitrary duration up to the specified inertial delay can be modeled by adding a reject clause to the assignment statement. A statement of the form

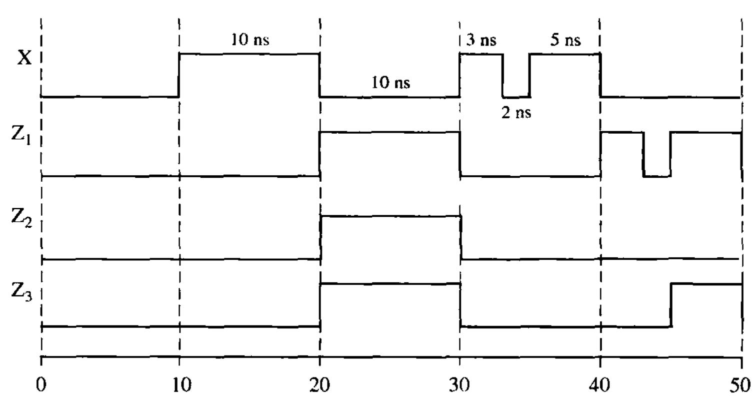
```
signal_name <= reject pulse-width inertial expression after
  delay-time
```

evaluates the expression, rejects any pulses whose width is less than pulse-width, and then sets the signal equal to the result after a delay of delay-time. In statements of this type, the rejection pulse width must be less than the delay time.

The second type of VHDL delay is **transport delay**, which is intended to model the delay introduced by wiring, simply delays an input signal by the specified delay time. In order to model this delay, the key word **transport** must be specified in the code. Figure 2-23 illustrates the difference between transport and inertial delays. Consider the following VHDL statements:

```
Z1 <= transport X after 10 ns;  -- transport delay
Z2 <= X after 10 ns;            -- inertial delay
Z3 <= reject 4 ns inertial X after 10 ns;
   -- inertial delay with specified rejection pulse width
```

FIGURE 2-23: Transport and Inertial Delays



Z_1 is the same as X, except that it is shifted 10 ns in time. Z_2 is similar to Z_1, except the pulses in X shorter than 10 ns are filtered out and do not appear in Z_2. Z_3 is the same as Z_2, except that only the pulses of width less than 4 ns have been rejected.

In general, using **reject** is equivalent to using a combination of an inertial delay and a transport delay. The statement for Z_3 given here could be replaced with the concurrent statements:

```
Zm <= X after 4 ns; -- inertial delay rejects short pulses
Z3 <= transport Zm after 6 ns;   -- total delay is 10 ns
```

Note that these delays are relevant only for simulation. Understanding how inertial delay works can remove a lot of frustration in your initial experience with VHDL simulation. The pulse rejection associated with inertial delay can inhibit many output changes. In simulations with basic gates and simple circuits, one should make sure that test sequences that you apply are wider than the inertial delays of the modeled devices.

2.9 Compilation, Simulation, and Synthesis of VHDL Code

After describing a digital system in VHDL, simulation of the VHDL code is important for two reasons. First, we need to verify the VHDL code correctly implements the intended design, and second, we need to verify that the design meets its specifications. We first simulate the design and then synthesize it to the target technology (e.g., FPGA or custom ASIC). In this section, first we describe steps in simulation and then introduce synthesis. As illustrated in Figure 2-24, there are three phases in the simulation of VHDL code: **analysis (compilation), elaboration**, and **simulation.**

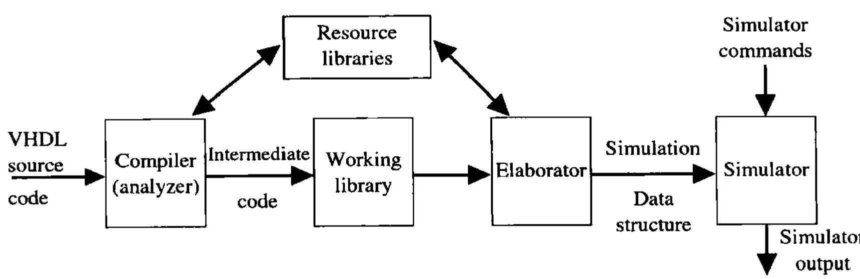


FIGURE 2-24: Compilation, Elaboration, and Simulation of VHDL Code

Before the VHDL model of a digital system can be simulated, the VHDL code must first be compiled. The VHDL compiler, also called an **analyzer**, first checks the VHDL source code to see that it conforms to the syntax and semantic rules of VHDL. If there is a syntax error, such as a missing semicolon, or if there is a semantic error, such as trying to add two signals of incompatible types, the compiler will output an

error message. The compiler also checks to see that references to libraries are correct. If the VHDL code conforms to all of the rules, the compiler generates intermediate code, which can be used by a simulator or by a synthesizer.

In preparation for simulation, the VHDL intermediate code must be converted to a form which can be used by the simulator. This step is referred to as **elaboration**. During elaboration, a *driver* is created for each signal. Each driver holds the current value of a signal and a queue of future signal values. Each time a signal is scheduled to change in the future, the new value is placed in the queue along with the time at which the change is scheduled. In addition, ports are created for each instance of a component; memory storage is allocated for the required signals; the interconnections among the port signals are specified; and a mechanism is established for executing the VHDL statements in the proper sequence. The resulting data structure represents the digital system being simulated.

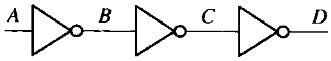
The simulation process consists of an **initialization phase** and actual **simulation**. The simulator accepts simulation commands, which control the simulation of the digital system and which specify the desired simulator output. VHDL simulation uses what is known as **discrete event simulation**. The passage of time is simulated in discrete steps in this method of simulation. The initialization phase is used to give an initial value to the signal. During simulation, the VHDL statements are executed and corresponding actions are scheduled. These actions are called transactions, and the process is called **scheduling a transaction**. The scheduled action happens, not necessarily when the statement executes, but when the scheduled time has been reached. A transaction does not mean that there is a change in the value of a signal. The new value for the signal after the transaction may be the same as the old value. If a change in the value occurs, we say that an **event** has taken place.

To facilitate correct initialization, the initial value can be specified in the VHDL model. In the absence of any specifications of the initial values, some simulator packages will assign an initial value depending on the type of the signal. Please note that this initialization is only for simulation and not for synthesis. During initialization, simulation time is set to zero and each process is activated. The process "executes," scheduling corresponding transactions; however, the scheduled transactions do not happen until one reaches the time at which the scheduled transaction is to occur. Execution of a process happens once, and then the process waits for a signal in the sensitivity list to change.

Understanding the role of the delta (Δ) time delays is important when interpreting output from a VHDL simulator. Although the delta delays do not show up on waveform outputs from the simulator, they show up on listing outputs. The simulator uses delta delays to make sure that signals are processed in the proper sequence. Basically, the simulator works as follows: Whenever a component input changes, the output is scheduled to change after the specified delay, or after Δ if no delay is specified. When all input changes have been processed, simulated time is advanced to the next time at which an output change is specified. When time is advanced by a finite amount (1 ns for example), the Δ counter is reset and simulation resumes. Real time does not advance again until all Δ delays associated with the current simulation time have been processed.

The following example illustrates how the simulator works for the circuit of Figure 2-25. Suppose that A changes at time = 3 ns. Statement 1 executes and B is

FIGURE 2-25: Illustration of Delta Delays during Simulation of Concurrent Statements




```
1 B <= not A;
2 C <= not B;
3 D <= not C after 5 ns;
```

ns	delta	A	B	C	D
0	+0	0	1	0	1
3	+0	1	1	0	1
3	+1	1	0	0	1
3	+2	1	0	1	1
8	+0	1	0	1	0

scheduled to change at time $3 + \Delta$. Then time advances to $3 + \Delta$, and statement 2 executes. C is scheduled to change at time $3 + 2\Delta$. Time advances to $3 + 2\Delta$, and statement 3 executes. D is then scheduled to change at 8 ns. You might think the change should occur at $(3 + 2\Delta + 5)$ ns. However, when time advances a finite amount (as opposed to Δ, which is infinitesimal), the Δ counter is reset. For this reason, when events are scheduled a finite time in the future, the Δ's are ignored. Since no further changes are scheduled after 8 ns, the simulator goes to an idle mode and waits for another input change. The table gives the simulator output listing.

2.9.1 Simulation with Multiple Processes

If a model contains more than one process, all processes execute concurrently with other processes. If there are concurrent statements outside processes, they also execute concurrently. Statements inside of each process execute sequentially. A process takes no time to execute unless it has wait statements in it. (Examples: `wait for 10 ns`, `wait for 0 ns`, and `wait on E`.) Signals take delta time to update when no delay is specified.

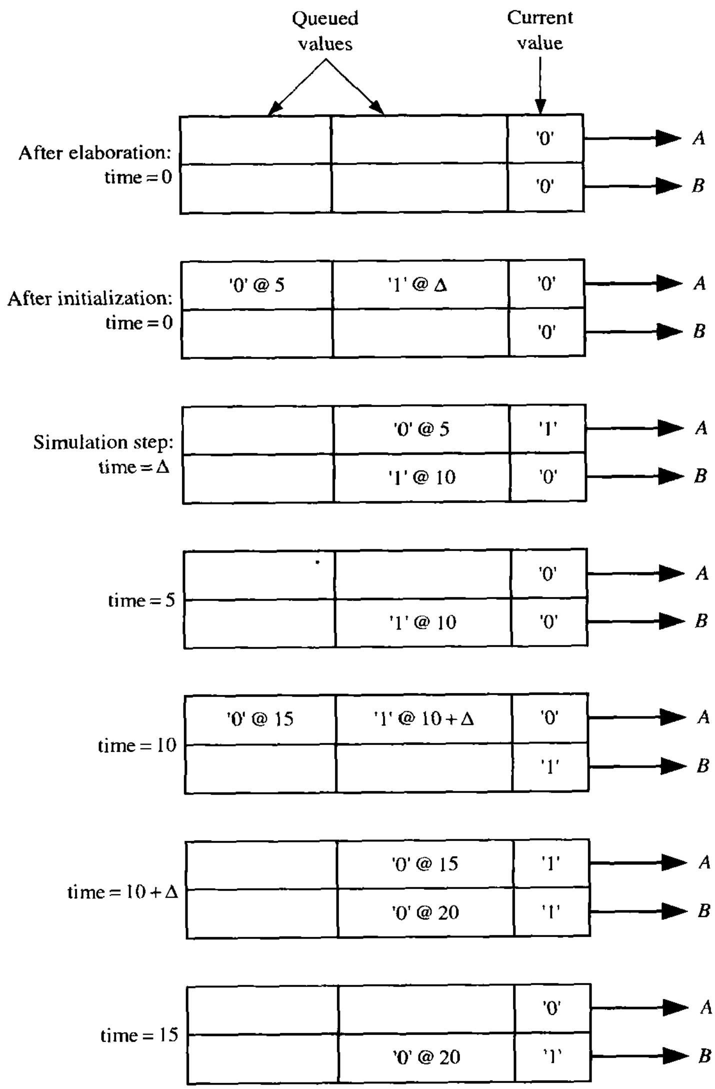
As an example of simulation of multiple processes, we trace execution of the VHDL code shown in Figure 2-26. The keyword **transport** specifies the type of delay as transport delay.

FIGURE 2-26: VHDL Code to Illustrate Process Simulation

```
entity simulation_example is
end simulation_example;

architecture test1 of simulation_example is
signal A,B: bit;
begin
  P1: process(B)
  begin
    A <= '1';
    A <= transport '0' after 5 ns;
  end process P1;

  P2: process(A)
  begin
    if A = '1' then B <= not B after 10 ns; end if;
  end process P2;
end test1;
```

Figure 2-27 shows the drivers for the signals A and B as the simulation progresses. After elaboration is finished, each driver holds '0', since this is the default initial value for a bit. When simulation begins, initialization takes place. Both processes are executed simultaneously one time through, and then the processes wait until a signal on the sensitivity list changes. When process P_1 executes at zero time, two changes in A are scheduled (A changes to '1' at time Δ and back to '0' at time = 5 ns). Meanwhile, process P_2 executes at zero time, but no change in B occurs, since A is still '0' during execution at time 0 ns. Time advances to Δ, and A changes to '1'. The change in A causes process P_2 to execute, and since A = '1', B is scheduled to change to '1' at time 10 ns. The next scheduled change occurs at time = 5 ns, when A changes

FIGURE 2-27: Signal Drivers for Simulation Example

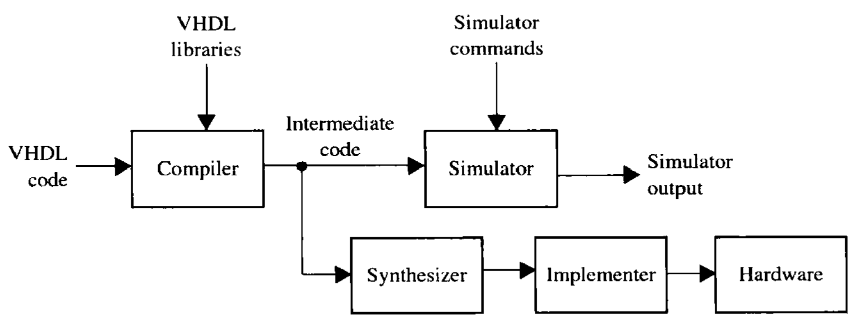


to '0'. This change causes P_2 to execute, but B does not change. B changes to '1' at time = 10 ns. The change in B causes P_1 to execute, and 2 changes in A are scheduled. When A changes to '1' at time 10 + Δ, process P_2 executes, and B is scheduled to change at time 20 ns. Then A changes at time 15 ns, and the simulation continues in this manner until the run-time limit is reached. It should be understood that A changes at 15 ns and not at 15 + Δ. The Δ delay comes into the picture only when no time delay is specified.

VHDL simulators use event-driven simulation, as illustrated in the preceding example. A change in a signal is referred to as an *event*. Each time an event occurs, any processes that have been waiting on the event are executed in zero time, and any resulting signal changes are queued up to occur at some future time. When all the active processes are finished executing, simulation time is advanced to the time for which the next event is scheduled, and the simulator processes that event. This continues until either no more events have been scheduled or the simulation time limit is reached.

When VHDL was originally created, simulation was the primary purpose; however, nowadays, one of the most important uses of VHDL is to synthesize or automatically create hardware from a VHDL description. The **synthesis** software for VHDL translates the VHDL code to a circuit description that specifies the needed components and the connections between the components. The initial steps (analysis and elaboration) in Figure 2-24 are common whether VHDL is used for simulation or synthesis. The simulation and synthesis processes are shown in Figure 2-28.

FIGURE 2-28: Compilation, Simulation, and Synthesis of VHDL Code



Although synthesis can be done in parallel to simulation, synthesis follows simulation because designers would normally want to catch errors before attempting to synthesize. After the VHDL code for a digital system has been simulated to verify that it works correctly, the VHDL code can be synthesized to produce a list of required components and their interconnections. The synthesizer output can then be used to implement the digital system using specific hardware, such as a CPLD or FPGA, or an ASIC. The CAD software used for implementation generates the necessary information to program the CPLD or FPGA hardware. In the case of an ASIC, it generates the mask required to create the ASIC. Synthesis and implementation of digital logic from VHDL code is discussed in more detail later.

2.10 VHDL Data Types and Operators

2.10.1 Data Types

VHDL has several predefined data types. Signals can have these predefined data types, or they can have a user-defined type. Some of the predefined types are as follows:

bit	'0' or '1'
boolean	FALSE or TRUE
integer	an integer in the range $-(2^{31}-1)$ to $+(2^{31}-1)$ (some implementations support a wider range)
real	floating-point number in the range $-1.0\text{E}38$ to $+1.0\text{E}38$
character	any legal VHDL character including upper- and lowercase letters, digits, and special characters (each printable character must be enclosed in single quotes; e.g., 'd', '7', '+')
time	an integer with units fs, ps, ns, us, ms, sec, min, or hr

Note that the integer range for VHDL is symmetrical, even though the range for a 32-bit 2's complement integer is -2^{31} to $+(2^{31}-1)$.

Users can define and create their own data types. A common user-defined type is the **enumeration** type in which all of the values are enumerated. For example, the declarations

```
type state_type is (S0, S1, S2, S3, S4, S5);
signal state: state_type := S1;
```

define a signal called *state* that can have any one of the values S_0, S_1, S_2, S_3, S_4, or S_5 and is initialized to S_1. If no initialization is given, the default initialization is the leftmost element in the enumeration list, S_0 in this example.

VHDL is a **strongly typed language**, so signals and variables of different types generally cannot be mixed in the same assignment statement, and no automatic type conversion is performed. Thus, the statement

```
A <= B or C;
```

is valid only if A, B, and C all have the same type or closely related types. If types do not match, explicit type conversions should be performed, or "overloaded operators" should be created. Operator overloading is described in Sections 2.13 and 3.4. The overloaded operators in the IEEE packages are presented in Section 2.13.

2.10.2 VHDL Operators

Predefined VHDL operators can be grouped into seven classes:

1. Binary logical operators: **and or nand nor xor xnor**
2. Relational operators: = /= < <= > >=
3. Shift operators: **sll srl sla sra rol ror**
4. Adding operators: + − & (concatenation)

5. Unary sign operators: + −
6. Multiplying operators: * / **mod rem**
7. Miscellaneous operators: **not abs** **

When parentheses are not used, operators in class 7 have highest precedence and are applied first, followed by class 6, then class 5, and so on. Class 1 operators have lowest precedence and are applied last. Operators in the same class have the same precedence and are applied from left to right in an expression. The precedence order can be changed by using parentheses. Consider the following expression, where *A*, *B*, *C*, and *D* are bit_vectors:

```
(A & not B or C ror 2 and D) = "110010"
```

Note that this is a relational expression performing an equality test; it is not an assignment statement.

To evaluate the expression, the operators are applied in the order

```
not, &, ror, or, and, =
```

If A = "110", B = "111", C = "011000", and D = "111011", the computation proceeds as follows:

```
not B = "000" (bit-by-bit complement)
A & not B = "110000" (concatenation)
C ror 2 = "000110" (rotate right 2 places)
(A & not B) or (C ror 2) = "110110" (bit-by-bit or)
(A & not B or C ror 2) and D = "110010" (bit-by-bit and)
[(A & not B or C ror 2 and D) = "110010"] = TRUE (the parentheses
force the equality test to be done last and the result is TRUE)
```

The binary logical operators (class 1) as well as **not** can be applied to bits, booleans, bit_vectors, and boolean_vectors. The class 1 operators require 2 operands of the same type, and the result is of that type.

The result of applying a relational operator (class 2) is always a Boolean (FALSE or TRUE). Equals (=) and not equals (/=) can be applied to almost any type. The other relational operators can be applied to any numeric or enumerated type as well as to some array types. For example, if $A = 5$, $B = 4$, and $C = 3$, the expression `(A <= B) and (B <= C)` evaluates to FALSE.

The shift operators can be applied to any bit_vector or boolean_vector. In the following examples, *A* is a bit_vector equal to "10010101":

```
A sll 2 is "01010100"    (shift left logical, filled with '0')
A srl 3 is "00010010"    (shift right logical, filled with '0')
A sla 3 is "10101111"    (shift left arithmetic, filled with
                         right bit)
A sra 2 is "11100101"    (shift right arithmetic, filled with
                         left bit)
A rol 3 is "10101100"    (rotate left)
A ror 5 is "10101100"    (rotate right)
```

The + and − operators can be applied to integer or real numeric operands. The + and − operators are not defined for bits or bit-vectors. That is why we had to make a full adder by specifically creating carry and sum bits for each bit (Figure 2-12). However, several standard libraries do provide functions for + and − that can work on bit-vectors. If we use such a library, we can perform addition using the statement $C <= A + B$. Some of the popular libraries are described in Section 2.13.

The & operator can be used to concatenate two vectors (or an element and a vector, or two elements) to form a longer vector. For example, "010" & '1' is "0101" and "ABC" & "DEF" is "ABCDEF".

The * and / operators perform multiplication and division on integer or floating-point operands. The rem and mod operators calculate the remainder and modulus for integer operands. The ** operator raises an integer or floating-point number to an integer power, and abs finds the absolute value of a numeric operand.

2.11 Simple Synthesis Examples

Synthesis tools try to infer the hardware components needed by "looking" at the VHDL code. In order for code to synthesize correctly, certain conventions must be followed. When writing VHDL code, you should always keep in mind that you are designing hardware, not simply writing a computer program. Each VHDL statement implies certain hardware requirements. So, poorly written VHDL code may result in poorly designed hardware. Even if VHDL code gives the correct result when simulated, it may not result in hardware that works correctly when synthesized. Timing problems may prevent the hardware from working properly even though the simulation results are correct.

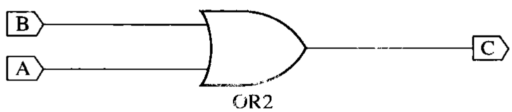
Consider the VHDL code in Figure 2-29. (Note that *B* is missing from the process sensitivity list.) This code will simulate as follows: Whenever *A* changes, it will cause the process to execute once. The value of *C* will reflect the values of *A* and *B* when the process began. If *B* changes now, that will not cause the process to execute.

FIGURE 2-29: VHDL Code Example where Simulation and Synthesis Results in Different Outputs

```
entity Q1 is
  port(A, B: in bit;
       C: out bit);
end Q1;

architecture circuit of Q1 is
begin
  process(A)
  begin
    C <= A or B after 5 ns;
  end process;
end circuit;
```

If this code is synthesized, most synthesizers will output an OR gate as in Figure 2-30. The synthesizer will warn you that *B* is missing from the sensitivity list, but will go ahead and synthesize the code properly. The synthesizer will also ignore the 5-ns delay on the above statement. If you want to model an exact 5-ns delay, you will have to use counters. The simulator output will not match the synthesizer's output since the process will not execute when *B* changes. This is an example of where the synthesizer guessed a little more than what you wrote; it assumed that you probably meant an OR gate and created that circuit (accompanied by a warning). But this circuit functions differently from what simulated before synthesis. It is important that you always check for synthesizer warnings of missing signals in the sensitivity list. Perhaps the synthesizer helped you; perhaps it created hardware that you did not intend to.

FIGURE 2-30: Synthesizer Output for Code in Figure 2-29



Now, consider the VHDL code in Figure 2-31. What hardware will you get if you synthesized this code?

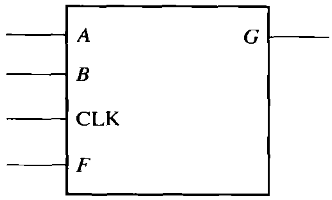
FIGURE 2-31: Example VHDL Code

```
entity Q3 is
  port(A,B,F, CLK: in bit;
       G: out bit);
end Q3;

architecture circuit of Q3 is
signal C: bit;
begin
  process(Clk)
  begin
    if (Clk = '1' and Clk'event) then
      C <= A and B; -- statement 1
      G <= C or F; -- statement 2
    end if;
  end process;
end circuit;
```

Let us think about the block diagram of the circuit represented by this code without worrying about the details inside. The block diagram is as shown in Figure 2-32. The ability to hide details and use abstractions is an important part of good system design.

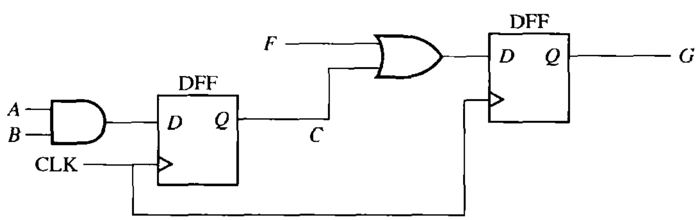
Note that *C* is an internal signal, and therefore it does not show up in the block diagram.

FIGURE 2-32: Block Diagram for VHDL Code in Figure 2-31



Now, let us think about the details of the circuit inside this block. This circuit is not two cascaded gates; the signal assignment statements are in a process. An edge-triggered clock is implied by the use of `clk'event` in the clock statement preceding the signal assignment. Since the values of C and G need to be retained after the clock edge, flip-flops are required for both C and G. Please note that a change in the value of C from statement 1 will not be considered during the execution of statement 2 in that pass of the process. It will be considered only in the next pass, and the flip-flop for C makes this happen in the hardware also. Hence the code implies hardware shown in Figure 2-33.

FIGURE 2-33: Hardware Corresponding to VHDL Code in Figure 2-31



We saw earlier that the following code represents a D-latch:

```
process(G, D)
begin
  if G = '1' then Q <= D; end if;
end process;
```

Let us understand why this code does not represent an AND gate with G and D as inputs. If G = '1', an AND gate will result in the correct output to match the **if** statement. However, what happens if currently Q = '1' and then G changes to '0'? When G changes to '0', an AND gate would propagate that to the output; however, the device we have modeled here should not. It is expected to make no changes to the output if G is not equal to '1'. Hence, it is clear that this device has to be a D-latch and not an AND gate.

In order to infer flip-flops or registers that change state on the rising edge of a clock signal, an **if**-clause of the form

```
if clock'event and clock = '1' then . . . end if;
```

is required by most synthesizers. For every assignment statement between **then** and **end if** above, a signal on the left side of the assignment will cause creation of a

register or flip-flop. The moral to this story is, if you don't want to create unnecessary flip-flops, don't put the signal assignments in a clocked process. If `clock'event` is omitted, the synthesizer may produce latches instead of flip-flops.

Now consider the VHDL code in Figure 2-34. If you attempt to synthesize this code, the synthesizer will generate an empty block diagram. This is because D, the output of the above block, is never assigned. It will generate warnings that

```
Input <CLK> is never used.
Input <A> is never used.
Input <B> is never used.
Output <D> is never assigned.
```

FIGURE 2-34: Example VHDL Code That Will Not Synthesize

```
entity no_syn is
  port(A,B, CLK: in bit;
       D: out bit);
end no_syn;

architecture no_synthesis of no_syn is
  signal C: bit;
begin
  process(Clk)
  begin
    if (Clk='1' and Clk'event) then
      C <= A and B;
    end if;
  end process;
end no_synthesis;
```

2.12 VHDL Models for Multiplexers

A multiplexer is a combinational circuit and can be modeled using concurrent statements only or using processes. A conditional signal assignment statement such as **when** or a selective signal assignment statement using **with select** can be used to model a multiplexer without processes. A case statement within a process can also be used to make a model for a multiplexer.

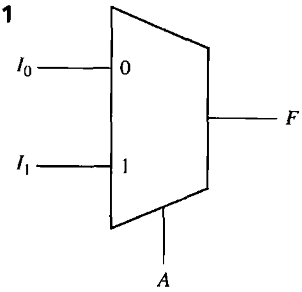
2.12.1 Using Concurrent Statements

Figure 2-35 shows a 2-to-1 multiplexer (MUX) with two data inputs and one control input. The MUX output is $F = A' \cdot I_0 + A \cdot I_1$. The corresponding VHDL statement is

```
F <= (not A and I0) or (A and I1);
```

Here, the MUX can be modeled as a single concurrent signal assignment statement. Alternatively, we can represent the MUX by a conditional signal assignment statement as shown in Figure 2-35. This statement executes whenever A, I_0, or I_1

FIGURE 2-35: 2-to-1 Multiplexer




```
-- conditional signal assignment statement

 F <= I0 when A = '0' else I1;
```

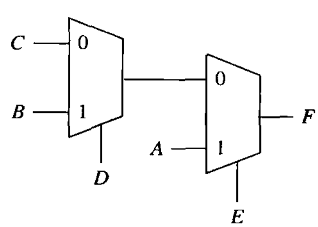
changes. The MUX output is I_0 when A = '0', and otherwise it is I_1. In the conditional statement, I_0, I_1, and F can either be bits or bit-vectors.

The general form of a conditional signal assignment statement is

```
signal_name <= expression1 when condition1
         else expression2 when condition2
         [else expressionN];
```

This concurrent statement is executed whenever a change occurs in a signal used in one of the expressions or conditions. If `condition1` is true, `signal_name` is set equal to the value of `expression1`, otherwise if `condition2` is true, `signal_name` is set equal to the value of `expression2`, and so on. The line in square brackets is optional. Figure 2-36 shows how two cascaded MUXes can be represented by a conditional signal assignment statement. The output MUX selects A when E = '1'; otherwise, it selects the output of the first MUX, which is B when D = '1', or it is C.

FIGURE 2-36: Cascaded 2-to-1 MUXes




```
F <= A when E = '1'
     else B when D = '1'
     else C;
```

Figure 2-37 shows a 4-to-1 multiplexer (MUX) with four data inputs and two control inputs, A and B. The control inputs select which one of the data inputs is transmitted to the output. The logic equation for the 4-to-1 MUX is

$$F = A'B'I_0 + A'BI_1 + A\,B'I_2 + A\,B\,I_3$$

Thus, one way to model the MUX is with the VHDL statement

```
F <= (not A and not B and I0) or (not A and B and I1) or
     (A and not B and I2) or (A and B and I3);
```

Another way to model the 4-to-1 MUX is to use a conditional assignment statement:

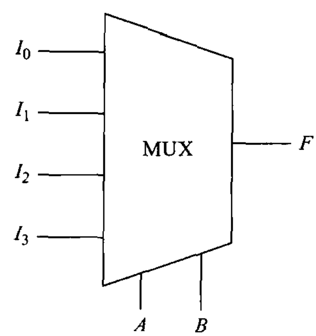
```
F <= I0 when A&B = "00"
else I1 when A&B = "01"
else I2 when A&B = "10"
else I3;
```

The expression A&B means that A is concatenated with B; that is, the two bits A and B are merged together to form a 2-bit vector. This bit-vector is tested and the appropriate MUX input is selected. For example, if A = '1' and B = '0', A&B = "10" and I_2 is selected. Instead of concatenating A and B, we could use a more complex condition:

```
F <= I0 when A = '0' and B = '0'
else I1 when A = '0' and B = '1'
else I2 when A = '1' and B = '0'
else I3;
```

A third way to model the MUX is to use a selected signal assignment statement, as shown in Figure 2-37. A&B cannot be used in this type of statement, so we concatenate A and B to create sel. The value of sel then selects the MUX input that is assigned to F.

FIGURE 2-37: 4-to-1 Multiplexer




```
sel <= A&B;
--selected signal assignment statement
with sel select
  F <= I0 when "00",
       I1 when "01",
       I2 when "10",
       I3 when "11",
```

The general form of a selected signal assignment statement is

```
with expression_s select
  signal_s <= expression1 [after delay-time] when choice1,
              expression2 [after delay-time] when choice2,
              . . .
              [expression_n [after delay-time] when others];
```

This concurrent statement executes whenever a signal changes in any of the expressions. First, `expression_s` is evaluated. If it equals `choice1, signal_s` is set equal to `expression1;` if it equals `choice2, signal_s` is set equal to `expression2;` and so on. If all possible choices for the value of `expression_s` are

given, the last line should be omitted; otherwise, the last line is required. When it is present, if `expression_s` is not equal to any of the enumerated choices, `signal_s` is set equal to `expression_n`. Then `signal_s` is updated after the specified `delay-time`, or after Δ if the **after** `delay-time` is omitted.

2.12.2 Using Processes

If a MUX model is used inside a process, a concurrent statement cannot be used. As an alternative, the MUX can be modeled using a **case** statement:

```
case Sel is
  when 0 => F <= I0;
  when 1 => F <= I1;
  when 2 => F <= I2;
  when 3 => F <= I3;
end case;
```

The case statement has the general form

```
case expression is
  when choice1 => sequential statements1
  when choice2 => sequential statements2
  . . .
  [when others => sequential statements]
end case;
```

The `expression` is evaluated first. If it is equal to `choice1`, then `sequential statements1` are executed; if it is equal to `choice2`, then `sequential statements2` are executed; and so on. All possible values of the expression must be included in the choices. If all values are not explicitly given, a **when others** clause is required in the **case** statement.

One might notice that combinational circuits can be described using concurrent or sequential statements. Sequential circuits generally require a **process** statement. **Process** statements can be used to make sequential or combinational circuits.

2.13 VHDL Libraries

VHDL libraries and packages are used to extend the functionality of VHDL by defining types, functions, components, and overloaded operators. In standard VHDL, some operations are valid only for certain data types. If those operations are desired for other data types, one has to use function "overloading" to create an "overloaded" operator. The concept of "function overloading" exists in many general-purpose languages. It means that two or more functions may have the same name, so long as the parameter types are sufficiently different enough to distinguish which function is actually intended. Overloaded functions can also be created to handle operations involving heterogeneous data types.

In the initial days of CAD, every tool vendor used to create its own libraries and packages. Porting designs from one environment to another became a problem under those conditions. The IEEE has developed standard libraries and packages to make design portability easier. The original VHDL standard only defines 2-valued logic (bits and bit-vectors). One of the earliest extensions was to define multivalued logic as an IEEE standard. The package IEEE.std_logic_1164 defines a std_logic type that has nine values, including '0', '1', 'X' (unknown), and 'Z' (high impedance). The package also defines std_logic_vectors, which are vectors of the std_logic type. This standard defines logic operations and other functions for working with std_logic and std_logic_vectors, but it does not provide for arithmetic operations. The std_logic_1164 package and its use for simulation and synthesis will be described in more detail in Chapter 3.

When VHDL became more widely used for synthesis, the IEEE introduced two packages to facilitate writing synthesizable code: IEEE.numeric_bit and IEEE.numeric_std. The former uses bit_vectors to represent unsigned and signed binary numbers, and the latter uses std_logic_vectors. Both packages define overloaded logic and arithmetic operators for unsigned and signed numbers. Prior to Chapter 3, we will use the numeric_bit package and unsigned numbers for arithmetic operations.

To access functions and components from a library, you need a library statement and a use statement. The statement

```
library IEEE;
```

allows your design to access all packages in the IEEE library. The statement

```
use IEEE.numeric_bit.all;
```

allows your design to use the entire numeric_bit package, which is found in the IEEE library. Whenever a package is used in a module, the library and use statements must be placed before the entity in that module period.

The numeric_bit package defines unsigned and signed types as unconstrained arrays of bits:

```
type unsigned is array (natural range <>) of bit;
type   signed is array (natural range <>) of bit;
```

Signed numbers are represented in 2's complement form. The package contains overloaded operators for arithmetic, relational, logical, and shifting operations on unsigned and signed numbers.

Unsigned and signed types are basically bit-vectors. However, overloaded operators are defined for these types and not for bit-vectors. The statement

```
C <= A + B;
```

will cause a compiler error if *A*, *B*, and *C* are bit_vectors. If these signals are of type unsigned or signed, the compiler will invoke the appropriate overloaded operator to carry out the addition.

The numeric_bit package defines the following overloaded operators:

arithmetic: +, −, *, /, rem, mod
relational:: =, /=, >, <, >=, <=
logical: not, and, or, nand, nor, xor, xnor
shifting: shift_left, shift_right, rotate_left, rotate_right, sll, srl, rol, ror

The arithmetic, relational, and logical operators (except not) each require a left operand and a right operand. For arithmetic and relational operators, the following left and right operand pairs are acceptable: unsigned and unsigned, unsigned and natural, natural and unsigned, signed and signed, signed and integer, integer and signed. For logical operators (except not), left and right operands must either both be unsigned or both signed. When the + and – operators are used with unsigned operands of different lengths, the shortest operand will be extended by filing in 0's on the left. Any carry is discarded so that the result has the same number of bits as the longest operand. For example, when working with unsigned numbers

"1011" + "110" = "1011" + "0110" = "0001" and the carry is discarded.

The numeric_bit package provides an overloaded operator to add an integer to an unsigned, but not to add a bit to an unsigned type. Thus, if A and B are unsigned, `A+B+1` is allowed, but a statement of the form

```
Sum <= A + B + carry;
```

is not allowed when carry is of type bit. The carry must be converted to unsigned before it can be added to the unsigned vector A+B. The notation **`unsigned'(0 => carry)`** will accomplish the necessary conversion.

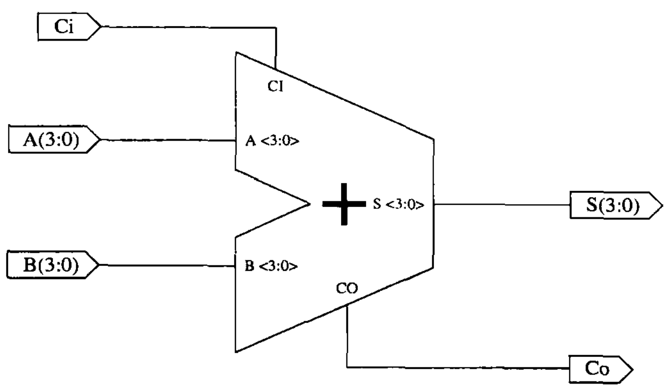
Figure 2-38 shows behavioral VHDL code that uses overloaded operators from the numeric_bit package to describe a 4-bit adder with a carry input. The entity declaration is the same as in Figure 2-12, except type unsigned is used instead of

FIGURE 2-38: VHDL Code for 4-Bit Adder Using Unsigned Vectors

```
library IEEE;
use IEEE.numeric_bit.all;

entity Adder4 is
  port(A, B: in unsigned(3 downto 0); Ci: in bit;  -- Inputs
       S: out unsigned(3 downto 0); Co: out bit);  -- Outputs
end Adder4;

architecture overload of Adder4 is
signal Sum5: unsigned(4 downto 0);
begin
  Sum5 <= '0' & A + B + unsigned'(0=>Ci);  -- adder
  S <= Sum5(3 downto 0);
  Co <= Sum5(4);
end overload;
```

bit_vector. Because adding two 4-bit numbers produces a 5-bit sum, a 5-bit signal (Sum5) is declared within the architecture. If we compute $A + B$, the result is only 4 bits. Since we want a 5-bit result, we must extend A to 5 bits by concatenating '0' and A. (B will automatically be extended to match.) After Sum5 is calculated using the overloaded operators from the numeric_bit package, it is split into a 4-bit sum (S) and a carry (C_o). Most synthesis tools will implement the code of Figure 2-38 as an adder with a carry input and output. One version of the Xilinx synthesizer produces the result shown in Figure 2-39.

FIGURE 2-39: Synthesizer Output for VHDL Code of Figure 2-38



Useful conversion functions found in the numeric_bit package include the following:

`TO_INTEGER(A)`: converts an unsigned vector A to an integer
`TO_UNSIGNED(B, N)`: converts an integer to an unsigned vector of length N
`UNSIGNED(A)`: causes the compiler to treat a bit_vector A as an unsigned vector
`BIT_VECTOR(B)`: causes the compiler to treat an unsigned vector B as a bit_vector

If multivalued logic is desired, one can use the IEEE standard numeric_std package instead of the numeric_bit package. The numeric_std package defines unsigned and signed types as std_logic vectors instead of bit_vectors. Three statements are required to use this package:

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;
```

This package defines the same set of overloaded operators and functions on unsigned and signed numbers as the numeric_bit package.

Another popular VHDL package used for simulation and synthesis with multivalued logic is the std_logic_arith package developed by Synopsis. This package defines unsigned and signed types and overloaded operators similarly to the IEEE

numeric_std package; however, the conversion functions have different names and there are some other differences. A major deficiency of the std_logic_arith package is that it does not define logic operations for unsigned or signed vectors. This package is not an IEEE standard even though it is commonly placed in the IEEE library.

Yet another option is to use the std_logic_unsigned package, also developed by Synopsis. This package does not define unsigned types, but instead it defines some overloaded arithmetic operators for std_logic_vectors. These operators std_logic_vectors as if they were unsigned numbers. When used in conjunction with the std_logic_1164 package, both arithmetic and logic operations can be performed on std_logic_vectors because the 1164 package defines the logic operations. The std_logic_unsigned package is not an IEEE standard even though it is commonly placed in the IEEE library. The VHDL code for the 4-bit adder of Figure 2-38 is rewritten in Figure 2-40 using the std_logic_unsigned package. Because the package provides an overloaded operator to add a std_logic bit to a std_logic_vector, type conversion is not needed. The result of synthesizing this code is the same as that for Figure 2-38.

FIGURE 2-40: VHDL Code for 4-Bit Adder Using the std_logic_unsigned Package

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.std_logic_unsigned.all;
entity Adder4 is
  port(A, B: in std_logic_vector(3 downto 0); Ci: in std_logic; --Inputs
       S: out std_logic_vector(3 downto 0); Co: out std_logic); --Outputs
end Adder4;

architecture overload of Adder4 is
signal Sum5: std_logic_vector(4 downto 0);
begin
  Sum5 <= '0' & A + B + Ci;  --adder
  S <= Sum5(3 downto 0);
  Co <= Sum5(4);
end overload;
```

In this section, we have discussed four different packages, which provide overloaded operators for arithmetic and relational operations. We will initially use the numeric_bit package because it is easiest to use and it is an IEEE standard. Starting in Chapter 3, we will use the IEEE numeric_std package because it is an IEEE standard, provides multivalued signals, and is similar in functionality to the numeric_bit package. We have chosen not to use the std_logic_arith and std_logic_unsigned packages because they are not IEEE standards and they have less functionality than the IEEE numeric_std package.

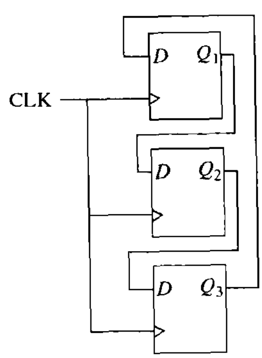
2.14 Modeling Registers and Counters Using VHDL Processes

When several flip-flops change state on the same clock edge, statements representing these flip-flops can be placed in the same clocked process. Figure 2-41 shows three flip-flops connected as a cyclic shift register. These flip-flops all change state following the rising edge of the clock. We have assumed a 5-ns propagation delay between the clock edge and the output change. Immediately following the clock edge, the three statements in the process execute in sequence with no delay. The new values of the Q's are then scheduled to change after 5 ns. If we omit the delay and replace the sequential statements with

```
Q1 <= Q3;     Q2 <= Q1;     Q3 <= Q2;
```

the operation is basically the same. The three statements execute in sequence in zero time, and then the Q's values change after a delta delay. In both cases, the old values of Q_1, Q_2, and Q_3 are used to compute the new values. This may seem strange at first, but that is the way the hardware works. At the rising edge of the clock, all of the D inputs are loaded into the flip-flops, but the state change does not occur until after a propagation delay.

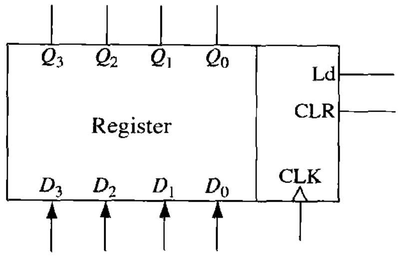
FIGURE 2-41: Cyclic Shift Register




```
process(CLK)
begin
  if CLK'event and CLK = '1' then
    Q1 <= Q3 after 5 ns;
    Q2 <= Q1 after 5 ns;
    Q3 <= Q2 after 5 ns;
  end if;
end process;
```

Figure 2-42 shows a simple register that can be loaded or cleared on the rising edge of the clock. If CLR = '1', the register is cleared, and if Ld = '1', the D inputs are loaded into the register. This register is fully synchronous so that the Q outputs only change in response to the clock edge and not in response to a change in Ld or CLR. In the VHDL code for the register, Q and D are bit-vectors dimensioned 3 **downto** 0. Since the register outputs can only change on the rising edge of the clock, CLR is not on the sensitivity list. It is tested after the rising edge of the clock. If $CLR = Ld$ = '0', no change of Q occurs. Since CLR is tested before Ld, if CLR = '1', the **elsif** prevents Ld from being tested and CLR overrides Ld.

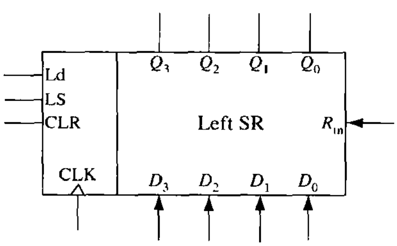
FIGURE 2-42: Register with Synchronous Clear and Load




```
process(CLK)
begin
  if CLK'event and CLK = '1' then
    if CLR = '1' then Q <= "0000";
    elsif Ld = '1' then Q <= D;
    end if;
  end if;
end process;
```

Next, we will model a left shift register using a VHDL process. The register in Figure 2-43 is similar to that in Figure 2-42, except that we have added a left shift control input (LS). When LS is '1', the contents of the register are shifted left and the rightmost bit is set equal to R_{in}. The shifting is accomplished by taking the rightmost 3 bits of Q, `Q(2 downto 0)`, and concatenating them with R_{in}. For example, if Q = "1101" and R_{in} = '0', then `Q(2 downto 0) & Rin` = "1010", and this value is loaded back into the Q register on the rising edge of CLK. The code implies that if $CLR = Ld = LS$ = '0', then Q remains unchanged.

FIGURE 2-43: Left Shift Register with Synchronous Clear and Load

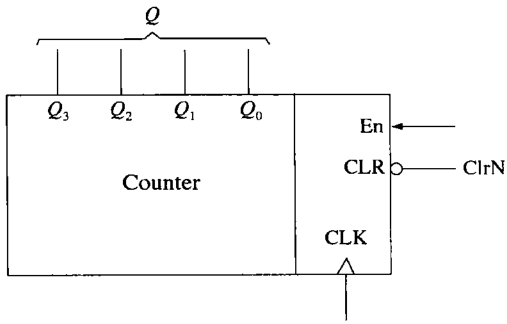



```
process(CLK)
begin
  if CLK'event and CLK = '1' then
    if CLR = '1' then Q <= "0000";
      elsif Ld = '1' then Q <= D;
      elsif LS = '1' then Q <= Q(2 downto 0) & Rin;
    end if;
  end if;
end process;
```

Figure 2-44 shows a simple synchronous counter. On the rising edge of the clock, the counter is cleared when $ClrN$ = '0', and it is incremented when $ClrN = En$ = '1'. In this example, the signal Q represents the 4-bit value stored in the counter. Since addition is not defined for bit-vectors, we have declared Q to be of type unsigned. Then we can increment the counter using the overloaded " + " operator that is defined in the ieee.numeric_bit package. The statement `Q <= Q + 1;` increments the counter. When the counter is in state "1111", the next increment takes it back to state "0000".

Now, let us create a VHDL model for a generic counter, the 74163. It is a 4-bit fully synchronous binary counter, which is available in both TTL and CMOS logic families. Although rarely used in new designs at present, it represents a general type of counter that is found in many CAD design libraries. In addition to performing the

FIGURE 2-44: VHDL Code for a Simple Synchronous Counter




```
signal Q: unsigned (3 downto 0);
-----------
process (CLK)
begin
  if CLK'event and CLK = '1' then
    if ClrN = '0' then Q <= "0000";
    elsif En = '1' then Q <= Q + 1;
    end if;
  end if;
end process;
```

counting function, it can be cleared or loaded in parallel. All operations are synchronized by the clock, and all state changes take place following the rising edge of the clock input. A block diagram of the counter is provided in Figure 2-45.

This counter has four control inputs—*ClrN*, *LdN*, *P*, and *T*. *P* and *T* are used to enable the counting function. Operation of the counter is as follows:

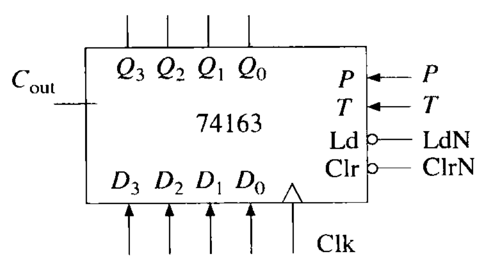
1. If *ClrN* = '0', all flip-flops are set to '0' following the rising clock edge.
2. If *ClrN* = '1' and *LdN* = '0', the *D* inputs are transferred in parallel to the flip-flops following the rising clock edge.
3. If *ClrN* = *LdN* = '1' and *P* = *T* = '1', the count is enabled and the counter state will be incremented by 1 following the rising clock edge.

If *T* = '1', the counter generates a carry (C_{out}) in state 15, so

$$C_{out} = Q_3\, Q_2\, Q_1\, Q_0\, T$$

The truth table in Figure 2-45 summarizes the operation of the counter. Note that *ClrN* overrides the load and count functions in the sense that when *ClrN* = '0',

FIGURE 2-45: 74163 Counter Operation



Control Signals			Next State				
ClrN	LdN	PT	Q_3^+	Q_2^+	Q_1^+	Q_0^+	
0	X	X	0	0	0	0	(clear)
1	0	X	D_3	D_2	D_1	D_0	(parallel load)
1	1	0	Q_3	Q_2	Q_1	Q_0	(increment count)
1	1	1	present state + 1				(no change)

clearing occurs regardless of the values of *LdN*, *P*, and *T*. Similarly, *LdN* overrides the count function. The *ClrN* input on the 74163 is referred to as a *synchronous* clear input because it clears the counter in synchronization with the clock, and no clearing can occur if no clock pulse is present.

The VHDL description of the counter is shown in Figure 2-46. Q represents the four flip-flops that comprise the counter. The counter output, Q_{out}, changes whenever *Q* changes. The carry output is computed whenever *Q* or *T* changes. The first if statement in the process tests for a rising edge of Clk. Since clear overrides load and count, the next **if** statement tests *ClrN* first. Since load overrides count, *LdN* is tested next. Finally, the counter is incremented if both *P* and *T* are '1'. Since *Q* is of type unsigned, we can use the overloaded "+" operator from the ieee.numeric_bit package to add 1 to increment the counter. The expression Q+1 would not be legal if *Q* were a bit-vector since addition is not defined for bit-vectors.

FIGURE 2-46: 74163 Counter Model

```
-- 74163 FULLY SYNCHRONOUS COUNTER

library IEEE;
use IEEE.numeric_bit.all;

entity c74163 is
  port(LdN, ClrN, P, T, Clk: in bit;
       D: in unsigned(3 downto 0);
       Cout: out bit; Qout: out unsigned(3 downto 0));
end c74163;

architecture b74163 of c74163 is
signal Q: unsigned(3 downto 0);    -- Q is the counter register
begin
  Qout <= Q;
  Cout <= Q(3) and Q(2) and Q(1) and Q(0) and T;
  process(Clk)
  begin
    if Clk'event and Clk = '1' then    -- change state on rising edge
      if ClrN = '0' then Q <= "0000";
      elsif LdN = '0' then Q <= D;
      elsif (P and T) = '1' then Q <= Q + 1;
      end if;
    end if;
  end process;
end b74163;
```

To test the counter, we have cascaded two 74163's to form an 8-bit counter (Figure 2-47). When the counter on the right is in state 1111 and T_1 = '1', *Carry1* = '1'. Then for the left counter, *PT* = '1' if *P* = '1'. If *PT* = '1', on the next clock the right counter is incremented to 0000 at the same time the left counter is incremented.

FIGURE 2-47: Two 74163 Counters Cascaded to Form an 8-Bit Counter

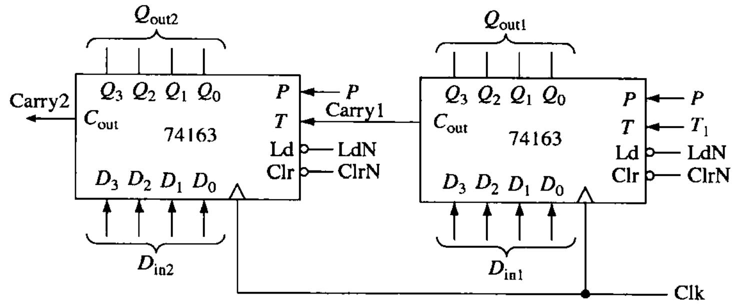


Figure 2-48 shows the VHDL code for the 8-bit counter. In this code we have used the c74163 model as a component and instantiated two copies of it. For convenience in reading the output, we have defined a signal *Count*, which is the integer equivalent of the 8-bit counter value. The function `to_integer` converts an unsigned vector to an integer.

Let us now synthesize the VHDL code for a left shift register (Figure 2-43). Before synthesis is started, we must specify a target device (e.g., a particular FPGA

FIGURE 2-48: VHDL for 8-Bit Counter

```
--Test module for 74163 counter

library IEEE;
use IEEE.numeric_bit.ALL;

entity eight_bit_counter is
  port(ClrN, LdN, P, T1, Clk: in bit;
       Din1, Din2: in unsigned(3 downto 0);
       Count: out integer range 0 to 255;
       Carry2: out bit);
end eight_bit_counter;

architecture cascaded_counter of eight_bit_counter is
component c74163
  port(LdN, ClrN, P, T, Clk: in bit;
       D: in unsigned(3 downto 0);
       Cout: out bit; Qout: out unsigned(3 downto 0));
end component;

signal Carry1: bit;
signal Qout1, Qout2: unsigned(3 downto 0);
begin
  ct1: c74163 port map (LdN, ClrN, P, T1, Clk, Din1, Carry1, Qout1);
  ct2: c74163 port map (LdN, ClrN, P, Carry1, Clk, Din2, Carry2, Qout2);
  Count <= to_integer(Qout2 & Qout1);
end cascaded_counter;
```

or CPLD) so that the synthesizer knows what components are available. Let us assume that the target is a CPLD or FPGA that has D flip-flops with clock enable (D-CE flip-flops). Q and D are 4-bit vectors. Because updates to Q follow "`CLK'event and CLK = '1' then`", this infers that Q must be a register composed of four flip-flops, which we will label Q_3, Q_2, Q_1, and Q_0. Since the flip-flops can change state when *Clr*, *Ld*, or *Ls* is '1', we connect the clock enables to an OR gate whose output is $Clr + Ld + Ls$. Then we connect gates to the D inputs to select the data to be loaded into the flip-flops. If Clr = '0' and Ld = '1', D is loaded into the register on the rising clock edge. If $Clr = Ld$ = '0' and Ls = '1', then Q_2 is loaded into Q_3, Q_1 is loaded into Q_2, and so on. Figure 2-49 shows the logic circuit for the first two flip-flops. If Clr = '1', the D flip-flop inputs are '0'and the register is cleared.

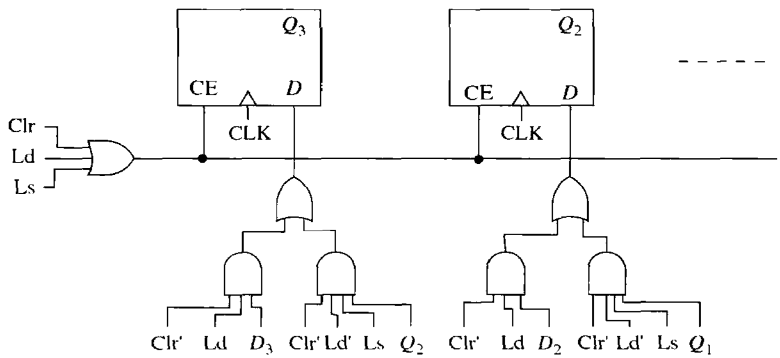


FIGURE 2-49: Synthesis of VHDL Code for Left Shift Register from Figure 2-43

A VHDL synthesizer cannot synthesize delays. Clauses of the form "`after time-expression`" will be ignored by most synthesizers, but some synthesizers require that **after** clauses be removed. Although initial values for signals may be specified in port and signal declarations, these initial values are ignored by the synthesizer. A reset signal should be provided if the hardware must be set to a specific initial state. Otherwise, the initial state of the hardware may be unknown and the hardware may malfunction. When an integer signal is synthesized, the integer is represented in hardware by its binary equivalent. If the range of an integer is not specified, the synthesizer will assume the maximum number of bits, usually 32. Thus

```
signal count: integer range 0 to 7;
```

would result in a 3-bit counter, but

```
signal count: integer;
```

could result in a 32-bit counter.

VHDL signals retain their current values until they are changed. This can result in creation of unwanted latches when the code is synthesized. For example, in a combinational process, the statement

```
if X = '1' then B <= 1; end if;
```

would create latches to hold the value of *B* when *X* changes to '0'. To avoid creation of unwanted latches in a combinational process, always include an **else** clause in every **if** statement. For example,

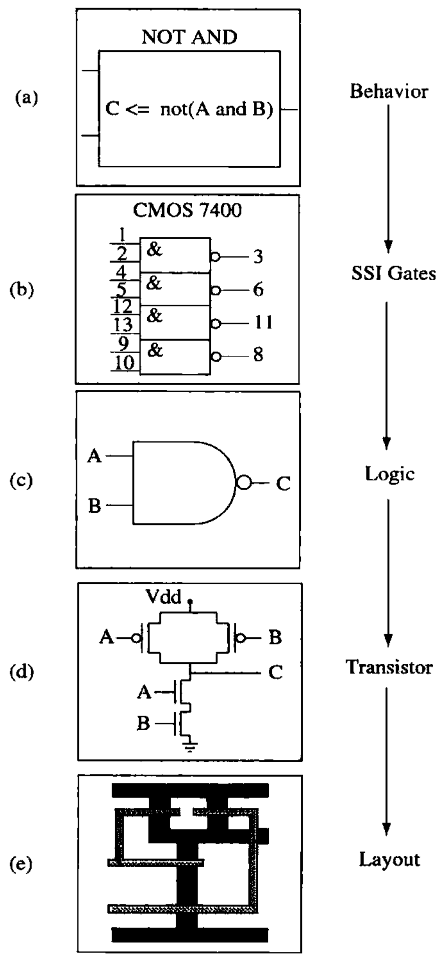
```
if X = '1' then B <= 1 else B <= 0; end if;
```

would create a MUX to switch the value of *B* from 1 to 0.

2.15 Behavioral and Structural VHDL

Any circuit or device can be represented in multiple forms of abstraction. Consider the different representations for a NAND gate, as illustrated in Figure 2-50. When hearing the term NAND, different designers, depending on the domain of their design level, think of these different representations of the same NAND device.

FIGURE 2-50: Different Levels of Abstraction of a NAND Device



Some would think of just a block representing the behavior of a NAND operator, as illustrated in Figure 2-50(a). Some others might think of the four gates in a CMOS 7400 chip, as in Figure 2-50(b). For designers who work at the logic level, they think of the logic symbol for a NAND gate, as in Figure 2-50(c). Transistor-level circuit designers think of the transistor-level circuit to achieve the NAND functionality, as in Figure 2-50(d). What passes through the mind of a physical level designer is the layout of a NAND gate, as in Figure 2-50(e). All of the figures represent the same device, but they differ in the amount of detail provided in the description.

Just as a NAND gate can be described in different ways, any logic circuit can be described with different levels of detail. Figure 2-51 indicates a behavioral level representation of the logic function $F = ab + bc$, whereas Figures 2-52 represents 2 equivalent structural representations. The functionality specified in the abstract description in Figure 2-51 can be achieved in different ways, two examples of which are by using two AND gates and one OR gate or three NAND gates. A structural description gives different descriptions for Figures 2-52(a) and 2-52(b), whereas the same behavioral description could result in either of these two representations. A structural description specifies more details, whereas the behavioral level description only specifies the behavior at a higher level of abstraction.

FIGURE 2-51: A Block Diagram with *A, B, C* as Inputs and *F = AB + BC* as Output

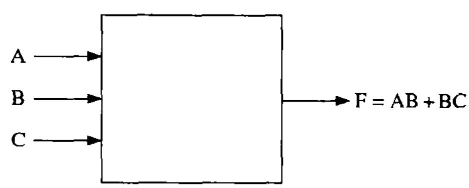
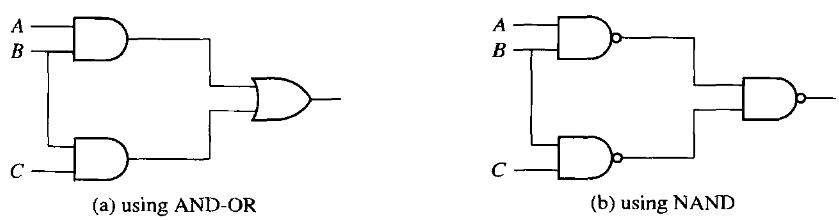


FIGURE 2-52: Two Implementations of *F = AB + BC*



You noticed that the same circuit can be described in different ways. Similarly, VHDL allows you to create design descriptions at multiple levels of abstraction. The most common ones are **behavioral models, dataflow (register transfer language [RTL]) models**, and **structural models**. Behavioral VHDL models describe the circuit or system at a high level of abstraction without implying any particular structure or technology. Only the overall behavior is specified. In contrast, in structural models, the components used and the structure of the interconnection between the components are clearly specified. Structural models may be detailed enough to specify use of particular gates and flip-flops from specific libraries/packages. The structural VHDL model is at a low level of abstraction. VHDL code can be written at an intermediate level of abstraction, at the dataflow level or RTL level, in addition to pure behavioral

level or structural level. Register transfer languages have been used for decades to describe the behavior of synchronous systems where a system is viewed as registers plus control logic required to perform loading and manipulation of registers. In the dataflow model, data path and control signals are specified. The working of the system is described in terms of the data transfer between registers.

If designs are specified at higher levels of abstraction, they need to get converted to the lower levels in order to get implemented. In the early days of design automation, there were not enough automatic software tools to perform this conversion; hence, designs needed to be specified at the lower levels of abstraction. Designs were entered using schematic capture or lower levels of abstraction. Nowadays, synthesis tools perform very efficient conversion of behavioral level designs into target technologies.

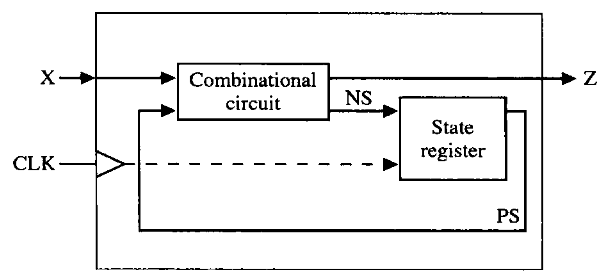
Behavioral and structural design techniques are often combined. Different parts of the design are often done with different techniques. State-of-the-art design automation tools generate efficient hardware for logic and arithmetic circuits; hence, a large part of those designs is done at the behavioral level. However, memory structures often need manual optimizations and are done by custom design, as opposed to automatic synthesis.

2.15.1 Modeling a Sequential Machine

In this section, we discuss several ways of writing VHDL descriptions for sequential machines. Let us assume that we have to write a **behavioral** model for a Mealy sequential circuit represented by the state table in Figure 2-53 (note that this is the BCD to excess-3 code converter designed in Chapter 1). A block diagram of this state machine is also shown in Figure 2-53. This view of the circuit can be used to write its entity description. Please note that the current state and next state are not visible externally.

FIGURE 2-53: State Table and Block Diagram of Sequential Machine

	NS		Z	
PS	$X=0$	$X=1$	$X=0$	$X=1$
S_0	S_1	S_2	1	0
S_1	S_3	S_4	1	0
S_2	S_4	S_4	0	1
S_3	S_5	S_5	0	1
S_4	S_5	S_6	1	0
S_5	S_0	S_0	0	1
S_6	S_0	—	1	—



There are several ways to model this sequential machine. One approach would be to use two processes to represent the two parts of the circuit. One process models the combinational part of the circuit and generates the next state information and outputs. The other process models the state register and updates the state at the appropriate edge of the clock. Figure 2-54 illustrates such a model for this Mealy machine. The first process represents the combinational circuit. At the behavioral level, we will represent the state and next state of the circuit by integer signals initialized to 0. Please remember that this initialization is meaningful only for simulations. Since the circuit outputs, Z and *Nextstate*, can change when either the *State* or X changes, the sensitivity list includes both *State* and X. The case statement tests the

value of *State*, and depending on the value of *X*, *Z* and *Nextstate* are assigned new values. The second process represents the state register. Whenever the rising edge of the clock occurs, *State* is updated to the value of *Nextstate*, so *CLK* appears in the sensitivity list. The second process will simulate correctly if written as

```
process(CLK)                    -- State Register
begin
  if CLK = '1' then             -- rising edge of clock (simulation)
    State <= Nextstate;
  end if;
end process;
```

but in order to synthesize with edge-triggered flip-flops, the `clk'event` attribute must be used, as in

```
process(CLK)                            -- State Register
begin                                   -- (synthesis)
  if CLK'event and CLK = '1' then       -- rising edge of clock
    State <= Nextstate;
  end if;
end process;
```

In Figure 2-54, *State* is an integer with range 0 to 6. The statement **when others => null** is not actually needed here because the outputs and next states of all possible values of *State* are explicitly specified; however, it should be included whenever the **else** clause of any **if** statement is omitted or when actions for all possible values of *State* are not specified. The **null** implies no action, which is appropriate since the other values of *State* should never occur. If else clauses are omitted or actions for any conditions are unspecified, synthesis typically results in creation of latches.

FIGURE 2-54: Behavioral Model for Excess-3 Code Converter

```
-- This is a behavioral model of a Mealy state machine (Figure 2-53)
-- based on its state table. The output (Z) and next state are
-- computed before the active edge of the clock. The state change
-- occurs on the rising edge of the clock.

entity Code_Converter is
  port(X, CLK: in bit;
       Z: out bit);
end Code_Converter;

architecture Behavioral of Code_Converter is
signal State, Nextstate: integer range 0 to 6;
begin
  process(State, X)             -- Combinational Circuit
  begin
    case State is
      when 0 =>
        if X = '0' then Z <= '1'; Nextstate <= 1;
```

```
        else Z <= '0'; Nextstate <= 2; end if;
      when 1 =>
        if X = '0' then Z <= '1'; Nextstate <= 3;
        else Z <= '0'; Nextstate <= 4; end if;
      when 2 =>
        if X = '0' then Z <= '0'; Nextstate <= 4;
        else Z <= '1'; Nextstate <= 4; end if;
      when 3 =>
        if X = '0' then Z <= '0'; Nextstate <= 5;
        else Z <= '1'; Nextstate <= 5; end if;
      when 4 =>
        if X = '0' then Z <= '1'; Nextstate <= 5;
        else Z <= '0'; Nextstate <= 6; end if;
      when 5 =>
        if X = '0' then Z <= '0'; Nextstate <= 0;
        else Z <= '1'; Nextstate <= 0; end if;
      when 6 =>
        if X = '0' then Z <= '1'; Nextstate <= 0;
        else Z <= '0'; Nextstate <= 0; end if;
      when others => null;          -- should not occur
    end case;
  end process;

  process(CLK)                      -- State Register
  begin
    if CLK'EVENT and CLK = '1' then -- rising edge of clock
      State <= Nextstate;
    end if;
  end process;
end Behavioral;
```

A simulator command file that can be used to test Figure 2-54 is as follows:

```
add wave CLK X State NextState Z
force CLK 0 0, 1 100 -repeat 200
force X 0 0, 1 350, 0 550, 1 750, 0 950, 1 1350
run 1600
```

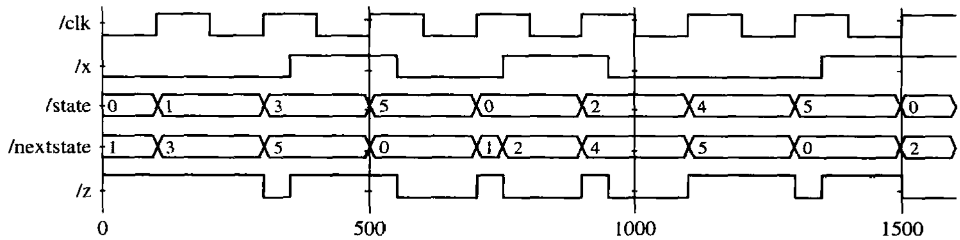
The first command specifies the signals that are to be included in the waveform output. The next command defines a clock with a period of 200 ns. *CLK* is '0' at time 0 ns, is '1' at time 100 ns, and repeats every 200 ns. In a command of the form

```
force signal_name v1 t1, v2 t2, . . .
```

`signal_name` gets the value `v1` at time `t1`, the value `v2` at time `t2`, and so on. *X* is '0' at time 0 ns, changes to '1' at time 350 ns, changes to '0' at time 550 ns, and so on. The *X* input corresponds to the sequence 0010 1001, and only the times at which *X* changes are specified. Execution of the preceding command file produces the waveforms shown in Figure 2-55.

In Chapter 1, we manually designed this state machine (Figure 1-26). This circuitry contained three flip-flops, four 3-input NAND gates, two 3-input NAND

FIGURE 2-55: Simulator Output for Excess-3 Code Converter



gates, and one inverter. The behavioral model of Figure 2-54 may not result in exactly that circuit. In fact, when we synthesized it using Xilinx ISE tools, we got a circuit that contains seven D-flip-flops, fifteen 2-input AND gates, three 2-input OR gates, and one 7-input OR gate. Apparently, the Xilinx synthesis tool may be using one-hot design by default, instead of encoded design. One-hot design is a popular approach for FPGAs, where flip-flops are abundant.

Figure 2-56 shows an alternative behavioral model for the code converter that uses a single process instead of two processes. The next state is not computed explicitly, but instead the state register is updated directly to the proper next state value on the rising edge of the clock. Since Z can change whenever State or X changes, Z should not be computed in the clocked process. Instead, we have used a conditional assignment statement to compute Z. If Z were updated in the clocked process, then a flip-flop would be created to store Z and Z would be updated at the wrong time. In general, the two-process model for a state machine is preferable to the one-process model, since the former corresponds more closely to the hardware implementation which uses a combinational circuit and a state register.

FIGURE 2-56: Behavioral Model for Code Converter Using a Single Process

```
-- This is a behavioral model of the Mealy state machine for BCD to
-- Excess-3 Code Converter based on its state table. The state change
-- occurs on the rising edge of the clock. The output is computed by a
-- conditional assignment statement whenever State or Z changes.

entity Code_Converter is
  port(X, CLK: in bit;
       Z: out bit);
end Code_Converter;

architecture one_process of Code_Converter is
signal State: integer range 0 to 6 := 0;
begin
  process(CLK)
  begin
    if CLK'event and CLK = '1' then
      case State is
        when 0 =>
          if X = '0' then State <= 1; else State <= 2; end if;
```

```
      when 1 =>
        if X = '0' then State <= 3; else State <= 4; end if;
      when 2 =>
        State <= 4;
      when 3 =>
        State <= 5;
      when 4 =>
        if X = '0' then State <= 5; else State <= 6; end if;
      when 5 =>
        State <= 0;
      when 6 =>
        State <= 0;
    end case;
  end if;
end process;
Z <= '1' when (State = 0 and X = '0') or (State = 1 and X = '0')
          or (State = 2 and X = '1') or (State = 3 and X = '1')
          or (State = 4 and X = '0') or (State = 5 and X = '1')
          or State = 6
     else '0';
end one_process;
```

Another way to model this Mealy machine is using the **dataflow** approach (i.e., using equations). The dataflow VHDL model of Figure 2-57 is based on the next state and output equations, which are derived in Chapter 1 (Figure 1-25). The flip-flops are updated in a process that is sensitive to *CLK*. When the rising edge of the clock occurs, Q_1, Q_2, and Q_3 are all assigned new values. A 10-ns delay is included to represent the propagation delay between the active edge of the clock and the change of the flip-flop outputs. Even though the assignment statements in the process are executed sequentially, Q_1, Q_2, and Q_3 are all scheduled to be updated at the same time, $T + \Delta$, where T is the time at which the rising edge of the clock occurred. Thus,

FIGURE 2-57: Sequential Machine Model Using Equations

```
-- The following is a description of the sequential machine of
-- the BCD to Excess-3 code converter in terms of its next state
-- equations. The following state assignment was used:
-- S0-->0; S1-->4; S2-->5; S3-->7; S4-->6; S5-->3; S6-->2

entity Code_Converter is
  port(X, CLK: in bit;
       Z: out bit);
end Code_Converter;

architecture Equations of Code_Converter is
signal Q1, Q2, Q3: bit;
begin
  process(CLK)
```

```
  begin
    if CLK = '1' and CLK'event then     -- rising edge of clock
      Q1 <= not Q2 after 10 ns;
      Q2 <= Q1 after 10 ns;
      Q3 <= (Q1 and Q2 and Q3) or (not X and Q1 and not Q3) or
            (X and not Q1 and not Q2) after 10 ns;
    end if;
  end process;
  Z <= (not X and not Q3) or (X and Q3) after 20 ns;
end Equations;
```

the old value of Q_1 is used to compute Q_2^+, and the old values of Q_1, Q_2, and Q_3 are used to compute Q_3^+. The concurrent assignment statement for Z causes Z to be updated whenever a change in X or Q_3 occurs. The 20-ns delay represents two gate delays. Note that in order to do VHDL modeling at this level, we need to perform state assignments, derive next state equations, and so on. In contrast, at the behavioral level, the state table was sufficient to create the VHDL model.

Yet another approach to creating a VHDL model of the aforementioned Mealy machine is to create a **structural** model describing the gates and flip-flops in the circuit. Figure 2-58 shows a structural VHDL representation of the circuit of Figure 1-26. Note that the designer had to manually perform the design and obtain the gate level circuitry here in order to create a model as in Figure 2-58. Seven NAND gates, three D flip-flops, and one inverter are used in the design presented in Chapter 1. When primitive components like gates and flip-flops are required, each of these components can be defined in a separate VHDL module. Depending on which CAD tools are used, the component modules can be included in the same file as the main VHDL description, or they be inserted as separate files in a VHDL project. The code in Figure 2-58 requires component modules DFF, Nand3, Nand2, and Inverter. CAD tools might include packages with similar components. If such packages are used, one should use the exact component names and port-map statements that match the input-output signals of the component in the package. The DFF module is as follows:

```
--D Flip-Flop
entity DFF is
  port(D, CLK: in bit;
       Q: out bit; QN: out bit := '1');
-- initialize QN to '1' since bit signals are defaulted to '0'
end DFF;
architecture SIMPLE of DFF is
begin
  process(CLK)            -- process is executed when CLK changes
  begin
    if CLK'event and CLK = '1' then    -- rising edge of clock
      Q <= D after 10 ns;
      QN <= not D after 10 ns;
    end if;
  end process;
end SIMPLE;
```

FIGURE 2-58: Structural Model of Sequential Machine

```
-- The following is a STRUCTURAL VHDL description of
-- the circuit to realize the BCD to Excess-3 code Converter.
-- This circuit was illustrated in Figure 1-26.
-- Uses components NAND3, NAND2, INVERTER and DFF
-- The component modules can be included in the same file
-- or they can be inserted as separate files.

entity Code_Converter is
  port(X,CLK: in bit;
       Z: out bit);
end Code_Converter;

architecture Structure of Code_Converter is
component DFF
  port(D, CLK: in bit; Q: out bit; QN: out bit := '1');
end component;
component Nand2
  port(A1, A2: in bit; Z: out bit);
end component;
component Nand3
  port(A1, A2, A3: in bit; Z: out bit);
end component;
component Inverter
  port(A: in bit; Z: out bit);
end component;
signal A1, A2, A3, A5, A6, D3: bit;
signal Q1, Q2, Q3: bit;
signal Q1N, Q2N, Q3N, XN: bit;
begin
  I1:  Inverter port map (X, XN);
  G1:  Nand3 port map (Q1, Q2, Q3, A1);
  G2:  Nand3 port map (Q1, Q3N, XN, A2);
  G3:  Nand3 port map (X, Q1N, Q2N, A3);
  G4:  Nand3 port map (A1, A2, A3, D3);
  FF1: DFF port map (Q2N, CLK, Q1, Q1N);
  FF2: DFF port map (Q1, CLK, Q2, Q2N);
  FF3: DFF port map (D3, CLK, Q3, Q3N);
  G5:  Nand2 port map (X, Q3, A5);
  G6:  Nand2 port map (XN, Q3N, A6);
  G7:  Nand2 port map (A5, A6, Z);
end Structure;
```

The Nand3 module is as follows:

```
--3 input NAND gate
entity Nand3 is
  port(A1, A2, A3: in bit; Z: out bit);
end Nand3;
```

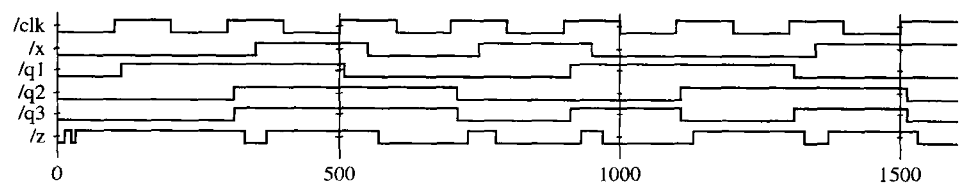
```
architecture concur of Nand3 is
begin
  Z <= not (A1 and A2 and A3) after 10 ns;
end concur;
```

The Nand2 and Inverter modules are similar except for the number of inputs. We have assumed a 10-ns delay in each component, and this can easily be changed to reflect the actual delays in the hardware being used.

Since Q_1, Q_2, and Q_3 are initialized to '0', the complementary flip-flop outputs (Q_1N, Q_2N, and Q_3N) are initialized to '1'. G_1 is a three-input NAND gate with inputs Q_1, Q_2, Q_3, and output A_1. FF_1 is a D flip-flop with the D input connected to Q_2N. Executing the simulator command file given next produces the waveforms of Figure 2-59, which are very similar to Figure 1-39.

```
add wave CLK X Q1 Q2 Q3 Z
force CLK 0 0, 1 100 -repeat 200
force X 0 0, 1 350, 0 550, 1 750, 0 950, 1 1350
run 1600
```

FIGURE 2-59: Waveforms for Code Converter



If we synthesized this structural description, we would get exactly the same circuit that we had in mind. Now the circuit includes only three D-flip-flops, three 2-input NAND gates, and four 3-input NAND gates. Compare it against the seven D-flip-flops, fifteen 2-input AND gates, three 2-input OR gates, and one 7-input OR gate generated when Figure 2-54 was synthesized. When the designer specified all components and their interconnections, the synthesizer tool did not have to infer or "guess."

Those who have developed C code with assembly inlining may feel some similarity to the phenomenon occurring here. By inlining the assembly code, you can precisely describe what microprocessor instruction sequence you want to be used, and the compiler gives you that. In a similar way, the synthesizer does not actually have to translate any structural descriptions that the designer wrote; it simply gives the hardware that the designer specified in a structural fashion. Some optimizing tools are capable of optimizing imperfect circuits that you might have specified. In general, you have more control of the generated circuitry when you use structural coding. However, it takes a lot more effort to produce a structural model because one needs to perform state assignments, derive next-state equations, and so on. **Time-to-market** is an important criterion for success in the IC market, and hence designers often use behavioral design in order to achieve quick time-to-market. Additionally, CAD tools have matured significantly during the past decade, and most synthesis tools are capable of producing efficient hardware for arithmetic and logic circuits.

2.16 Variables, Signals, and Constants

So far, we have used only signals in the VHDL code and have not used variables. VHDL also provides variables as in other general-purpose high-level languages. Variables may be used for local storage in processes. They can also be used in procedures and functions (which are yet to be introduced). A large part of what is described in this section is relevant only for simulation.

A variable declaration has the form

```
variable list_of_variable_names: type_name [ := initial_value];
```

Variables must be declared within the process in which they are used and are local to that process. (An exception to this rule is *shared* variables, which are not discussed in this text.) Signals, on the other hand, must be declared outside of a process. Signals declared at the start of an architecture can be used anywhere within that architecture. A signal declaration has the form

```
signal list_of_signal_names: type_name [ := initial_value];
```

Variables are updated using a variable assignment statement of the form

```
variable_name := expression;
```

When this statement is executed, the variable is instantaneously updated with no delay, not even a delta delay. In contrast, consider a signal assignment of the form

```
signal_name <= expression [after delay];
```

The expression is evaluated when this statement is executed, and the signal is scheduled to change after delay. If no delay is specified, then the signal is scheduled to be updated after a delta delay.

It is incorrect to use

```
variable_name <= expression [after delay];
```

> **When to Use a Signal versus Variable:** If whatever you are modeling actually corresponds to some physical signal in your circuit, you should use a signal. If whatever you are modeling is simply a temporary value that you are using for convenience of programming, a variable will be sufficient. Values represented using variables will not appear on any physical wire in the implied circuit. If you would like them to appear, you should use signals.

The examples in Figures 2-60 and 2-61 illustrate the difference between using variables and signals in a process. The variables must be declared and initialized inside the process, whereas the signals must be declared and initialized outside the process. In Figure 2-60, if *trigger* changes at time = 10 ns, Var1, Var2, and Var3 are computed sequentially and updated instantly, and then Sum is computed using the

FIGURE 2-60: Process Using Variables and Corresponding Simulation Output

```
entity dummy is
end dummy;

architecture var of dummy is
signal trigger, sum: integer:=0;
begin
  process
  variable var1: integer:=1;
  variable var2: integer:=2;
  variable var3: integer:=3;
  begin
    wait on trigger;
    var1 := var2 + var3;
    var2 := var1;
    var3 := var2;
    sum <= var1 + var2 + var3;
  end process;
end var;
```

Simulation Output of 2-60

ns	delta	trigger	Var1	Var2	Var3	Sum
0	+0	0	1	2	3	0
0	+1	0	1	2	3	0
10	+0	1	5	5	5	0
10	+1	1	5	5	5	15

new variable values. The sequence is Var1 = 2 + 3 = 5, Var2 = 5, Var3 = 5. Then Sum = 5 + 5 + 5 is computed. Since Sum is a signal, it is updated Δ time later, so Sum = 15 at time = 10 + Δ. In summary, variables work just as variables you are used to in another language, whereas signals get updated with time delays. In Figure 2-61, if *trigger* changes at time = 10 ns, signals Sig1, Sig2, Sig3, and Sum are all computed at time 10 ns, but the signals are not updated until time 10 + Δ. The old values of Sig1 and Sig2 are used to compute Sig2 and Sig3. Therefore, at time = 10 + Δ, Sig1 = 5, Sig2 = 1, Sig3 = 2, and Sum = 6.

FIGURE 2-61: Process Using Signals and Corresponding Simulation Output

```
entity dummy is
end dummy;

architecture sig of dummy is
signal trigger, sum: integer:=0;
signal sig1: integer:=1;
signal sig2: integer:=2;
signal sig3: integer:=3;
begin
  process
  begin
    wait on trigger;
    sig1 <= sig2 + sig3;
    sig2 <= sig1;
    sig3 <= sig2;
    sum <= sig1 + sig2 + sig3;
  end process;
end sig;
```

Simulation Output of 2-61

ns	delta	trigger	Sig1	Sig2	Sig3	Sum
0	+0	0	1	2	3	0
0	+1	0	1	2	3	0
10	+0	1	1	2	3	0
10	+1	1	5	1	2	6

During simulation, initialization makes the process execute once, and it stops when wait statements are encountered. Hence, simulation outputs can vary depending on whether the wait statements are put at the beginning of the process, end of the process, or whether a sensitivity list is used. Figures 2-62 and 2-63 illustrate various possibilities. Please remember that these differences are not important when VHDL is used for synthesis of hardware. These are subtle differences that only affect simulation of behavioral VHDL.

FIGURE 2-62: Process Using Variables and Corresponding Simulation Output

```
entity dummy is
end dummy;

architecture var of dummy is
signal trigger, sum: integer:=0;
begin
  process(trigger)
  variable var1: integer:=1;
  variable var2: integer:=2;
  variable var3: integer:=3;
  begin
    var1 := var2 + var3;
    var2 := var1;
    var3 := var2;
    sum <= var1 + var2 + var3;
  end process;
end var;
```

Simulation Output of 2-62

ns	delta	trigger	Var1	Var2	Var3	Sum
0	+0	0	1	2	3	0
0	+1	0	5	5	5	15
10	+0	1	10	10	10	15
10	+1	1	10	10	10	30

FIGURE 2-63: Process Using Signals and Corresponding Simulation Output

```
entity dummy is
end dummy;

architecture sig of dummy is
signal trigger, sum: integer:=0;
signal sig1: integer:=1;
signal sig2: integer:=2;
signal sig3: integer:=3;
begin
  process(trigger)
  begin
    sig1 <= sig2 + sig3;
    sig2 <= sig1;
    sig3 <= sig2;
    sum <= sig1 + sig2 + sig3;
  end process;
end sig;
```

Simulation Output of 2-63

ns	delta	trigger	Sig1	Sig2	Sig3	Sum
0	+0	0	1	2	3	0
0	+1	0	5	1	2	6
10	+0	1	5	1	2	6
10	+1	1	3	5	1	8

2.16.1 Constants

Like variables, constants are also used for convenience of programming.

A common form of constant declaration is

```
constant constant_name: type_name := constant_value;
```

A constant *delay1* of type time, having the value of 5 ns, can be defined as

```
constant delay1: time := 5 ns;
```

Constants declared at the start of an architecture can be used anywhere within that architecture, but constants declared within a process are local to that process.

Variables, signals, and constants can have any one of the predefined VHDL types, or they can have a user-defined type.

2.17 Arrays

Digital systems often use memory arrays. VHDL arrays can be used to specify the values to be stored in these arrays. A key feature of VLSI circuits is the repeated use of similar structures. Arrays in VHDL can be used while modeling the repetition.

In order to use an array in VHDL, we must first declare an array type and then declare an array object. For example, the following declaration defines a one-dimensional array type named SHORT_WORD:

```
type SHORT_WORD is array (15 downto 0) of bit;
```

An array of this type has an integer index with a range from 15 **downto** 0, and each element of the array is of type bit. SHORT_WORD is the name of the newly created data type. We may note that SHORT_WORD is nothing but a bit_vector of size 16.

Now, we can declare array objects of type SHORT_WORD as follows:

```
signal     DATA_WORD:   SHORT_WORD;
variable   ALT_WORD:    SHORT_WORD := "0101010101010101";
constant   ONE_WORD:    SHORT_WORD := (others => '1');
```

Three different arrays are defined by the preceding statements. *DATA_WORD* is a signal array of 16 bits, indexed 15 **downto** 0, which is initialized (by default) to all '0' bits. *ALT_WORD* is a variable array of 16 bits, which is initialized to alternating 0's and 1's. *ONE_WORD* is a constant array of 16 bits; all bits are set to 1 by `(others => '1')`.

We can reference individual elements of the defined array by specifying an index value. For example, *ALT_WORD(0)* accesses the rightmost bit of *ALT_WORD*. We can also specify a portion of the array by specifying an index range: *ALT_WORD(5* **downto** *0)* accesses the low-order 6 bits of *ALT_WORD*, which have an initial value of "010101".

The array type and array object declarations illustrated here have the general forms

```
type array_type_name is array index_range of element_type;
signal array_name: array_type_name [ := initial_values];
```

In the preceding declaration, **signal** may be replaced with **variable** or **constant.**

2.17.1 Matrices

Multidimensional array types may also be defined with two or more dimensions. The following example defines a two-dimensional array variable, which is a matrix of integers with four rows and three columns:

```
type matrix4x3 is array (1 to 4, 1 to 3) of integer;
variable matrixA: matrix4x3 := ((1, 2, 3), (4, 5, 6), (7, 8, 9),
      (10, 11, 12));
```

The variable *matrixA* will be initialized to

1	2	3
4	5	6
7	8	9
10	11	12

The array element *matrixA(3, 2)* references the element in the third row and second column, which has a value of 8.

When an array type is declared, the dimensions of the array may be left undefined. This is referred to as an *unconstrained array type.* For example,

```
type intvec is array (natural range <>) of integer;
```

declares intvec as an array type that defines a one-dimensional array of integers with an unconstrained index range of natural numbers. The default type for array indices is integer, but another type may be specified. Since the index range is not specified in the unconstrained array type, the range must be specified when the array object is declared. For example,

```
signal intvec5: intvec(1 to 5) := (3, 2, 6, 8, 1);
```

defines a signal array named *intvec5* with an index range of 1 to 5 that is initialized to 3, 2, 6, 8, 1. The following declaration defines matrix as a two-dimensional array type with unconstrained row and column index ranges:

```
type matrix is array (natural range <>, natural range <>) of
      integer;
```

Example

Parity bits are often used in digital communication for error detection and correction. The simplest of these involve transmitting one additional bit with the data, a parity bit. Use VHDL arrays to represent a parity generator that generates a 5-bit-odd-parity generation for a 4-bit input number using the look-up table (LUT) method.

Answer

The input word is a 4-bit binary number. A 5-bit odd-parity representation will contain exactly an odd number of 1's in the output word. This can be accomplished by the read-only memory (ROM) method using a look-up table of size 16 entries × 5 bits. The look-up table is indicated in Figure 2-64.

FIGURE 2-64: LUT Contents for a Parity Code Generator

Input (LUT Address)				Output (LUT Data)				
A	B	C	D	P	Q	R	S	T
0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	1	0
0	0	1	0	0	0	1	0	0
0	0	1	1	0	0	1	1	1
0	1	0	0	0	1	0	0	0
0	1	0	1	0	1	0	1	1
0	1	1	0	0	1	1	0	1
0	1	1	1	0	1	1	1	0
1	0	0	0	1	0	0	0	0
1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	1
1	0	1	1	1	0	1	1	0
1	1	0	0	1	1	0	0	1
1	1	0	1	1	1	0	1	0
1	1	1	0	1	1	1	0	0
1	1	1	1	1	1	1	1	1

The VHDL code for the parity generator is illustrated in Figure 2-65. The IEEE numeric bit package is used here. *X* and *Y* are defined to be unsigned vectors. The first four bits of the output are identical to the input. Hence, instead of storing all five bits of the output, we might store only the parity bit and then concatenate it to the input bits. In the VHDL code (Figure 2-65), a new data type `OutTable` is defined to be an array of 16 bits. A constant table of type `OutTable` is defined using the following statement:

```
type OutTable is array(0 to 15) of bit;
```

The index of this array is an integer in the range `0 to 15`. Hence, unsigned vector *X* needs to be converted to an integer first, which can be done using the `to_integer` function defined in the library.

FIGURE 2-65: Parity Code Generator Using the LUT Method

```
library IEEE;
use IEEE.numeric_bit.all;

entity parity_gen is
  port(X: in unsigned(3 downto 0);
       Y: out unsigned(4 downto 0));
end parity_gen;

architecture Table of parity_gen is
type OutTable is array(0 to 15) of bit;
signal ParityBit: bit;
```

```
constant OT: OutTable := ('1','0','0','1','0','1','1','0',
                          '0','1','1','0','1','0','0','1');
begin
  ParityBit <= OT(to_integer(X));
  Y <= X & ParityBit;
end Table;
```

Predefined unconstrained array types in VHDL include bit_vector and string, which are defined as follows:

```
type bit_vector is array (natural range <>) of bit;
type string is array (positive range <>) of character;
```

The characters in a string literal must be enclosed in double quotes. For example, "This is a string." is a string literal. The following example declares a constant *string1* of type string:

```
constant string1: string(1 to 29) :=
   "This string is 29 characters."
```

A bit_vector literal may be written either as a list of bits separated by commas or as a string. For example, ('1','0','1','1','0') and "10110" are equivalent forms. The following declares a constant *A* that is a bit_vector with a range `0 to 5`:

```
constant A: bit_vector(0 to 5) := "101011";
```

After a type has been declared, a related subtype can be declared to include a subset of the values specified by the type. For example, the type SHORT_WORD, which was defined at the start of this section, could have been defined as a subtype of bit_vector:

```
subtype SHORT_WORD is bit_vector (15 downto 0);
```

Two predefined subtypes of type integer are POSITIVE, which includes all positive integers, and NATURAL, which includes all positive integers and 0.

2.18 Loops in VHDL

Often, we encounter systems where some activity is happening in a repetitive fashion. VHDL loop statements can be used to express this behavior. A loop statement is a sequential statement. VHDL has several kinds of loop statements including **for** loops and **while** loops.

1. ***infinite loop***

   Infinite loops are undesirable in common computer languages, but they can be useful in hardware modeling where a device works continuously and continues to work until the power is off.

The general form for an infinite loop is

```
[loop-label:] loop
  sequential statements
end loop [loop-label];
```

An exit statement of the form

```
exit; or exit when condition;
```

may be included in the loop. The loop will terminate when the exit statement is executed, provided that the condition is TRUE.

2. **for loop**

One way to augment the basic loop is the **for** loop, where the number of invocations of the loop can be specified.

The general form of a **for** loop is

```
[loop-label:] for loop-index in range loop
  sequential statements
end loop [loop-label];
```

The `loop-index` is automatically defined when the loop is entered, and it should not explicitly be declared. It is initialized to the first value in the range and then the `sequential statements` are executed. The range is specified, for example as 0 **to** n, where n can be a constant or variable. The `loop-index` can be used within the `sequential statements` inside the loop, but it cannot be changed within the loop. When the end of the loop is reached, the `loop-index` is set to the next value in the `range` and the `sequential statements` are executed again. This process continues until the loop has been executed for every value in the `range`, and then the loop terminates. After the loop terminates, the `loop-index` is no longer available.

We could use this type of a loop in behavioral models. The following excerpt models a 4-bit adder. The loop index (i) will be initialized to 0 when the **for** loop is entered, and the sequential statements will be executed. Execution will be repeated for $i = 1$, $i = 2$, and $i = 3$; then the loop will terminate. The carry out from one iteration (*cout*) is copied to the carry in (*cin*) before the end of the loop. Since variables are used for the sum and carry bits, the update of carry out happens instantaneously. Code like this often appears in VHDL functions and procedures (described in Chapter 3):

```
loop1: for i in 0 to 3 loop
  cout := (A(i) and B(i)) or (A(i) and cin) or (B(i) and cin);
  sum(i) := A(i) xor B(i) xor cin;
  cin := cout;
end loop loop1;
```

You could also use the for loop construct to create multiple copies of a basic cell. When the preceding code is synthesized, the synthesizer typically provides four copies of a 1-bit adder connected in a ripple carry fashion.

3. **while loop**

In the **for** loop, the loop index cannot be changed by the programmer. However, in the **while** loop, the loop index can be manipulated by the programmer. So incrementing the loop index by 2 can be done in the **while** loop. As in **while** loops in most languages, a condition is tested before each iteration. The loop is terminated if the condition is false. The general form of a **while** loop is

```
[loop-label:] while condition loop
  sequential statements
end loop [loop-label];
```

This construct is primarily for simulation.

Figure 2-66 illustrates a **while** loop that models a down counter. We use the **while** statement to continue the decrementing process until the stop is encountered or the counter reaches 0. The counter is decremented on every rising edge of `clk` until either the `count` is 0 or `stop` is 1.

FIGURE 2-66: Use of While Loop

```
while stop = '0' and count /= 0 loop
  wait until clk'event and clk = '1';
    count <= count - 1 ;
  wait for 0 ns;
end loop;
```

2.19 Assert and Report Statements

Once a VHDL model for a system is made, the next step is to test it. A model must be tested and validated before it can be successfully used. VHDL provides some special statements, such as **assert, report**, and **severity**, to aid in the testing and validation process.

The **assert** statement checks to see if a certain condition is true, and, if not, it causes an error message to be displayed. One form of the assert statement is

```
assert boolean-expression
  report string-expression
  [severity severity-level;]
```

The **assert** statement specifies a Boolean expression which indicates the condition to be met. If the condition has not been met, an assertion violation has occurred. If an assertion violation occurs during simulation, the simulator reports it with the `string-expression` provided in the **report** clause. If the `boolean-expression` is false, then the `string-expression` is displayed on the monitor along with the `severity-level`. If the `boolean-expression` is true, no message is displayed.

There are four possible `severity-levels`: note, warning, error, and failure. We can include one of these to indicate the degree to which the violation of the particular assertion affects the operation of the model. For instance, a serious violation may have to be flagged as a failure, whereas some minor violation only needs to be flagged as a note or warning. The action taken for these severity-levels depends on the simulator. The severity-level is optional.

If the **assert** clause is omitted, then the report is always made. Thus, the statement

```
report "ALL IS WELL";
```

will display the message "ALL IS WELL" whenever the statement is executed.

Assert and **report** statements are very useful for creation of **test benches**. A test bench is a piece of VHDL code that can provide input combinations to test a VHDL model for the system under test. It provides stimuli to the system/circuit under test. Test benches are frequently used during simulation to provide sequences of inputs to the circuit/VHDL model under test. Figure 2-67 shows a test-bench for testing the 4-bit binary adder that we created earlier in this chapter. The adder we are testing will be treated as a component and embedded in the test bench program. The signals generated within the test bench are interfaced to the adder, as shown in Figure 2-67. The test bench code in Figure 2-68 uses constant arrays to define the test inputs for the adder and the expected outputs. It uses a **for** loop to select the inputs from the

FIGURE 2-67: Interfacing of Signals while Using a Test Bench to Test a 4-Bit Adder

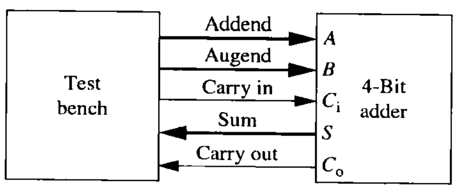


FIGURE 2-68: Test Bench for 4-Bit Adder

```
entity TestAdder is
end TestAdder;

architecture test1 of TestAdder is
component Adder4
  port(A, B: in bit_vector(3 downto 0); Ci: in bit;
       S: out bit_vector(3 downto 0); Co: out bit);
end component;
constant N: integer := 11;
type bv_arr is array(1 to N) of bit_vector(3 downto 0);
type bit_arr is array(1 to N) of bit;
constant addend_array: bv_arr := ("0111", "1101", "0101", "1101",
         "0111", "1000", "0111", "1000", "0000", "1111", "0000");
constant augend_array: bv_arr := ("0101", "0101", "1101", "1101",
         "0111", "0111", "1000", "1000", "1101", "1111", "0000");
constant cin_array: bit_arr := ('0', '0', '0', '0', '1', '0', '0',
         '0', '1', '1', '0');
```

```
constant sum_array: bv_arr := ("1100", "0010", "0010", "1010",
         "1111", "1111", "1111", "0000", "1110", "1111", "0000");
constant cout_array: bit_arr := ('0', '1', '1', '1', '0', '0', '0',
         '1', '0', '1', '0');
signal addend, augend, sum: bit_vector(3 downto 0);
signal cin, cout: bit;
begin
  process
  begin
    for i in 1 to N loop
      addend <= addend_array(i);
      augend <= augend_array(i);
      cin <= cin_array(i);
      wait for 40 ns;
      assert (sum = sum_array(i) and cout = cout_array(i))
        report "Wrong Answer"
        severity error;
    end loop;
    report "Test Finished";
  end process;
  add1: adder4 port map (addend, augend, cin, sum, cout);
end test1;
```

arrays. It uses **assert** and **report** statements to check the outputs and report whether the output matched the expected output for the particular combination of inputs. The **assert** statement is meaningful only for simulation. During synthesis, the synthesizer may simply assume that the assertion violation does not exist.

We will provide another example to illustrate how a waveform input can be provided in a test bench. In earlier examples in this chapter, we used simulator commands to test VHDL models. Figure 2-69 illustrates a piece of VHDL code that

FIGURE 2-69: Generating a Test Sequence for Testing VHDL Model for Code Converter

```
entity test_code_conv is
end test_code_conv;

architecture tester of test_code_conv is
signal X, CLK, Z: bit;
component Code_Converter is
  port(X, CLK: in bit;
       Z: out bit);
end component;
begin
  clk <= not clk after 100 ns;
  X <= '0', '1' after 350 ns, '0' after 550 ns, '1' after
       750 ns, '0' after 950 ns, '1' after 1350 ns;
  CC: Code_Converter port map (X, clk, Z);
end tester;
```

accomplishes exactly the same testing that was done using simulator commands in Figure 2-55. A time-varying signal is provided to input X using the statement

```
X <= '0', '1' after 350 ns, '0' after 550ns, '1' after 750 ns, '0'
     after 950 ns, '1' after 1350 ns;
```

In this chapter, we have covered the basics of VHDL. We have shown how to use VHDL to model combinational logic and sequential machines. Since VHDL is a hardware description language, it differs from an ordinary programming language in several ways. Most importantly, VHDL statements execute concurrently, since they must model real hardware in which the components are all in operation at the same time. Statements within a process execute sequentially, but the processes themselves operate concurrently. VHDL signals model actual signals in the hardware, but variables may be used for internal computation that is local to processes, procedures, and functions. We will cover more advanced features of VHDL in Chapter 3.

2.20 Problems

2.1 **(a)** What do the acronyms VHDL and VHSIC stand for?
(b) How does a hardware description language like VHDL differ from an ordinary programming language?
(c) What are the advantages of using a hardware description language as compared with schematic capture in the design process?

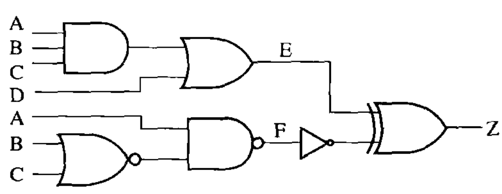
2.2 **(a)** Which of the following are legal VHDL identifiers? `123A`, `A_123`, `_A123`, `A123_`, `c1__c2`, `and`, `and1`
(b) Which of the following identifiers are equivalent? `aBC`, `ABC`, `Abc`, `abc`

2.3 Given the concurrent VHDL statements:

```
B <= A and C after 3ns;
C <= not B after 2ns;
```

(a) Draw the circuit the statements represent.
(b) Draw a timing diagram if initially $A = B =$ '0' and $C =$ '1', and A changes to '1' at time 5 ns.

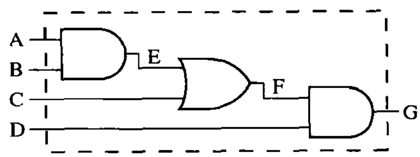
2.4 Write a VHDL description of the following combinational circuit using concurrent statements. Each gate has a 5-ns delay, excluding the inverter, which has a 2-ns delay.



2.5 **(a)** Write VHDL code for a full subtracter using logic equations.
(b) Write VHDL code for a 4-bit subtracter using the module defined in (a) as a component.

2.6 Write VHDL code for the following circuit. Assume that the gate delays are negligible.

(a) Use concurrent statements.
(b) Use a process with sequential statements.



2.7 In the following VHDL code, A, B, C, and D are integers that are 0 at time 10 ns. If D changes to 1 at 20 ns, specify the times at which A, B, and C will change and the values they will take.

```
process(D)
begin
  A <= 1 after 5 ns;
  B <= A + 1;              -- executes before A changes
  C <= B after 10 ns;      -- executes before B changes
end process;
```

2.8 **(a)** What device does the following VHDL code represent?

```
process(CLK, Clr, Set)
begin
  if Clr = '1' then Q <= '0';
  elsif Set = '1' then Q <= '1';
  elsif CLK'event and CLK <= '0' then
    Q <= D;
  end if;
end process;
```

(b) What happens if Clr = Set = '1' in the device in part (a)?

2.9 Write a VHDL description of an S-R latch using a process.

2.10 An M-N flip-flop responds to the falling clock edge as follows:

If $M = N$ = '0', the flip-flop changes state.
If M = '0' and N = '1', the flip-flop output is set to '1'.
If M = '1' and N = '0', the flip-flop output is set to '0'.
If $M = N$ = '1', no change of flip-flop state occurs.
The flip-flop is cleared asynchronously if $CLRn$ = '0'.

Write a complete VHDL module that implements an M-N flip-flop.

2.11 A DD flip-flop is similar to a D flip-flop, except that the flip-flop can change state ($Q^+ = D$) on both the rising edge and falling edge of the clock input. The flip-flop has a direct reset input, R, and R = '0' resets the flip-flop to Q = '0' independent of the clock. Similarly, it has a direct set input, S, that sets the flip-flop to '1' independent of the clock. Write a VHDL description of a DD flip-flop.

2.12 An inhibited toggle flip-flop has inputs $I0$, $I1$, T, and *Reset*, and outputs Q and QN. *Reset* is active high and overrides the action of the other inputs. The flip-flop works as follows. If $I0$ = '1', the flip-flop changes state on the rising edge of T; if $I1$ = '1', the flip-flop changes state on the falling edge of T. If $I0$ = $I1$ = '0', no state change occurs (except on reset). Assume the propagation delay from T to output is 8 ns and from reset to output is 5 ns.

(a) Write a complete VHDL description of this flip-flop.
(b) Write a sequence of simulator commands that will test the flip-flop for the input sequence $I1$ = '1', toggle T twice, $I1$ = '0', $I0$ = '1', toggle T twice.

2.13 In the following VHDL process A, B, C, and D are all integers that have a value of 0 at time = 10 ns. If E changes from '0' to '1' at time = 20 ns, specify the time(s) at which each signal will change and the value to which it will change. List these changes in chronological order (20, 20 + Δ, 20 + 2Δ, etc.).

```
p1: process
begin
  wait on E;
  A <= 1 after 5 ns;
  B <= A + 1;
  C <= B after 10 ns;
  wait for 0 ns;
  D <= B after 3 ns;
  A <= A + 5 after 15 ns;
  B <= B + 7;
end process p1;
```

2.14 In the following VHDL process A, B, C, and D are all integers that have a value of 0 at time = 10 ns. If E changes from '0' to '1' at time = 20 ns, specify the time(s) at which each signal will change and the value to which it will change. List these changes in chronological order (20, 20 + Δ, 20 + 2Δ, etc.).

```
p2: process(E)
begin
  A <= 1 after 5 ns;
  B <= A + 1;
  C <= B after 10 ns;

  D <= B after 3 ns;
  A <= A + 5 after 15 ns;
  B <= B + 7;
end process p2;
```

2.15 For the following VHDL code, assume that D changes to '1' at time 5 ns. Give the values of A, B, C, D, E, and F each time a change occurs. That is, give the values at time 5 ns, $5 + \Delta$, $5 + 2\Delta$, and so on. Carry this out until either 20 steps have occurred, until no further change occurs, or until a repetitive pattern emerges.

```
entity prob is
  port(D: inout bit);
end prob;

architecture q1 of prob is
  signal A, B, C, E, F: bit;
begin
  C <= A;
  A <= (B and not E) or D;
  P1: process (A)
  begin
    B <= A;
  end process P1;
  P2: process
  begin
    wait until A = '1';
    wait for 0 ns;
    E <= B after 5 ns;
    D <= '0';
    F <= E;
  end process P2;
end architecture q1;
```

2.16 Assuming B is driven by the simulator command

```
force B 0 0, 1 10, 0 15, 1 20, 0 30, 1 35
```

draw a timing diagram illustrating A, B, and C if the following concurrent statements are executed:

```
A <= transport B after 5 ns;
C <= B after 8 ns;
```

2.17 Assuming B is driven by the simulator command

```
force B 0 0, 1 4, 0 10, 1 15, 0 20, 1 30, 0 40
```

draw a timing diagram illustrating A, B, and C if the following concurrent statements are executed:

```
A <= transport B after 5 ns;
C <= B after 5 ns;
```

2.18 In the following VHDL code, A, B, C, and D are bit signals that are '0' at time = 4 ns. If A changes to 1 at time 5 ns, make a table showing the values of A, B, C, and D as

a function of time until time = 18 ns. Include deltas. Indicate the times at which each process begins executing.

```
P1: process(A)
begin
  B <= A after 5 ns;
  C <= B after 2 ns;
end process;
P2: process
begin
  wait on B;
  A <= not B;
  D <= not A xor B;
end process;
```

2.19 If A = "101", B = "011", and C = "010", what are the values of the following statements?

(a) `(A & B) or (B & C)`
(b) `A ror 2`
(c) `A sla 2`
(d) `A & not B = "111110"`
(e) `A or B and C`

2.20 Consider the following VHDL code:

```
entity Q3 is
  port(A, B, C, F, Clk: in bit;
       E: out bit);
end Q3;

architecture Qint of Q3 is
  signal D, G: bit;
begin
  process(Clk)
  begin
    if Clk'event and Clk = '1' then
      D <= A and B and C;
      G <= not A and not B;
      E <= D or G or F;
    end if;
  end process;
end Qint;
```

(a) Draw a block diagram for the circuit (no gates—at block level only).
(b) Give the circuit generated by the preceding code (at the gate level)

2.21 Implement the following VHDL code using these components: D flip-flops with clock enable, a multiplexer, an adder, and any necessary gates. Assume that *Ad* and *Ora* will never be '1' at the same time, and only enable the flip-flops when *Ad* or *Ora* is '1'.

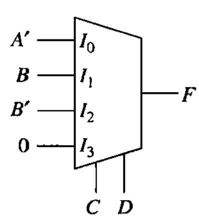
```
library IEEE;
use IEEE.numeric_bit.all;

entity module1 is
port(A, B: in unsigned (2 downto 0);
     Ad, Ora, clk: in bit;
     C: out unsigned (2 downto 0));
end module1;

architecture RT of module1 is
begin
  process(clk)
  begin
    if clk = '1' and clk'event then
      if Ad = '1' then C <= A + B; end if;
      if Ora = '1' then C <= A or B; end if;
    end if;
  end process;
end RT;
```

2.22 Draw the circuit represented by the following VHDL process. Use only two gates.

```
process(clk, clr)
begin
  if clr = '1' then Q <= '0';
  elsif clk'event and clk = '0' and CE = '1' then
    if C = '0' then Q <= A and B;
    else Q <= A or B; end if;
  end if;
end process;
```

Why is *clr* on the sensitivity list but *C* is not?

2.23 **(a)** Write a selected signal assignment statement to represent the 4-to-1 MUX shown below. Assume that there is an inherent delay in the MUX that causes the change in output to occur 10 ns after a change in input.
(b) Repeat (a) using a conditional signal assignment statement.
(c) Repeat (a) using a process and a case statement.



2.24 **(a)** Write a VHDL process that is equivalent to the following concurrent statement:

```
A <= B1 when C = 1 else B2 when C = 2 else B3 when C = 3 else 0;
```

(b) Draw a circuit to implement the following VHDL statement:

```
A <= B1 when C1 = '1' else B2 when C2 = '1' else
     B3 when C3 = '1' else '0';
```

where all signals are of type bit.

2.25 Write a VHDL description of an SR latch.

(a) Use a conditional assignment statement.
(b) Use the characteristic equation.
(c) Use logic gates.

2.26 For the VHDL code of Figure 2-38, what will be the values of S and Co if A = "1101", B = "111", and Ci = '1'?

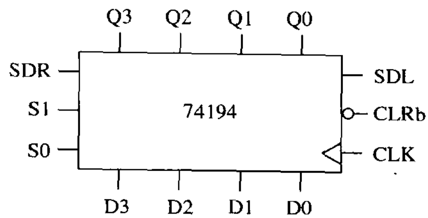
2.27 Write VHDL code to add a positive integer B ($B < 16$) to a 4-bit bit-vector A to produce a 5-bit bit-vector as a result. Use an overloaded operator in the IEEE numeric bit package to do the addition. Use calls to conversion functions as needed. The final result should be a bit-vector, not an unsigned vector.

2.28 A 4-bit magnitude comparator chip (e.g., 74LS85) compares two 4-bit numbers A and B and produces outputs to indicate whether $A < B$, $A = B$, or $A > B$. There are three output signals to indicate each of the above conditions. Note that exactly one of the output lines will be high and the other two lines will be low at any time. The chip is a cascadable chip and has three inputs, $A > B.IN$, $A = B.IN$, and $A < B.IN$, in order to allow cascading the chip to make 8-bit or bigger magnitude comparators.

(a) Draw block diagram of a 4-bit magnitude comparator
(b) Draw a block diagram to indicate how you can construct an 8-bit magnitude comparator using two 4-bit magnitude comparators.
(c) Write behavioral VHDL description for the 4-bit comparator.
(d) Write VHDL code for the 8-bit comparator using two 4-bit comparators as components.

2.29 Write a VHDL module that describes a 16-bit serial-in, serial-out shift register with inputs SI (serial input), EN (enable), and CK (clock, shifts on rising edge) and a serial output (SO).

2.30 A description of a 74194 four-bit bidirectional shift register follows:
The $CLRb$ input is asynchronous and active low and overrides all the other control inputs. All other state changes occur following the rising edge of the clock. If the control inputs $S_1 = S_0 = 1$, the register is loaded in parallel. If $S_1 = 1$ and $S_0 = 0$, the register is shifted right and SDR (serial data right) is shifted into Q_3. If $S_1 = 0$ and $S_0 = 1$, the register is shifted left and SDL is shifted into Q_0. If $S_1 = S_0 = 0$, no action occurs.



(a) Write a behavioral-level VHDL model for the 74194.

(b) Draw a block diagram and write a VHDL description of an 8-bit bidirectional shift register that uses two 74194's as components. The parallel inputs and outputs to the 8-bit register should be X(7 **downto** 0) and Y(7 **downto** 0). The serial inputs should be RSD and LSD.

2.31 A synchronous (4-bit) up/down decade counter with output Q works as follows: All state changes occur on the rising edge of the CLK input, except the asynchronous clear (CLR). When $CLR = 0$, the counter is reset regardless of the values of the other inputs.

If the $LOAD$ input is 0, the data input D is loaded into the counter.
If $LOAD = ENT = ENP = UP = 1$, the counter is incremented.
If $LOAD = ENT = ENP = 1$ and $UP = 0$, the counter is decremented.
If $ENT = UP = 1$, the carry output (CO) = 1 when the counter is in state 9.
If $ENT = 1$ and $UP = 0$, the carry output (CO) = 1 when the counter is in state 0.

(a) Write a VHDL description of the counter.

(b) Draw a block diagram and write a VHDL description of a decimal counter that uses two of the above counters to form a two-decade decimal up/down counter that counts up from 00 to 99 or down from 99 to 00.

(c) Simulate for the following sequence: load counter with 98, increment three times, do nothing for two clocks, decrement four times, and clear.

2.32 Write a VHDL model for a 74HC192 synchronous 4-bit up/down counter. Ignore all timing data. Your code should contain a statement of the form `process(DOWN, UP, CLR, LOADB)`

2.33 Consider the following 8-bit bi-directional synchronous shift register with parallel load capability. The notation used to represent the input/output pins is explained below.

CLR	Asynchronous Clear, overrides all other inputs
$Q(7:0)$	8-bit output
$D(7:0)$	8-bit input
$S0, S1$	mode control inputs
LSI	serial input for left shift
RSI	serial input for right shift

The mode control inputs work as follows:

S0	*S1*	Action
0	0	No action
0	1	Right shift
1	0	Left shift
1	1	Load parallel data (i.e., $Q = D$)

(a) Write an entity description for this shift register.
(b) Write an architecture description of this shift register.
(c) Draw a block diagram illustrating how two of these can be connected to form a 16-bit cyclic shift register, which is controlled by signals L and R. If L = '1' and R = '0', then the 16-bit register is cycled left. If L = '0' and R = '1', the register is cycled right. If $L = R$ = '1', the 16-bit register is loaded from $X(15{:}0)$. If $L = R$ = '0', the register is unchanged.
(d) Write an entity description for the module in part (c).
(e) Write an architecture description using the module from parts (a) and (b).

2.34 Complete the following VHDL code to implement a counter that counts in the following sequence: Q = 1000, 0111, 0110, 0101, 0100, 0011, 1000, 0111, 0110, 0101, 0100, 0011, . . . (repeats). The counter is synchronously loaded with 1000 when *Ld8* = '1'. It goes through the prescribed sequence when *Enable* = '1'. The counter outputs *S5* = '1' whenever it is in state 0101. Do not change the entity in any way. Your code must be synthesizable.

```
library IEEE;
use IEEE.numeric_bit.all;

entity countQ1 is
  port(clk, Ld8, Enable: in bit; S5: out bit;
       Q: out unsigned(3 downto 0));
end countQ1;
```

2.35 A synchronous 4-bit UP/DOWN binary counter has a synchronous clear signal *CLR* and a synchronous load signal *LD*. *CLR* has higher priority than *LD*. Both *CLR* and *LD* are active high. D is a 4-bit input to the counter and Q is the 4-bit output from the counter. *UP* is a signal that controls the direction of counting. If *CLR* and *LD* are not active and *UP* = 1, the counter increments. If *CLR* and *LD* are not active and UP 0, the counter decrements. All changes occur on the falling edge of the clock.

(a) Write a behavioral VHDL description of the counter.
(b) Use the above UP/DOWN counter to implement a synchronous modulo 6 counter that counts from 1 to 6. This modulo 6 counter has an external reset which, if applied, makes the count = 1. A count enable signal *CNT* makes it count in the sequence 1, 2, 3, 4, 5, 6, 1, 2, . . . incrementing once for each clock pulse. You should use any necessary logic to make the counter go to count = 1 after count = 6. The modulo 6 counter only counts in the UP sequence. Provide a textual/pictorial description of your approach.
(c) Write a behavioral VHDL description for the modulo-6 counter in part (b).

2.36 Examine the following VHDL code and complete the following exercises:

```
entity Problem
  port(X, CLK: in bit;
       Z1, Z2: out bit);
end Problem;

architecture Table of Problem is
  signal State, Nextstate: integer range 0 to 3 := 0;
begin
  process(State, X)       --Combinational Circuit
  begin
    case State is
      when 0 =>
        if X = '0' then Z1 < = '1'; Z2 <= '0'; Nextstate < = 0;
        else Z1 < = '0'; Z2 < = '0'; Nextstate < = 1; end if;
      when 1 =>
        if X = '0' then Z1 < = '0'; Z2 <= '1'; Nextstate < = 1;
        else Z1 < = '0'; Z2 < = '1'; Nextstate < = 2; end if;
      when 2 =>
        if X = '0' then Z1 < = '0'; Z2 <= '1'; Nextstate < = 2;
        else Z1 < = '0'; Z2 < = '1'; Nextstate < = 3; end if;
      when 3 =>
        if X = '0' then Z1 < = '0'; Z2 <= '0'; Nextstate < = 0;
        else Z1 < = '1'; Z2 < = '0'; Nextstate < = 1; end if;
    end case;
  end process;
  process(CLK)            --State Register
  begin
    if CLK'event and CLK = '1' then      --rising edge of clock
      State <= Nextstate;
    end if;
  end process;
end Table;
```

(a) Draw a block diagram of the circuit implemented by this code.
(b) Write the state table that is implemented by this code.

2.37 (a) Write a behavioral VHDL description of the state machine you designed in Problem 1.13. Assume that state changes occur on the falling edge of the clock pulse. Instead of using if-then-else statements, represent the state table and output table by arrays. Compile and simulate your code using the following test sequence:

$$X = 1101\ 1110\ 1111$$

X should change 1/4 clock period after the rising edge of the clock.

(b) Write a data flow VHDL description using the next state and output equations to describe the state machine. Indicate on your simulation output at which times S and V are to be read.

(c) Write a structural model of the state machine in VHDL that contains the interconnection of the gates and D flip-flops.

2.38 (a) Write a behavioral VHDL description of the state machine that you designed in Problem 1.14. Assume that state changes occur on the falling edge of the clock pulse. Use a case statement together with if-then-else statements to represent the state table. Compile and simulate your code using the following test sequence:

$$X = 1011\ 0111\ 1000$$

X should change 1/4 clock period after the falling edge of the clock.

(b) Write a data flow VHDL description using the next state and output equations to describe the state machine. Indicate on your simulation output at which times D and B should be read.

(c) Write a structural model of the state machine in VHDL that contains the interconnection of the gates and J-K flip-flops.

2.39 A Moore sequential machine with two inputs (X_1 and X_2) and one output (Z) has the following state table:

Present State	Next State $X_1X_2 = 00$	01	10	11	Output (Z)
1	1	2	2	1	0
2	2	1	2	1	1

Write VHDL code that describes the machine at the behavioral level. Assume that state changes occur 10 ns after the falling edge of the clock, and output changes occur 10 ns after the state changes.

2.40 Write VHDL code to implement the following state table. Use two processes. State changes should occur on the falling edge of the clock. Implement the Z_1 and Z_2 outputs using concurrent conditional statements. Assume that the combinational part of the sequential circuit has a propagation delay of 10 ns, and the propagation delay between the rising-edge of the clock and the state register output is 5 ns.

Present State	Next state $X_1X_2 = 00$	01	11	Output (Z_1Z_2)
1	3	2	1	00
2	2	1	3	10
3	1	2	3	01

2.41 In the following code, *state* and *nextstate* are integers with a range of 0 to 2.

```
process(state, X)
begin
  case state is
    when 0 => if X = '1' then nextstate <= 1;
    when 1 => if X = '0' then nextstate <= 2;
```

```
    when 2 => if X = '1' then nextstate <= 0;
  end case;
end process;
```

(a) Explain why a latch would be created when the code is synthesized.
(b) What signal would appear at the latch output?
(c) Make changes in the code which would eliminate the latch.

2.42 For the process given below, A, B, C, and D are all integers that have a value of 0 at time = 10 ns. If E changes from '0' to '1' at time 20 ns, specify all resulting changes. Indicate the time at which each change will occur, the signal/variable affected, and the value to which it will change.

```
process
  variable F: integer: =1; variable A: integer: =0;
begin
  wait on E;
  A := 1;
  F := A + 5;
  B <= F + 1 after 5 ns;
  C <= B + 2 after 10 ns;
  D <= C + 5 after 15 ns;
  A := A + 5;
end process;
```

2.43 What is wrong with the following model of a 4-to-1 MUX? (It is not a syntax error.)

```
architecture mux_behavioral of 4to1mux is
signal sel: integer range 0 to 3;
begin
  process(A, B, I0, I1, I2, I3)
  begin
    sel <= 0;
    if A = '1' then sel <= sel + 1; end if;
    if B = '1' then sel <= sel + 2; end if;
    case sel is
      when 0 => F <= I0;
      when 1 => F <= I1;
      when 2 => F <= I2;
      when 3 => F <= I3;
    end case;
  end process;
end mux_behavioral;
```

2.44 When the following VHDL code is simulated, A is changed to '1' at time 5 ns. Make a table that shows all changes in A, B, and D and the times at which they occur through time = 40 ns.

```
entity Q1F00 is
  port(A: inout bit);
end Q1F00;
```

```
architecture Q1F00 of Q1F00 is
  signal B, D: bit;
begin
  D <= A xor B after 10 ns;
  process(D)
    variable C: bit;
  begin
    C := not D;
    if C = '1' then
      A <= not A after 15 ns;
    end if;
    B <= D;
  end process;
end Q1F00;
```

2.45 What device does the following VHDL code represent?

```
process(CLK, RST)
  variable Qtmp: bit;
begin
  if RST '1' then Qtmp := '0';
  elsif CLK'event and CLK = '1' then
    if T = '1' then
      Qtmp := not Qtmp;
    end if;
  end if;
  Q <= Qtmp;
end process;
```

2.46 **(a)** Write a VHDL module for a LUT with four inputs and three outputs. The 3-bit output should be a binary number equal to the number of 1's in the LUT input.
(b) Write a VHDL module for a circuit that counts the number of 1's in a 12-bit number. Use three of the modules from (a) along with overloaded addition operators.
(c) Simulate your code and test if for the following data inputs:

111111111111, 010110101101, 100001011100

2.47 Implement a 3-to-8 decoder using a LUT. Give the LUT truth table and write the VHDL code. The inputs should be A, B, and C and the output should be an 8-bit unsigned vector.

2.48 $A(1$ *to* $20)$ is an array of 20 integers. Write VHDL code that finds the largest integer in the array

(a) Using a **for** loop
(b) Using a **while** loop

2.49 Write VHDL code to test a Mealy sequential circuit with one input (X) and one output (Z). The code should include the Mealy circuit as a component. Assume the Mealy circuit changes state on the rising edge of CLK. Your test code should generate a clock with 100 ns period. The code should apply the following test sequence:

$$X = 0, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 0$$

X should change 10 ns after the rising edge of CLK. Your test code should read Z at an appropriate time and verify that the following output sequence was generated:

$$Z = 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0$$

Report an error if the output sequence from the Mealy circuit is incorrect; otherwise, report "sequence correct." Complete the following architecture for the tester:

```
architecture test1 of tester is
  component Mealy
    -- sequential circuit to be tested; assume this component
    -- is available in your design; do NOT write code for the
    -- component
    port(X, CLK: in bit; Z: out bit);
  end component;
  signal XA: bit_vector(0 to 11) := "011011011100";
  signal ZA: bit_vector(0 to 11) := "100110110110";
```

2.50 Write a VHDL test bench that will test the VHDL code for the sequential circuit of Figure 2-58. Your test bench should generate all ten possible input sequences (0000, 1000, 0100, 1100, . . .) and verify that the output sequences are correct. Remember that the components have a 10-ns delay. The input should be changed 1/4 of a clock period after the rising edge of the clock and the output should be read at the appropriate time. Report "Pass" if all sequences are correct; otherwise, report "Fail."

2.51 Write a test bench to test the counter of Problem 2.34. The test bench should generate a clock with a 100-ns period. The counter should be loaded on the first clock; then it should count for five clocks; then it should do nothing for two clocks; then it should continue counting for ten clocks. The test bench port should output the current time (in time units, not the count) whenever $S5$ = '1'. Use only concurrent statements in your test bench.

2.52 Complete the following VHDL code to implement a test bench for the sequential circuit SMQ1. Assume that the VHDL code for the SMQ1 sequential circuit module is already available. Use a clock with a 50-ns half-period. Your test bench should test the circuit for the input sequence $X = 1, 0, 0, 1, 1$. Assume that the correct output sequence for this input sequence is 1, 1, 0, 1, 0. Use a single concurrent statement to generate the X sequence. The test bench should read the values of output Z at the proper times and compare them with the correct values of Z. The correct answer is stored as a bit-vector constant:

$$answer(1\ to\ 5) = \text{"11010"};$$

The port signal *correct* should be set to TRUE if the answer is correct; otherwise, it should be set to FALSE. Make sure that your read Z at the correct time. Use wait statements in your test bench.

```
entity testSMQ1 is
  port(correct: out Boolean);
end testSMQ1;
architecture testSM of testSMQ1 is
  component SMQ1  -- the sequential circuit module
    port(X, CLK: in bit; Z: out bit);
  end component;
  constant answer: bit_vector(1 to 5) := "11010";
begin
```

CHAPTER 3 Additional Topics in VHDL

Up to this point, we have described the basic features of VHDL and how they can be used in the digital system design process. In this chapter, we describe additional features of VHDL that illustrate its power and flexibility. VHDL functions and procedures are presented. Several additional features, such as attributes, function overloading, and generic and generate statements, are also presented. The IEEE multivalued logic system and principles of signal resolution are described. A simple memory model is presented to illustrate the use of tristate signals.

3.1 VHDL Functions

A key feature of VLSI circuits is the repeated use of similar structures. VHDL provides functions and procedures to easily express repeated invocation of the same functionality or the repeated use of structures. We describe functions in this section. Functions can return only a single value through a return statement. Procedures are more general and complex than functions. They can return any number of values using output parameters. Procedures are described in the next section.

A function executes a sequential algorithm and returns a single value to the calling program. When the following function is called, it returns a bit-vector equal to the input bit-vector (*reg*) rotated one position to the right:

```
function rotate_right (reg: bit_vector)
  return bit_vector is
begin
  return reg ror 1;
end rotate_right;
```

A function call can be used anywhere that an expression can be used. For example, if A = "10010101", the statement

```
B <= rotate_right(A);
```

would set B equal to "11001010", and leave A unchanged.

The general form of a function declaration is

```
function function-name (formal-parameter-list)
  return return-type is
[declarations]
begin
  sequential statements   -- must include return return-value;
end function-name;
```

The general form of a function call is

```
function_name(actual-parameter-list)
```

The number and type of parameters on the `actual-parameter-list` must match the `formal-parameter-list` in the function declaration. The parameters are treated as input values and cannot be changed during the execution of the function.

Example Write a VHDL function for generating an even parity bit for a 4-bit number. The input is a 4-bit number and the output is a code word that contains the data and the parity bit. Figure 3-1 shows the solution.

FIGURE 3-1: Parity Generation Using a Function

```
-- Function example code without a loop
-- This function takes a 4-bit vector
-- It returns a 5-bit code with even parity

function parity (A: bit_vector(3 downto 0))
  return bit_vector is

variable parity: bit;
variable B: bit_vector(4 downto 0);
begin
  parity := a(0) xor a(1) xor a(2) xor a(3);
  B := A & parity;
  return B;
end parity;
```

If parity circuits are used in several parts in a system, we could call the function each time it is desired.

Figure 3-2 illustrates a function using a **for** loop. In Figure 3-2, the loop index (i) will be initialized to 0 when the **for** loop is entered, and the sequential statements will be executed. Execution will be repeated for $i = 1, i = 2$, and $i = 3$; then the loop will terminate.

If A, B, and C are integers, the statement C <= A + B will set C equal to the sum of A and B. However, if A, B, and C are bit-vectors, this statement will not work, since the "+" operation is not defined for bit-vectors. However, we can write a function to perform bit-vector addition. The function given in Figure 3-2 adds two 4-bit

FIGURE 3-2: Add Function

```
-- This function adds two 4-bit vectors and a carry.
-- Illustrates function creation and use of loop
-- It returns a 5-bit sum

function add4 (A, B: bit_vector(3 downto 0); carry: bit)
  return bit_vector is

variable cout: bit;
variable cin: bit := carry;
variable sum: bit_vector(4 downto 0) := "00000";
begin
loop1: for i in 0 to 3 loop
  cout := (A(i) and B(i)) or (A(i) and cin) or (B(i) and cin);
  sum(i) := A(i) xor B(i) xor cin;
  cin := cout;
end loop loop1;
sum(4) := cout;
return sum;
end add4;
```

vectors plus a carry and returns a 5-bit vector as the sum. The function name is *add4*; the formal parameters are *A*, *B*, and *carry*; and the return type is a bit-vector. Variables *cout* and *cin* are defined to hold intermediate values during the calculation. The variable *sum* is used to store the value to be returned. When the function is called, *cin* will be initialized to the value of the carry. The **for** loop adds the bits of *A* and *B* serially in the same manner as a serial adder. The first time through the loop, *cout* and *sum*(0) are computed using *A*(0), *B*(0), and *cin*. Then the *cin* value is updated to the new *cout* value, and execution of the loop is repeated. During the second time through the loop, *cout* and *sum*(1) are computed using *A*(1), *B*(1), and the new *cin*. After four times through the loop, all values of *sum(i)* have been computed and *sum* is returned. The total simulation time required to execute the *add4* function is zero. Not even delta time is required, since all the computations are done using variables, and variables are updated instantaneously.

The function call is of the form

```
add4(A, B, carry)
```

A and *B* may be replaced with any expressions that evaluate to bit-vectors with dimensions 3 **downto** 0, and *carry* may be replaced with any expression that evaluates to a bit. For example, the statement

```
Z <= add4(X, not Y, '1');
```

calls the function *add4*. Parameters *A*, *B*, and *carry* are set equal to the values of *X*, **not** *Y*, and '1', respectively. *X* and *Y* must be bit-vectors dimensioned 3 **downto** 0. The function computes

$$Sum = A + B + carry = X + \textbf{not}\ Y + \text{'1'}$$

and returns this value. Since *Sum* is a variable, computation of *Sum* requires zero time. After delta time, *Z* is set equal to the returned value of *Sum*. Since **not** *Y* + '1' equals the 2's complement of *Y*, the computation is equivalent to subtracting by adding the 2's complement. If we ignore the carry stored in *Z*(4), the result is *Z*(3 **downto** 0) = *X* − *Y*.

Functions can be used to return an array. As an example, we will write a function that inputs an array of numbers and returns an array which contains the square of the input numbers. Figure 3-3 illustrates the function as well as the function call. The number of input numbers is provided as a parameter to the function. In the illustrated call to the function, the numbers are 4 bits wide.

FIGURE 3-3: A Function to Compute Squares of an Array of Unsigned Numbers and Its Call

```
library IEEE;
use IEEE.numeric_bit.all;

entity test_squares is
  port(CLK: in bit);
end test_squares;

architecture test of test_squares is
type FourBitNumbers is array (0 to 4) of unsigned (3 downto 0);
type squareNumbers is array (0 to 4) of unsigned (7 downto 0);
constant FN: FourBitNumbers := ("0001", "1000", "0011", "0010", "0101");
signal answer: squareNumbers;
signal length: integer := 4;

function squares (Number_arr: FourBitNumbers; length: positive)
  return squareNumbers is

variable SN: squareNumbers;
begin
  loop1: for i in 0 to length loop
         SN(i) := Number_arr(i) * Number_arr(i);
  end loop loop1;
return SN;
end squares;

begin
  process(CLK)
  begin
    if CLK = '1' and CLK'EVENT then
     answer <= squares(FN, length);
    end if;
  end process;
end test;
```

Functions are frequently used to do type conversions. We already came across type conversion functions in the IEEE numeric_bit library: `to_integer(A)` and `to_unsigned(B, N)`. The first one converts an unsigned-vector to an integer, and the second one converts an integer to an unsigned-vector with the specified number of bits.

3.2 VHDL Procedures

Procedures facilitate decomposition of VHDL code into modules. Unlike functions, which return only a single value through a return statement, procedures can return any number of values using output parameters. The form of a procedure declaration is

```
procedure procedure_name (formal-parameter-list) is
[declarations]
begin
  sequential statements
end procedure_name;
```

The `formal-parameter-list` specifies the inputs and outputs to the procedure and their types. A procedure call is a sequential or concurrent statement of the form

```
procedure_name(actual-parameter-list);
```

As an example we will write a procedure *Addvec*, which will add two *N*-bit vectors and a carry, and return an *N*-bit sum and a carry. We will use a procedure call of the form

```
Addvec(A, B, Cin, Sum, Cout, N);
```

where *A, B*, and *Sum* are *N*-bit vectors, *Cin* and *Cout* are bits, and *N* is an integer.

Figure 3-4 gives the procedure definition. *Add1, Add2*, and *Cin* are input parameters, and *Sum* and *Cout* are output parameters. *N* is a positive integer that specifies the number of bits in the bit-vectors. The addition algorithm is essentially the same as the one used in the *add4* function. *C* must be a variable, since the new value of *C* is needed each time through the loop; however, *Sum* can be a signal since *Sum* is not used within the loop. After *N* times through the loop, all the values of the signal *Sum* have been computed, but *Sum* is not updated until a delta time after exiting from the loop.

Within the procedure declaration, the class, mode, and type of each parameter must be specified in the `formal-parameter-list`. The class of each parameter can be **signal, variable**, or **constant**. If the class of an input parameter is omitted, **constant** is used as the default. If the class is a **signal**, then the actual parameter in the procedure call must be a **signal** of the same type. Similarly, for a formal parameter of class **variable**, the actual parameter must be a **variable** of the same type. However, for a **constant** formal parameter, the actual parameter can be any expression that evaluates to a constant of the proper type. This constant value is used inside the procedure and cannot be changed; thus, a **constant** formal parameter is always of mode **in**. Signals and variables can be of mode **in, out**, or **inout**. Parameters of mode **out** and **inout** can be changed in the procedure, so they are used to return values to the caller.

FIGURE 3-4: Procedure for Adding Bit-Vectors

```
-- This procedure adds two n-bit bit_vectors and a carry and
-- returns an n-bit sum and a carry. Add1 and Add2 are assumed
-- to be of the same length and dimensioned n-1 downto 0.

procedure Addvec (Add1, Add2: in bit_vector; Cin: in bit;
                  signal Sum: out bit_vector; signal Cout: out bit;
                  n: in positive) is

variable C: bit;
begin
  C := Cin;
  for i in 0 to n-1 loop
    Sum(i) <= Add1(i) xor Add2(i) xor C;
    C := (Add1(i) and Add2(i)) or (Add1(i) and C) or (Add2(i) and C);
  end loop;
  Cout <= C;
end Addvec;
```

In procedure *Addvec*, parameters *Add1, Add2*, and *Cin* are, by default, of class constant. Therefore, in the procedure call, *Add1, Add2*, and *Cin* can be replaced with any expressions that evaluate to constants of the proper type and dimension. Since *Sum* and *Cout* change within the procedure and are used to return values, they have been declared as class **signal**. Thus, in the procedure call, *Sum* and *Cout* can be replaced only with signals of the proper type and dimension.

The `formal-parameter-list` in a **function** declaration is similar to that of a **procedure**, except parameters of class **variable** are not allowed. Furthermore, all parameters must be of mode **in**, which is the default mode. Parameters of mode **out** or **inout** are not allowed, since a function returns only a single value, and this value cannot be returned through a parameter. Table 3-1 summarizes the modes and classes that may be used for procedure and function parameters. A procedure can have output parameters of mode **out** or **inout**. They can be signals or variables. They obviously cannot be constants because constants cannot be modified.

TABLE 3-1: Parameters for Subprogram Calls

		Actual Parameter	
Mode	Class	Procedure Call	Function Call
In[1]	Constant[2]	Expression	Expression
	Signal	Signal	Signal
	Variable	Variable	n/a
Out/inout	Signal	Signal	n/a
	Variable[3]	Variable	n/a

[1] Default mode for functions

[2] Default for in mode

[3] Default for out/inout mode

NOTE: n/a = "not applicable"

3.3 Attributes

An important feature of the VHDL language is attributes. Attributes can be associated with signals. They can also be associated with arrays.

3.3.1 Signal Attributes

You have already used a signal attribute, the 'EVENT attribute, for creating edge-triggered clocks. As you know, CLOCK'EVENT (read as "CLOCK tick EVENT") returns a value of TRUE if a change in signal *CLOCK* has just occurred. VHDL has two types of attributes: (1) attributes that return a value and (2) attributes that return a signal.

Table 3-2 gives several examples of attributes that return a value. In this table, *S* represents a signal name, and *S* is separated from an attribute name by a tick mark (single quote). In VHDL, an event on a signal means a change in the signal. Thus, S'ACTIVE (read as "S tick ACTIVE") returns a value of TRUE if a transaction in *S* has just occurred. A transaction occurs on a signal every time it is evaluated, regardless of whether the signal changes or not. Consider the concurrent VHDL statement A <= B **and** C. If $B = 0$, then a transaction occurs on *A* every time *C* changes, since *A* is recomputed every time *C* changes. If $B = 1$, then an event and a transaction occur on *A* every time *C* changes. S'ACTIVE returns TRUE if *S* has just been re-evaluated, even if *S* does not change. In contrast, S'EVENT returns TRUE only if a change has occurred in *S*. If *S* changes at time *T*, then S'EVENT is true at time *T* but false at time $T + \Delta$.

TABLE 3-2: Signal Attributes That Return a Value

Attribute	Returns
S'ACTIVE	True if a transaction occurred during the current delta, else false
S'EVENT	True if an event occurred during the current delta, else false
S'LAST_EVENT	Time elapsed since the previous event on *S*
S'LAST_VALUE	Value of S before the previous event on *S*
S'LAST_ACTIVE	Time elapsed since previous transaction on *S*

Table 3-3 gives signal attributes that create a signal. The brackets around (time) indicate that (time) is optional. If (time) is omitted, then one delta is used. The attribute S'DELAYED(time) creates a signal identical to *S*, except it is shifted by the amount of time specified. The example in Figure 3-5 illustrates use of the attributes listed in Table 3-3. The signal *C_delayed5* is the same as *C* shifted right by 5 ns. The signal *A_trans* toggles every time *B* or *C* changes, since *A* has a transaction whenever *B* or *C* changes. The initial computation of A <= B **and** C produces a transaction on *A* at time = Δ, so *A_trans* changes to '1' at that time. The signal A'STABLE(time) is true if *A* has not changed during the preceding interval of length (time). Thus, *A_stable5* is false for 5 ns after *A* changes, and it is true otherwise. The signal A'QUIET(time) is true if *A* has had no transactions during the preceding interval of length (time). Thus, *A_quiet5* is false for 5 ns after *A* has had a

TABLE 3-3: Signal Attributes That Create a Signal

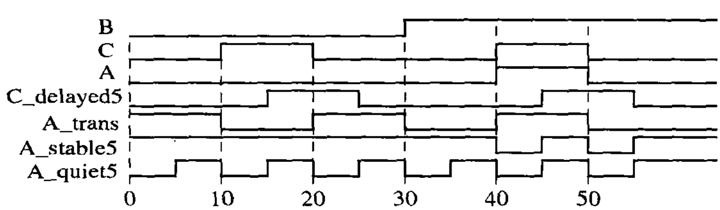
Attribute	Creates
S'DELAYED [(time)]*	Signal same as *S* delayed by specified time
S'STABLE [(time)]*	Boolean signal that is true if *S* had no events for the specified time
S'QUIET [(time)]*	Boolean signal that is true if *S* had no transactions for the specified time
S'TRANSACTION	Signal of type bit that changes for every transaction on *S*

*Delta is used if no time is specified.

FIGURE 3-5: Examples of Signal Attributes

```
entity attr_ex is
  port(B, C: in bit);
end attr_ex;

architecture test of attr_ex is
signal A, C_delayed5, A_trans: bit;
signal A_stable5, A_quiet5: boolean;
begin
  A <= B and C;
  C_delayed5 <= C'delayed(5 ns);
  A_trans <= A'transaction;
  A_stable5 <= A'stable(5 ns);
  A_quiet5 <= A'quiet(5 ns);
end test;
```

(a) VHDL code for attribute test



(b) Waveforms for attribute test

transaction. S'EVENT and **not** S'STABLE both return true if an event has occurred during the current delta; however, they cannot always be used interchangeably, since the former just returns a value and the latter returns a signal.

3.3.2 Array Attributes

Table 3-4 gives array attributes. In this table, *A* can either be an array name or an array type. In the examples, *ROM1* is a two-dimensional array for which the first index range is 0 **to** 15, and the second index range is 7 **downto** 0. ROM1'LEFT(2) is 7, since the left bound of the second index range is 7. Although *ROM1* is declared as

TABLE 3-4: Array Attributes `type ROM is array (0 to 15, 7 downto 0) of bit;`
`signal ROM1 : ROM;`

Attribute	Returns	Examples
A'LEFT(N)	left bound of Nth index range	ROM1'LEFT(1) = 0 ROM1'LEFT(2) = 7
A'RIGHT(N)	right bound of Nth index range	ROM1'RIGHT(1) = 15 ROM1'RIGHT(2) = 0
A'HIGH(N)	largest bound of Nth index range	ROM1'HIGH(1) = 15 ROM1'HIGH(2) = 7
A'LOW(N)	smallest bound of Nth index range	ROM1'LOW(1) = 0 ROM1'LOW(2) = 0
A'RANGE(N)	Nth index range	ROM1'RANGE(1) = 0 **to** 15 ROM1'RANGE(2) = 7 **downto** 0
A'REVERSE_RANGE(N)	Nth index range reversed	ROM1'REVERSE_RANGE(1) = 15 **downto** 0 ROM1'REVERSE_RANGE(2) = 0 **to** 7
A'LENGTH(N)	size of Nth index range	ROM1'LENGTH(1) = 16 ROM1'LENGTH(2) = 8

a signal, the array attributes also work with array constants and array variables. In the examples, the results are the same if *ROM1* is replaced with its type, ROM. For a vector (a one-dimensional array), *N* is 1 and can be omitted. If *A* is a bit-vector dimensioned 2 **to** 9, then A'LEFT is 2 and A'LENGTH is 8.

3.3.3 Use of Attributes

Attributes are often used together with assert statements (see Section 2.19) for error checking. The assert statement checks to see if a certain condition is true and, if not, causes an error message to be displayed. We present two examples: one illustrating use of signal attributes and another one illustrating array attributes.

Use of Signal Attributes

Consider the process in Figure 3-6, which checks to see if the setup and hold times are satisfied for a D flip-flop. We will use attributes 'EVENT and 'STABLE. 'STABLE is an attribute that returns a Boolean signal if the signal has no events for a specified time (i.e., a TRUE signal returned by this indicates that the signal was stable for a specified time). For example, the signal A'STABLE(time) is true if *A* has not changed during the preceding interval of length (time). Thus, A'stable(5) is false for 5 ns after *A* changes, and it is true otherwise.

In the check process, after the active edge of the clock occurs, the *D* input is checked to see if it has been stable for the specified *setup_time*. If not, a setup-time violation is reported as an error. Then, after waiting for the *hold_time*, *D* is checked to see if it has been stable during the hold-time period. If not, a hold-time violation is reported as an error.

FIGURE 3-6: Process for Checking Setup and Hold Times

```
check: process
begin
  wait until (Clk'event and CLK = '1');
  assert (D'stable(setup_time))
    report ("Setup time violation")
    severity error;
  wait for hold_time;
  assert (D'stable(hold_time))
    report ("Hold time violation")
    severity error;
end process check;
```

Use of Array Attributes in Vector Addition

As an example of using the assert statement together with array attributes, consider the procedure illustrated in Figure 3-7 for adding bit-vectors. This procedure adds two vectors of arbitrary size. The vectors should, however, be of the same length. It is not required to pass the length of the arrays in the procedure call. Since vector lengths are not passed as a parameter to the procedure, the procedure uses array attributes and checks whether the lengths are equal. Figure 3-7 shows the code for the procedure *Addvec2*. The inputs to the procedure include the two input vectors and the carry in bit. The procedure creates a temporary variable, *C*, for the internal

FIGURE 3-7: Procedure for Adding Bit-Vectors

```
-- This procedure adds two bit_vectors and a carry and returns a sum
-- and a carry. Both bit_vectors should be of the same length.

procedure Addvec2 (Add1, Add2: in bit_vector; Cin: in bit;
                   signal Sum: out bit_vector;
                   signal Cout: out bit) is

variable C: bit := Cin;
alias n1: bit_vector(Add1'length-1 downto 0) is Add1;
alias n2: bit_vector(Add2'length-1 downto 0) is Add2;
alias S: bit_vector(Sum'length-1 downto 0) is Sum;
begin
  assert ((n1'length = n2'length) and (n1'length = S'length))
    report "Vector lengths must be equal!"
    severity error;
  for i in s'reverse_range loop   -- reverse range makes you start from LSB
    S(i) <= n1(i) xor n2(i) xor C;
    C := (n1(i) and n2(i)) or (n1(i) and C) or (n2(i) and C);
  end loop;
  Cout <= C;
end Addvec2;
```

carry and initializes it to the input carry, *Cin*. Then it creates aliases *n1*, *n2*, and *S*, which have the same length as *Add1*, *Add2*, and *Sum*, respectively. These aliases are dimensioned from their length minus 1 **downto** 0. Even though the ranges of *Add1*, *Add2*, and *Sum* might be **downto** or **to** and might not include 0, the ranges for the aliases are defined in a uniform manner to facilitate further computation. If the input vectors and *Sum* are not the same length, an error message is reported. The sum and carry are computed bit-by-bit in a loop. Since this loop must start with $i = 0$, the range of *i* is the reverse of the range for *S*. Finally, the carry output, *Cout*, is set equal to the corresponding temporary variable, *C*.

3.4 Creating Overloaded Operators

Let us understand how overloaded operators are created. Operator overloading means that we will extend the definition of the operator to other data types in addition to the default data types that have already been defined. The operator will implicitly call an appropriate function, which eliminates the need for an explicit function or procedure call. When the compiler encounters a function declaration in which the function name is an operator enclosed in double quotes, the compiler treats this function as an operator overloading function.

The VHDL arithmetic operators, + and −, are defined to operate on integers, but not on bit-vectors. We have been using the IEEE numeric_bit library in order to access the overloaded arithmetic operators for bit-vectors using the unsigned type. Let us create a "+" function for bit-vectors.

The package shown in Figure 3-8 illustrates the creation of a "+" function for bit-vectors. It adds two bit-vectors and returns a bit-vector. This function uses aliases so that it is independent of the ranges of the bit-vectors, but it assumes that the lengths of the vectors are the same. It uses a **for** loop to do the bit-by-bit addition. Without this overloaded function, the "+" function was not available for bit-vectors. The IEEE numeric_bit only provides it for the unsigned type.

FIGURE 3-8: VHDL Package with Overloaded Operators for Bit-Vectors

```
-- This package provides an overloaded function for the plus operator

package bit_overload is
  function "+" (Add1, Add2: bit_vector)
    return bit_vector;
end bit_overload;

package body bit_overload is
  -- This function returns a bit_vector sum of two bit_vector operands
  -- The add is performed bit by bit with an internal carry
  function "+" (Add1, Add2: bit_vector)
    return bit_vector is
```

```
  variable sum: bit_vector(Add1'length-1 downto 0);
  variable c: bit := '0';                 -- no carry in
  alias n1: bit_vector(Add1'length-1 downto 0) is Add1;
  alias n2: bit_vector(Add2'length-1 downto 0) is Add2;
  begin
    for i in sum'reverse_range loop
      sum(i) := n1(i) xor n2(i) xor c;
      c := (n1(i) and n2(i)) or (n1(i) and c) or (n2(i) and c);
    end loop;
    return (sum);
  end "+";
end bit_overload;
```

Overloading can also be applied to procedures and functions. Several procedures can have the same name, and the type of the actual parameters in the procedure call determines which version of the procedure is called. An examination of the IEEE numeric_bit library illustrates that several overloaded operators and functions are defined.

3.5 Multivalued Logic and Signal Resolution

In previous chapters, we have used 2-valued bit logic in our VHDL code. In order to represent tristate buffers and buses, it is necessary to be able to represent a third value, 'Z', which represents the high-impedance state. It is also at times necessary to have a fourth value, 'X', to represent an unknown state. This unknown state may occur if the initial value of a signal is unknown or if a signal is simultaneously driven to two conflicting values, such as '0' and '1'. If the input to a gate is 'Z', the gate output may assume an unknown value, 'X'.

We need multivalued logic in order to meet these requirements. The IEEE numeric_std and the IEEE standard logic use a 9-valued logic. Different CAD tool developers have defined 7-valued, 9-valued, and 11-valued logic conventions.

In this chapter, we will present two examples of multivalued logic, (1) a 4-valued logic system and (2) the **IEEE-1164 standard** 9-valued logic system. The 4-valued logic system is described in Section 3.5.1 and the 9-valued logic is explained in Section 3.6.

3.5.1 A 4-Valued Logic System

Signals in a 4-valued logic can assume the four values: 'X', '0', '1', and 'Z', where each of the symbols represent the following:

'X' Unknown
'0' 0
'1' 1
'Z' High impedance

The high-impedance state is used for modeling tristate buffers and buses. This unknown state can be used if the initial value of a signal is unknown or if a signal is simultaneously driven to two conflicting values, such as '0' and '1'.

Let us model tristate buffers using the 4-valued logic. Figure 3-9 shows two tristate buffers with their outputs tied together, and Figure 3-10 shows the corresponding VHDL representation. A new data type X01Z, which can assume the four values 'X', '0', '1', and 'Z' is assumed. The tristate buffers have an active-high output enable, so that when b = '1' and d = '0', $f = a$; when b = '0' and d = '1', $f = c$; and when $b = d$ = '0', the f output assumes the high-Z state. If $b = d$ = '1', an output conflict can occur. Two VHDL architecture descriptions are shown. The first one uses two concurrent statements, and the second one uses two processes. In either case, f is driven from two different sources, and VHDL uses a *resolution function* to determine the actual output. For example, if $a = c = d$ = '1' and b = '0', f is driven to 'Z' by one concurrent statement or process, and f is driven to '1' by the other concurrent statement or process. The resolution function is automatically called to determine that the proper value of f is '1'. The resolution function will supply a value of 'X' (unknown) if f is driven to both '0' and '1' at the same time.

FIGURE 3-9: Tristate Buffers with Active-High Output Enable

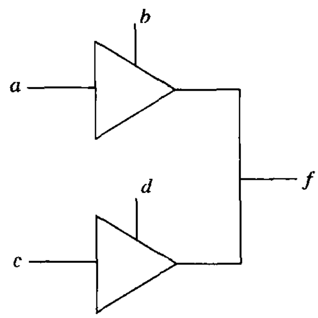


FIGURE 3-10: VHDL Code for Tristate Buffers

```
use WORK.fourpack.all; -- fourpack is a resolved package for 4-variable logic
                       -- more details on resolution in next subsection

entity t_buff_exmpl is
  port(a, b, c, d: in X01Z;   -- signals are four-valued
       f: out X01Z);
end t_buff_exmpl;

architecture t_buff_conc of t_buff_exmpl is
begin
  f <= a when b = '1' else 'Z';
  f <= c when d = '1' else 'Z';
end t_buff_conc;
```

```
architecture t_buff_bhv of t_buff_exmpl is
begin
  buff1: process(a, b)
  begin
    if (b = '1') then
      f <= a;
    else
      f <= 'Z';   -- "drive" the output high Z when not enabled
    end if;
  end process buff1;

  buff2: process(c, d)
  begin
    if (d = '1') then
      f <= c;
    else
      f <= 'Z';   -- "drive" the output high Z when not enabled
    end if;
  end process buff2;
end t_buff_bhv;
```

The code in Figure 3-10 utilizes a 4-valued logic package and corresponding signal resolution functions. Let us understand how to create signal resolution functions. A package, as described in the following subsection, is necessary to make the code in Figure 3-10 work.

3.5.2 Signal Resolution Functions

VHDL signals may either be resolved or unresolved. Signal resolution is necessary when different wires in a system are driving a common signal path. Signal resolution means arriving at a resulting value when two or more different signals are connected to the same point. VHDL with multivalued logic can be used to create resolutions when signals are connected.

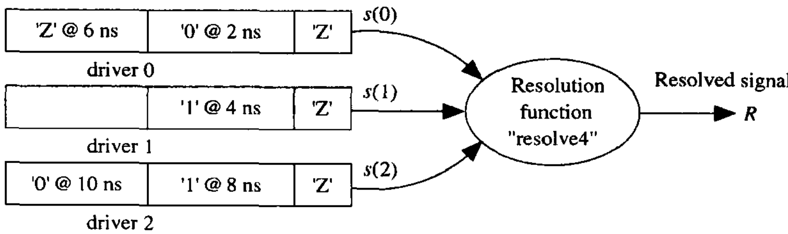
Resolved signals have an associated resolution function, and unresolved signals do not. We have previously used signals of type bit, which are unresolved. With unresolved signals, if we drive a bit signal B to two different values in two concurrent statements (or in two processes), the compiler will flag an error because there is no way to determine the proper value of B.

Consider the following three concurrent statements, where R is a resolved signal of type X01Z:

```
R <= transport '0' after 2 ns, 'Z' after 6 ns;
R <= transport '1' after 4 ns;
R <= transport '1' after 8 ns, '0' after 10 ns;
```

Assuming that R is initialized to 'Z', three drivers would be created for R, as shown in Figure 3-11. Each time one of the unresolved signals $s(0)$, $s(1)$, or $s(2)$ changes, the resolution function is automatically called to determine the value of the resolved signal, R.

FIGURE 3-11: Resolution of Signal Drivers



Since the X01Z logic has a symbol for high impedance, we can create resolution functions to model the wires when multiple signals are connected. Figure 3-12 shows how the resolution function for X01Z logic is defined in a package called *fourpack*. First, an unresolved logic type u_X01Z is defined, along with the corresponding

FIGURE 3-12: Resolution Function for X01Z Logic

```
package fourpack is
  type u_x01z is ('X', '0', '1', 'Z');          -- u_x01z is unresolved
  type u_x01z_vector is array (natural range <>) of u_x01z;
  function resolve4 (s: u_x01z_vector) return u_x01z;
  subtype x01z is resolve4 u_x01z;
  -- x01z is a resolved subtype which uses the resolution function resolve4
  type x01z_vector is array (natural range <>) of x01z;
end fourpack;

package body fourpack is
  type x01z_table is array (u_x01z, u_x01z) of u_x01z;
  constant resolution_table: x01z_table := (
    ('X','X','X','X'),
    ('X','0','X','0'),
    ('X','X','1','1'),
    ('X','0','1','Z'));

  function resolve4 (s:u_x01z_vector)
    return u_x01z is

  variable result: u_x01z := 'Z';
  begin
    if (s'length = 1) then
      return s(s'low);
    else
      for i in s'range loop
        result := resolution_table(result, s(i));
      end loop;
    end if;
    return result;
  end resolve4;
end fourpack;
```

unconstrained array type, u_X01Z_vector. Then a resolution function, named *resolve4*, is declared. Resolved X01Z logic is defined as a subtype of u_X01Z. The subtype declaration contains the function name *resolve4*. This implies that whenever a signal of type X01Z is computed, function *resolve4* is called to compute the correct value.

The resolution function, which is based on the operation of a tristate bus, is specified by the following table:

	'X'	'0'	'1'	'Z'
'X'	'X'	'X'	'X'	'X'
'0'	'X'	'0'	'X'	'0'
'1'	'X'	'X'	'1'	'1'
'Z'	'X'	'0'	'1'	'Z'

This table gives the resolved value of a signal for each pair of input values: 'Z' resolved with any value returns that value, 'X' resolved with any value returns 'X', and '0' resolved with '1' returns 'X'. The function *resolve4* has an argument, *s*, which represents a vector of one or more signal values to be resolved. If the vector is of length 1, then the first (and only) element of the vector is returned. Otherwise, the return value (the resolved signal) is computed iteratively by starting with *result* = 'Z' and recomputing *result* by a table look-up using each element of the *s* vector in turn. In the example of Figure 3-11, the *s* vector has three elements, and *resolve4* would be called at 0, 2, 4, 6, 8, and 10 ns to compute *R*. The following table shows the result:

Time	s(0)	s(1)	s(2)	R
0	'Z'	'Z'	'Z'	'Z'
2	'0'	'Z'	'Z'	'0'
4	'0'	'1'	'Z'	'X'
6	'Z'	'1'	'Z'	'1'
8	'Z'	'1'	'1'	'1'
10	'Z'	'1'	'0'	'X'

In order to write VHDL code using X01Z logic, we need to define the required operations for this type of logic. For example, AND and OR may be defined using the following tables:

AND	'X'	'0'	'1'	'Z'
'X'	'X'	'0'	'X'	'X'
'0'	'0'	'0'	'0'	'0'
'1'	'X'	'0'	'1'	'X'
'Z'	'X'	'0'	'X'	'X'

OR	'X'	'0'	'1'	'Z'
'X'	'X'	'X'	'1'	'X'
'0'	'X'	'0'	'1'	'X'
'1'	'1'	'1'	'1'	'1'
'Z'	'X'	'X'	'1'	'X'

The table on the left corresponds to the way an AND gate with 4-valued inputs would work. If one of the AND gate inputs is '0', the output is always '0'. If both inputs are '1', the output is '1'. In all other cases, the output is unknown ('X'), since

a high-Z gate input may act like either a '0' or '1'. For an OR gate, if one of the inputs is '1', the output is always '1'. If both inputs are '0', the output is '0'. In all other cases, the output is 'X'. AND and OR functions based on these tables can be included in the package *fourpack* to overload the AND and OR operators.

While this section illustrated how resolved signals can be created, fortunately you do not have to create such signals. Standard libraries with resolved data types are available. The IEEE 1164 standard and IEEE_numeric_std are examples of such multivalued logic libraries.

3.6 The IEEE 9-Valued Logic System

The **IEEE 1164** standard specifies a 9-valued logic system with signal resolution. The 9 logic values defined in this standard are

'U'	Uninitialized
'X'	Forcing unknown
'0'	Forcing 0
'1'	Forcing 1
'Z'	High impedance
'W'	Weak unknown
'L'	Weak 0
'H'	Weak 1
'–'	Don't care

The unknown, '0', and '1' values come in two strengths—forcing and weak. A forcing '1' means that the signal is as perfect as the power supply voltage. A 'weak 1', represented by 'H', means that the signal is logically high, but there is a voltage drop (e.g., output of a pull-up resistor). A forcing '0' represents a perfect ground, whereas a 'weak 0' represents a signal which is logically '0', but not exactly the ground voltage (e.g., the output of a pull-down resistor). The 9-valued system has the representation 'U' for denoting uninitialized signals. Don't care states can be represented by '–'.

If a forcing signal and a weak signal are tied together, the forcing signal dominates. For example, if '0' and 'H' are tied together, the result is '0'. The 9-valued logic is useful in modeling the internal operation of certain types of ICs. In this text, we will normally use only a subset of the IEEE values—'X', '0', '1', and 'Z'.

The IEEE-1164 standard defines the AND, OR, NOT, XOR, and other functions for 9-valued logic. The package IEEE.std_logic_1164 defines a **std_logic type** that uses the 9-valued logic. It also specifies a number of subtypes of the 9-valued logic, such as the X01Z subtype, which we have already been using. Analogous to bit-vectors, when vectors are created with the std_logic type, they are called **std_logic vectors**. When bit-vectors are used, typically they are initialized to '0', whereas when the std_logic type is used, the uninitilized value 'U' is the default value.

Table 3-5 shows the resolution function table for the IEEE 9-valued logic. The row index values have been listed as comments to the right of the table. The resolution function table for X01Z logic is a subset of this table, as indicated by the black rectangle.

TABLE 3-5: Resolution Function Table for IEEE 9-Valued Logic

```
CONSTANT resolution_table : stdlogic_table := (
--  --------------------------------------------------------------------
--  | U    X    0    1    Z    W    L    H    -              |   |
--  --------------------------------------------------------------------
    ( 'U', 'U', 'U', 'U', 'U', 'U', 'U', 'U', 'U' ), -- | U |
    ( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' ), -- | X |
    ( 'U', 'X', '0', 'X', '0', '0', '0', '0', 'X' ), -- | 0 |
    ( 'U', 'X', 'X', '1', '1', '1', '1', '1', 'X' ), -- | 1 |
    ( 'U', 'X', '0', '1', 'Z', 'W', 'L', 'H', 'X' ), -- | Z |
    ( 'U', 'X', '0', '1', 'W', 'W', 'W', 'W', 'X' ), -- | W |
    ( 'U', 'X', '0', '1', 'L', 'W', 'L', 'W', 'X' ), -- | L |
    ( 'U', 'X', '0', '1', 'H', 'W', 'W', 'H', 'X' ), -- | H |
    ( 'U', 'X', 'X', 'X', 'X', 'X', 'X', 'X', 'X' )  -- | - |
);
```

Table 3-6 shows the AND function table for the IEEE 9-valued logic. The row index values have been listed as comments to the right of the table. The AND function table for X01Z logic is a subset of this table, as indicated by the black rectangle. The IEEE-1164 standard first defines **std_ulogic** (unresolved standard logic); then it defines the std_logic type as a subtype of std_ulogic with the associated resolution function.

TABLE 3-6: AND Table for IEEE 9-Valued Logic

```
CONSTANT and_table : stdlogic_table := (
--  --------------------------------------------------------------------
--  | U    X    0    1    Z    W    L    H    -              |   |
--  --------------------------------------------------------------------
    ( 'U', 'U', '0', 'U', 'U', 'U', '0', 'U', 'U' ), -- | U |
    ( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | X |
    ( '0', '0', '0', '0', '0', '0', '0', '0', '0' ), -- | 0 |
    ( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | 1 |
    ( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | Z |
    ( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' ), -- | W |
    ( '0', '0', '0', '0', '0', '0', '0', '0', '0' ), -- | L |
    ( 'U', 'X', '0', '1', 'X', 'X', '0', '1', 'X' ), -- | H |
    ( 'U', 'X', '0', 'X', 'X', 'X', '0', 'X', 'X' )  -- | - |
);
```

The **and** functions given in Figure 3-13 use Table 3-6. These functions provide for operator overloading. This means that if we write an expression that uses the **and** operator, the compiler will automatically call the appropriate **and** function to evaluate the **and** operation depending on the type of the operands. If **and** is used with bit variables, the ordinary **and** function is used, but if **and** is used with std_logic variables, the std_logic **and** function is called. Operator overloading also automatically applies the appropriate **and** function to vectors. When **and** is used with bit-vectors, the ordinary bit-by-bit **and** is performed, but when **and** is applied to std_logic vectors, the std_logic **and** is applied on a bit-by-bit basis. The first **and** function in Figure 3-13 computes the **and** of the left (l) and right (r) operands by doing a table look-up. Although the **and** function is first defined for **std_ulogic**, it also works for std_logic since std_logic is a

subtype of std_ulogic. The second **and** function works with std_logic vectors. Aliases are used to make sure the index range is the same direction for both operands. If the vectors are not the same length, the **assert** false always causes the message to be displayed. Otherwise, each bit in the result vector is computed by table look-up.

FIGURE 3-13: AND Function for std_logic_vectors

```
function "and" (l: std_ulogic; r: std_ulogic) return UX01 is
begin
  return (and_table(l, r));
end "and"; -- end of function for unresolved standard logic

function "and" (l, r: std_logic_vector) return std_logic_vector is
  alias lv: std_logic_vector (1 to l'LENGTH) is l; --alias makes index range
  alias rv: std_logic_vector (1 to r'LENGTH) is r; -- in same direction
  variable result: std_logic_vector ( 1 to l'LENGTH );
begin
  if (l'LENGTH /= r'LENGTH) then
    assert FALSE
      report "arguments of overloaded 'and' operator are not of the same length"
      severity FAILURE;
  else
    for i in result'RANGE loop
      result(i) := and_table(lv(i), rv(i));
    end loop;
  end if;
  return result;
end "and";
```

If multivalued logic is desired, we can use the IEEE standard numeric_std package instead of the numeric_bit package that we have been using so far. The IEEE.numeric_std package is similar to the IEEE.numeric_bit package, but it defines unsigned and signed types as vectors of std_logic type instead of as vectors of bits. It also defines the same set of overloaded operators and functions on unsigned and signed numbers as the numeric_bit package.

A VHDL program that used vectors with the unsigned type can be ported to use vectors with 9-valued logic by simply replacing the statement

```
use IEEE.numeric_bit.all;
```

with the statements

```
use IEEE.std_logic_1164.all;  -- The IEEE.numeric_std package
                              -- uses the 1164 standard.
use IEEE.numeric_std.all;
```

The IEEE.numeric_std package uses the std_logic type from the 1164 standard. Hence, both the statements need to be included. With these statements, the unsigned type is considered to use 9-valued logic. No other changes in the program are

required. If the original program used the type bit, they should be converted to the std_logic type.

Other popular VHDL package used for simulation and synthesis with multivalued logic are the std_logic_arith package and the std_logic_unsigned package, developed by Synopsis. These packages can be invoked by the following statements:

```
use IEEE.std_logic_unsigned.all;
use IEEE.std_logic_arith.all;
```

In examples from now on, we will use the IEEE numeric_std package because it is an IEEE standard and it is similar in functionality to the numeric_bit package that we have been using so far. We have chosen not to use the std_logic_arith and std_logic_unsigned packages because they are not IEEE standards and they have less functionality than the IEEE numeric_std package.

3.7 SRAM Model Using IEEE 1164

In this section, we develop a VHDL model to represent the operation of a static RAM (SRAM). *RAM* stands for random-access memory, which means that any word in the memory can be accessed in the same amount of time as any other word. Strictly speaking, ROM memories are also random access, but historically, the term *RAM* is normally applied only to read-write memories. This model also illustrates the usefulness of the multivalued logic system. Multivalued logic is used to model tristate conditions on the memory data lines.

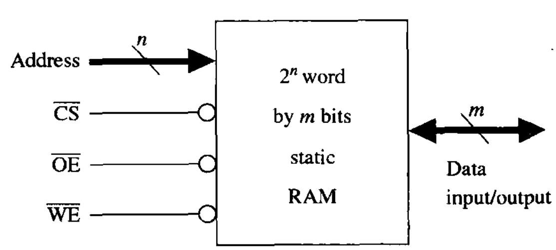
Figure 3-14 shows the block diagram of a static RAM with n address lines, m data lines, and three control lines. This memory can store 2^n words, each m bits wide. The data lines are bidirectional in order to reduce the required number of pins and the package size of the memory chip. When reading from the RAM, the data lines are outputs; when writing to the RAM, the data lines serve as inputs. The three control lines function as follows:

$\overline{\text{CS}}$ When asserted low, chip select selects the memory chip so that memory read and write operations are possible.

$\overline{\text{OE}}$ When asserted low, output enable enables the memory output onto an external bus.

$\overline{\text{WE}}$ When asserted low, write enable allows data to be written to the RAM.

FIGURE 3-14: Block Diagram of Static RAM



We say that a signal is asserted when it is in its active state. An active-low signal is asserted when it is low, and an active-high signal is asserted when it is high.

The truth table for the RAM (Table 3-7) describes its basic operation. High-Z in the I/O column means that the output buffers have high-Z outputs, and the data inputs are not used. In the read mode, the address lines are decoded to select *m* of the memory cells, and the data comes out on the I/O lines after the memory access time has elapsed. In the write mode, input data is routed to the latch inputs in the selected memory cells when $\overline{\text{WE}}$ is low, but writing to the latches in the memory cells is not completed until either $\overline{\text{WE}}$ goes high or the chip is deselected. The truth table does not take memory timing into account.

TABLE 3-7: Truth Table for Static RAM

$\overline{\text{CS}}$	$\overline{\text{OE}}$	$\overline{\text{WE}}$	Mode	I/O pins
H	X	X	not selected	high-Z
L	H	H	output disabled	high-Z
L	L	H	read	data out
L	X	L	write	data in

We now write a simple VHDL model for the memory that does not take timing considerations into account. In Figure 3-15, the RAM memory array is represented by an array of unsigned standard logic vectors (*RAM1*). This memory has 256 words, each of which are 8 bits. Since *Address* is typed as an unsigned bit-vector, it must be converted to an integer in order to index the memory array. The RAM process sets the I/O lines to high-Z if the chip is not selected. If *We_b* = '1', the RAM is in the read mode, and *IO* is the data read from the memory array. If *We_b* = '0', the memory is in the write mode, and the data on the I/O lines is stored in *RAM1* on the rising edge of *We_b*. If *Address* and *We_b* change simultaneously, the old value of *Address* should be used. *Address'delayed* is used as the array index to delay *Address* by one delta to make sure that the old address is used. *Address'delayed* uses one of the signal attributes described earlier in this chapter (Table 3-3). This is a RAM with asynchronous read and synchronous write.

FIGURE 3-15: Simple Memory Model

```
-- Simple memory model
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;

entity RAM6116 is
  port(Cs_b, We_b, Oe_b: in std_logic;
       Address: in unsigned(7 downto 0);
       IO: inout unsigned(7 downto 0));
end RAM6116;

architecture simple_ram of RAM6116 is
type RAMtype is array(0 to 255) of unsigned(7 downto 0);
```

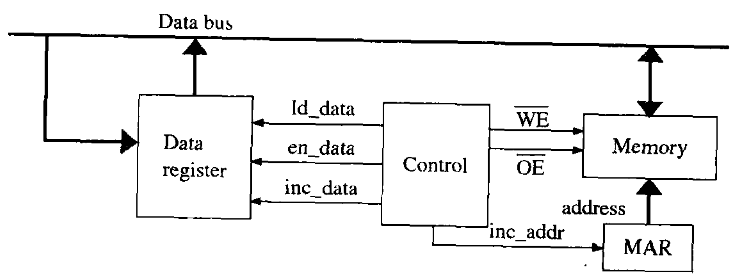
```
signal RAM1: RAMtype := (others => (others =>'0'));
                         -- Initialize all bits to '0'
begin
  IO <= "ZZZZZZZZ" when Cs_b = '1' or We_b = '0' or Oe_b = '1'
    else RAM1(to_integer(Address));   -- read from RAM
  process(We_b, Cs_b)
  begin
    if Cs_b = '0' and rising_edge(We_b) then  -- rising-edge of We_b
      RAM1(to_integer(Address'delayed)) <= IO; -- write
    end if;
  end process;
end simple_ram;
```

3.8 Model for SRAM Read/Write System

To illustrate further the use of multivalued logic, we present an example with a **bidirectional tristate bus**. We will design a memory read-write system that reads the content of 32 memory locations from a RAM, increments each data value, and stores it back into the RAM. A block diagram of the system is shown in Figure 3-16. In order to hold the word that we read from memory, we use a **data register**. In order to hold the memory address that we are accessing, we use a memory address register (**MAR**). The system reads a word from the RAM, loads it into the data register, increments the data register, stores the result back in the RAM, and then increments the memory address register. This process continues until the memory address equals 32.

The data bus is used as a **bidirectional bus**. During the read operation, the memory output appears on the bus, and the data register output to the data bus will be in a tristate condition. During the write operation, the data register output is on the data bus and the memory will use it as input data.

FIGURE 3-16: Block Diagram of RAM Read-Write System



Control signals required to operate the system are

ld_data	load data register from Data Bus
en_data	enable data register output onto Data Bus
inc_data	increment Data Register
inc_addr	increment MAR
$\overline{WE}$	Write Enable for SRAM
$\overline{OE}$	Output Enable for SRAM

Figure 3-17 shows the SM chart for the system. The SM chart uses four states. In the first state, the SRAM drives the memory data onto the bus and the memory data is loaded into the Data Register. The control signal $\overline{OE}$ and *ld_data* are true in this state. The Data Register is incremented in S_1. The *en_data* control signal is true in state S_2, and hence the Data Register drives the bus. Write enable $\overline{WE}$ is an active-low signal, which is asserted low only in S_2, so that $\overline{WE}$ is high in the other states. The contents of the data register thus get written to the RAM at the transition from S_2 to S_3. The memory address is incremented. The process continues until the address is 32. State S_3 checks this and produces a done signal when the address reaches 32.

FIGURE 3-17: SM Chart for RAM System

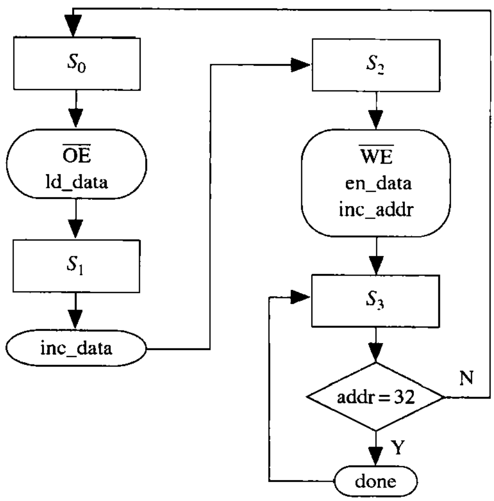


Figure 3-18 shows the VHDL code for the RAM system. The first process represents the SM chart, and the second process is used to update the registers on the rising edge of the clock. A short delay is added when the address is incremented to make sure the write to memory is completed before the address changes. A concurrent statement is used to simulate the tristate buffer, which enables the data register output onto the I/O lines.

FIGURE 3-18: VHDL Code for RAM System

```
-- SRAM Read-Write System model
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;

entity RAM6116_system is
end RAM6116_system;

architecture RAMtest of RAM6116_system is
component RAM6116 is
  port(Cs_b, We_b, Oe_b: in std_logic;
```

```
      Address: in unsigned(7 downto 0);
      IO: inout unsigned(7 downto 0));
end component RAM6116;

signal state, next_state: integer range 0 to 3;
signal inc_addr, inc_data, ld_data, en_data, Cs_b, clk, Oe_b, done:
      std_logic := '0';
signal We_b: std_logic := '1';                       -- initialize to read mode
signal Data: unsigned(7 downto 0);                   -- data register
signal Address: unsigned(7 downto 0) := "00000000";  -- address register
signal IO: unsigned(7 downto 0);                 -- I/O bus
begin
  RAM1: RAM6116 port map (Cs_b, We_b, Oe_b, Address, IO);
  control: process(state, Address)
  begin
    --initialize all control signals (RAM always selected)
    ld_data <= '0'; inc_data <= '0'; inc_addr <= '0'; en_data <= '0';
    done <= '0'; We_b <= '1'; Cs_b <= '0'; Oe_b <= '1';

    --start SM chart here
    case state is
      when 0 => Oe_b <= '0'; ld_data <= '1'; next_state <= 1;
      when 1 => inc_data <= '1'; next_state <= 2;
      when 2 => We_b <= '0'; en_data <= '1'; inc_addr <= '1'; next_state <= 3;
      when 3 =>
        if (Address = "00100000") then done <= '1'; next_state <= 3;
        else next_state <= 0;
        end if;
    end case;
  end process control;

  --The following process is executed on the rising edge of a clock.
  register_update: process(clk)   -- process to update data register
  begin
    if rising_edge(clk) then
      state <= next_state;
      if (inc_data = '1') then data <= data + 1; end if;
                             -- increment data in data register
      if (ld_data = '1') then data <= IO; end if;
                             -- load data register from bus
      if (inc_addr = '1') then Address <= Address + 1 after 1 ns; end if;
                             -- delay added to allow completion of memory write
    end if;
  end process register_update;

  -- Concurrent statements
  clk <= not clk after 100 ns;
  IO <= data when en_data = '1'
    else "ZZZZZZZZ";
end RAMtest;
```

This system can be modified to include all memory locations for testing the correctness of the entire SRAM. Memory systems are often tested by writing checkerboard patterns (alternate 0's and 1's) in all locations. For instance, we can write 01010101 (55 hexadecimal) into all odd addresses and 10101010 (hexadecimal AA) into all even addresses. Then the odd and even locations can be swapped. Developing VHDL code for such a system is left as an exercise problem.

3.9 Generics

Generics are commonly used to specify parameters for a component in such a way that the parameter values may be specified when the component is instantiated. For example, the rise and fall times for a gate could be specified as generics, and different numeric values for these generics could be assigned for each instance of the gate. The example of Figure 3-19 describes a two-input NAND gate whose rise and fall delay times depend on the number of loads on the gate. In the entity declaration, *Trise, Tfall,* and *load* are generics that specify the no-load rise time, the no-load fall time, and the number of loads. In the architecture, an internal *nand_value* is computed whenever *a* or *b* changes. If *nand_value* has just changed to a '1', a rising output has occurred, and the gate delay time is computed as

$$Trise + 3\text{ ns} * load$$

where 3 ns is the added delay for each load. Otherwise, a falling output has just occurred and the gate delay is computed as

$$Tfall + 2\text{ ns} * load$$

where 2 ns is the added delay for each load.

FIGURE 3-19: Rise/Fall Time Modeling Using Generic Statement

```
entity NAND2 is
  generic(Trise, Tfall: time; load: natural);
  port(a, b: in bit;
       c: out bit);
end NAND2;

architecture behavior of NAND2 is
signal nand_value: bit;
begin
  nand_value <= a nand b;
  c <= nand_value after (Trise + 3 ns * load) when nand_value = '1'
    else nand_value after (Tfall + 2 ns * load);
end behavior;

entity NAND2_test is
  port(in1, in2, in3, in4: in bit;
```

```
      out1, out2: out bit);
end NAND2_test;

architecture behavior of NAND2_test is
component NAND2 is
  generic(Trise: time := 3 ns; Tfall: time := 2 ns; load: natural := 1);
  port(a, b: in bit; c: out bit);
end component;
begin
  U1: NAND2 generic map (2 ns, 1 ns, 2) port map (in1, in2, out1);
  U2: NAND2 port map (in3, in4, out2);
end behavior;
```

The entity *NAND2_test* tests the NAND2 component. The component declaration in the architecture specifies default values for *Trise, Tfall,* and *load.* When *U1* is instantiated, the generic map specifies different values for *Trise, Tfall,* and *load.* When *U2* is instantiated, no generic map is included, so the default values are used.

3.10 Named Association

Up to this point, we have used *positional association* in the port maps and generic maps that are part of an instantiation statement. For example, assume that the entity declaration for a full adder is

```
entity FullAdder is
  port(X, Y, Cin: in bit; Cout, Sum: out bit);
end FullAdder;
```

The statement

```
FA0: FullAdder port map (A(0), B(0), '0', open, S(0));
```

creates a full adder and connects $A(0)$ to the X input of the adder, $B(0)$ to the Y input, '0' to the *Cin* input, leaves the *Cout* output unconnected, and connects $S(0)$ to the *Sum* output of the adder. The first signal in the port map is associated with the first signal in the entity declaration, the second signal with the second signal, and so on. In order to indicate no connection, the keyword **open** is used.

As an alternative, we can use *named association,* in which each signal in the port map is explicitly associated with a signal in the port of the component entity declaration. For example, the statement

```
FA0: FullAdder port map (Sum=>S(0), X=>A(0), Y=>B(0), Cin=>'0');
```

makes the same connections as the previous instantiation statement (i.e., *Sum* connects to $S(0)$, X connects to $A(0)$, etc). When named association is used, the order in which the connections are listed is not important, and any port signals not listed are left unconnected. Use of named association makes code easier to read, and it offers more flexibility in the order in which signals are listed.

When named association is used with a generic map, any unassociated generic parameter assumes its default value. For example, if we replace the statement in Figure 3-19 labeled U1 with

```
U1:NAND2 generic map (load => 3,Trise => 4ns) port map
(in1,in2,out1);
```

Tfall would assume its default value of 2 ns.

3.11 Generate Statements

In Chapter 2, we instantiated four full-adder components and interconnected them to form a 4-bit adder. Specifying the port maps for each instance of the full adder would become very tedious if the adder had 8 or more bits. When an iterative array of identical components is required, the **generate** statement provides an easy way of instantiating these components. The example of Figure 3-20 shows how a generate statement can be used to instantiate four 1-bit full adders to create a 4-bit adder. A 5-bit vector is used to represent the carries, with *Cin* the same as *C*(0) and *Cout* the same as *C*(4). The **for** loop generates four copies of the full adder, each with the appropriate **port map** to specify the interconnections between the adders.

Another example where the generate statement would have been very useful is the array multiplier. The VHDL code for the array multiplier (Chapter 4) used repeated use of **port map** statements in order to instantiate each component. They could have been replaced with generate statements.

FIGURE 3-20: Adder4 Using Generate Statement

```
entity Adder4 is
port(A, B: in bit_vector(3 downto 0); Ci: in bit;    -- Inputs
     S: out bit_vector(3 downto 0); Co: out bit);   -- Outputs
end Adder4;

architecture Structure of Adder4 is
component FullAdder
  port(X, Y, Cin: in bit;      -- Inputs
       Cout, Sum: out bit);    -- Outputs
end component;

signal C: bit_vector(4 downto 0);
begin
  C(0) <= Ci;
  -- generate four copies of the FullAdder
  FullAdd4: for i in 0 to 3 generate
  begin
    FAx: FullAdder port map (A(i), B(i), C(i), C(i+1), S(i));
  end generate FullAdd4;
  Co <= C(4);
end Structure;
```

In the preceding example, we used a generate statement of the form

```
generate_label: for identifier in range generate
[begin]
  concurrent statement(s)
end generate [generate_label];
```

At compile time, a set of concurrent statement(s) is generated for each value of the identifier in the given range. In Figure 3-20, one concurrent statement—a component instantiation statement—is used. A generate statement itself is defined to be a concurrent statement, so nested generate statements are allowed.

3.11.1 Conditional Generate

A generate statement with an if clause may be used to conditionally generate a set of concurrent statement(s). This type of generate statement has the form

```
generate_label: if condition generate
[begin]
  concurrent statement(s)
end generate [generate_label];
```

In this case, the concurrent statements(s) are generated at compile time only if the condition is true.

Figure 3-21 illustrates the use of conditional compilation using a generate statement with an **if** clause. An *N*-bit left-shift register is created if *Lshift* is true using the statement

```
genLS: if Lshift generate
  shifter <= Q(N-1 downto 1) & Shiftin;
end generate;
```

If *Lshift* is false, a right-shift register is generated using another conditional generate statement. The example also shows how generics and generate statements can be used together. It illustrates the use of generic parameters to write a VHDL model with parameters so that the size and function can be changed when it is instantiated.

FIGURE 3-21: Shift Register Using Conditional Compilation

```
entity shift_reg is
  generic(N: positive := 4; Lshift: Boolean := true);-- generic parameters used
  port(D: in bit_vector(N downto 1);
       Qout: out bit_vector(N downto 1);
       CLK, Ld, Sh, Shiftin: in bit);
end shift_reg;

architecture SRN of shift_reg is
signal Q, shifter: bit_vector(N downto 1);
begin
  Qout <= Q;
  genLS: if Lshift generate    -- conditional generate of left shift register
```

```
    shifter <= Q(N-1 downto 1) & Shiftin;
  end generate;
  genRS: if not Lshift generate -- conditional generate of right shift register
    shifter <= Shiftin & Q(N downto 2);
  end generate;
  process(CLK)
  begin
    if CLK'event and CLK = '1' then
      if LD = '1' then Q <= D;
      elsif Sh = '1' then Q <= shifter;
      end if;
    end if;
  end process;
end SRN;
```

3.12 Files and TEXTIO

The ability to input files and text is very valuable while testing large VHDL designs. This section introduces file input and output in VHDL. Files are frequently used with test benches to provide a source of test data and to provide storage for test results. VHDL provides a standard *TEXTIO* package that can be used to read or write lines of text from or to a file.

Before a file is used, it must be declared using a declaration of the form

```
file file-name: file-type [open mode] is "file-pathname";
```

For example,

```
file test_data: text open read_mode is "c:\test1\test.dat"
```

declares a file named *test_data* of type text that is opened in the read mode. The physical location of the file is in the test1 directory on the c: drive.

A file can be opened in *read_mode, write_mode*, or *append_mode*. In read_mode, successive elements in the file can be read using the read procedure. When a file is opened in write_mode, a new empty file is created by the host computer's file system, and successive data elements can be written to the file using the write procedure. To write to an existing file, the file should be opened in the append_mode.

A file can contain only one type of object, such as integers, bit-vectors, or text strings, as specified by the file type. For example, the declaration

```
type bv_file is file of bit_vector;
```

defines *bv_file* to be a file type that can contain only bit-vectors. Each file type has an associated implicit endfile function. A call of the form

```
endfile(file_name)
```

returns TRUE if the file pointer is at the end of the file.

The standard *TEXTIO* package that comes with VHDL contains declarations and procedures for working with files composed of lines of text. The package specification for *TEXTIO* (see Appendix C) defines a file type named text:

```
type text is file of string;
```

The *TEXTIO* package contains procedures for reading lines of text from a file of type text and for writing lines of text to a file.

Procedure `readline` reads a line of text and places it in a buffer with an associated pointer. The pointer to the buffer must be of type line, which is declared in the *TEXTIO* package as

```
type line is access string;
```

When a variable of type line is declared, it creates a pointer to a string. The code

```
variable buff: line;
. . .
readline(test_data, buff);
```

reads a line of text from *test_data* and places it in a buffer that is pointed to by *buff*. After reading a line into the buffer, we must call a version of the read procedure one or more times to extract data from the line buffer. The *TEXTIO* package provides overloaded read procedures to read data of types bit, bit-vector, boolean, character, integer, real, string, and time from the buffer. For example, if *bv4* is a bit_vector of length four, the call

```
read(buff, bv4);
```

extracts a 4-bit vector from the buffer, sets *bv4* equal to this vector, and adjusts the pointer *buff* to point to the next character in the buffer. Another call to read then extracts the next data object from the line buffer.

A call to read may be of one of two forms:

```
read(pointer, value);
read(pointer, value, good);
```

where *pointer* is of type line and *value* is the variable into which we want to read the data. In the second form, *good* is a boolean that returns TRUE if the read is successful and FALSE if it is not. The size and type of *value* determines which of the read procedures in the *TEXTIO* package is called. For example, if *value* is a string of length 5, then a call to read reads the next five characters from the line buffer. If *value* is an integer, a call to read skips over any spaces and then reads decimal digits until a space or other nonnumeric character is encountered. The resulting string is then converted to an integer. Characters, strings, and bit-vectors within files of type text are not delimited by quotes.

To write lines of text to a file, we must call a version of the write procedure one or more times to write data to a line buffer and then call writeline to write the line of data to a file. The *TEXTIO* package provides overloaded write procedures to write data of types bit, bit-vector, boolean, character, integer, real, string, and time to the buffer. For example, the code

```
variable buffw: line;
variable int1: integer;
variable bv8: bit_vector(7 downto 0);
. . .
write(buffw, int1, right, 6);
write(buffw, bv8, right, 10);
writeline(output_file, buffw);
```

converts *int1* to a text string, writes this string to the line buffer pointed to by *buffw*, and adjusts the pointer. The text will be right justified in a field six characters wide. The second call to write puts the bit_vector *bv8* in a line buffer, and adjusts the pointer. The 8-bit vector will be right justified in a field 10 characters wide. Then writeline writes the buffer to the *output_file*. Each call to write has four parameters: (1) a buffer pointer of type line; (2) a value of any acceptable type; (3) justification (left or right), which specifies the location of the text within the output field; and (4) field width, an integer that specifies the number of characters in the field.

As an example, we write a procedure to read data from a file and store the data in a memory array. This procedure will later be used to load instruction codes into a memory module for a computer system. The computer system can then be tested by simulating the execution of the instructions stored in memory. The data in the file will be of the following format:

```
address N comments
byte1 byte2 byte3 . . . byteN comments
```

The address consists of four hexadecimal digits, and *N* is an integer that indicates the number of bytes of code that will be on the next line. Each byte of code consists of two hexadecimal digits. Each byte is separated by one space, and the last byte must be followed by a space. Anything following the last space will not be read and will be treated as a comment. The first byte should be stored in the memory array at the given address, the second byte at the next address, and so forth. For example, consider the following file:

```
12AC 7 (7 hex bytes follow)
AE 03 B6 91 C7 00 0C
005B 2 (2 hex bytes follow)
01 FC<space>
```

When the *fill_memory* procedure is called using this file as an input, AE is stored in 12AC, 03 in 12AD, B6 in 12AE, 91 in 12AF, and so on.

Figure 3-22 gives VHDL code that calls the procedure *fill_memory* to read data from a file and store it in an array named *mem*. Since *TEXTIO* does not include a read procedure for hex numbers, the procedure *fill_memory* reads each hex value as a string of characters and then converts the string to an integer. Conversion of a single hex digit to an integer value is accomplished by table look-up. The constant named *lookup* is an array of integers indexed by characters in the range '0' to 'F'. This range includes the 23 ASCII characters: '0', '1', '2', . . . , '9', ':', ';', '<', '=', '>',

'?', '@', 'A', 'B', 'C', 'D', 'E', 'F'. The corresponding array values are 0, 1, 2, . . . , 9, −1, −1, −1, −1, −1, −1, −1, 10, 11, 12, 13, 14, 15. The −1 could be replaced with any integer value, since the seven special characters in the index range should never occur in practice. Thus, *lookup*('2') is the integer value 2, *lookup*('C') is 12, and so forth.

FIGURE 3-22: VHDL Code to Fill a Memory Array from a File

```
library IEEE;
use IEEE.numeric_bit.all;                    -- to use TO_UNSIGNED(int, size)
use std.textio.all;

entity testfill is
end testfill;

architecture fillmem of testfill is
type RAMtype is array (0 to 8191) of unsigned(7 downto 0);
signal mem: RAMtype := (others => (others => '0'));

procedure fill_memory(signal mem: inout RAMType) is
type HexTable is array (character range <>) of integer;
-- valid hex chars: 0, 1, . . . A, B, C, D, E, F (upper-case only)
constant lookup: HexTable('0' to 'F'): =
  (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, -1, -1, -1,
   -1, -1, -1, -1, 10, 11, 12, 13, 14, 15);
file infile: text open read_mode is "mem1.txt"; -- open file for reading
-- file infile: text is in "mem1.txt";         -- VHDL '87 version
variable buff: line;
variable addr_s: string(4 downto 1);
variable data_s: string(3 downto 1);           -- data_s(1) has a space
variable addr1, byte_cnt: integer;
variable data: integer range 255 downto 0;
begin
  while (not endfile(infile)) loop
    readline(infile, buff);
    read(buff, addr_s);                        -- read addr hexnum
    read(buff, byte_cnt);                      -- read number of bytes to read
    addr1 := lookup(addr_s(4)) * 4096 + lookup(addr_s(3)) * 256
             + lookup(addr_s(2)) * 16 + lookup(addr_s(1));
    readline(infile, buff);
    for i in 1 to byte_cnt loop
      read(buff, data_s);                    -- read 2 digit hex data and a space
      data := lookup(data_s(3)) * 16 + lookup(data_s(2));
      mem(addr1) <= TO_UNSIGNED(data, 8);
      addr1:= addr1 + 1;
    end loop;
  end loop;
end fill_memory;
```

```
begin
  testbench: process
  begin
    fill_memory(mem);
    -- insert code which uses memory data
  end process;
end fillmem;
```

Procedure *fill_memory* calls *readline* to read a line of text that contains a hex address and an integer. The first call to read reads the address string from the line buffer, and the second call to read reads an integer, which is the byte count for the next line. The integer *addr1* is computed using the look-up table for each character in the address string. The next line of text is read into the buffer, and a loop is used to read each byte. Since *data_s* is three characters long, each call to read reads two hex characters and a space. The hex characters are converted to an integer and then to an unsigned vector, which is stored in the memory array. The address is incremented before reading and storing the next byte. The procedure exits when the end of file is reached.

This chapter has introduced several important features of VHDL. Functions and procedures were introduced first. Attributes were presented next. Attributes associated with signals allow checking of setup and hold times and other timing specifications. Attributes associated with arrays allow us to write procedures that do not depend on the manner in which the arrays are indexed. Operator overloading can be used to extend the definition of VHDL operators so that they can be used with different types of operands. The IEEE Standard 1164 defines a system of 9-valued logic that is widely used with VHDL. Multivalued logic and the associated resolution functions allow us to model tristate buses and other systems where a signal is driven from more than one source. Generics enable us to specify parameter values for a component when the component is instantiated. Generate statements provide an efficient way to describe systems that have an iterative structure. The *TEXTIO* package provides a convenient way of doing file input and output.

3.13 Problems

3.1 Write a VHDL function that converts a 5-bit bit_vector to an integer. Note that the integer value of the binary number $a_4a_3a_2a_1a_0$ can be computed as

$$((((0 + a_4)*2 + a_3)*2 + a_2)*2 + a_1)*2 + a_0$$

How much simulated time will it take for your function to execute?

3.2 Write a VHDL function that will create the 2's complement of an n-bit vector. Use a call of the form `comp2(bit_vec, N)` where N is the length of the vector. State any assumptions you make about the range of bit_vec. Do the complement on a bit-by-bit basis using a loop.

3.3 Write a VHDL function which will return the largest integer in an array of N integers. The function call should be of the form `LARGEST(ARR, N)`.

3.4 A and B are bit-vectors that represent unsigned binary numbers. Write a VHDL function that returns TRUE if $A > B$. The function call should be of the form GT(A, B, N), where N is the length of the bit vectors. Do not call any functions of procedures from within your code. *Hint*: Start comparing the most significant bits of A and B first and proceed from left to right. As soon as you find a pair of unequal bits you can determine whether or not $A > B$. For example, if $A = 1011010$ and $B = 1010110$, you can determine that $A > B$ when you make the fourth comparison.

3.5 What are the major differences between VHDL functions and VHDL procedures?

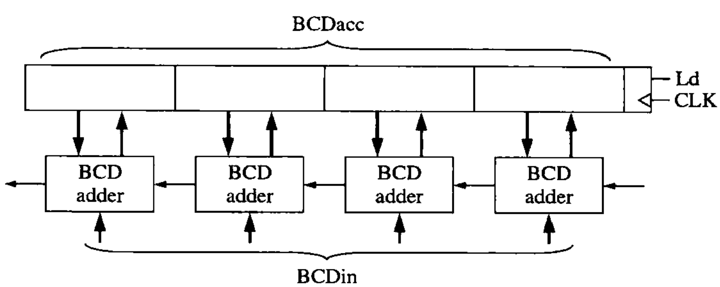
3.6 Write a VHDL procedure that counts the number of ones in an input bit-vector that is N bits long ($N \leq 31$). The output should be an unsigned vector that is 5 bits long.

3.7 X and Y are bit-vectors of length N that represent signed binary numbers, with negative numbers represented in 2's complement. Write a VHDL procedure that will compute $D = X - Y$. This procedure should also return the borrow from the last bit position (B) and an overflow flag (V). Do not call any other functions or procedures in your code. The procedure call should be of the form `SUBVEC(X, Y, D, B, V, N);`.

3.8 Write a VHDL module that implements a 4-digit BCD adder with accumulator (see block diagram below). If $LD = 1$, then the contents of $BCDacc$ are replaced with $BCDacc + BCDin$. Each four-digit BCD signal should be represented by an array of the following type:

```
type BCD4 is array (3 downto 0) of unsigned (3 downto 0);
```

Write a procedure that adds two BCD digits and a carry and returns a BCD digit and a carry. Call this procedure concurrently four times in your code.



3.9 For the following VHDL code, list the values of B and C at each time a change occurs. Include all deltas, and stop your listing when time > 8 ns. Assume that B is changed to "0110" at time 5 ns. Indicate the times at which procedure $P1$ is called.

```
entity Q1 is
  port(B, C: inout bit_vector(3 downto 0));
end Q1;

architecture Q1 of Q1 is
  procedure P1(signal A: inout bit_vector) is
  begin
    for i in 1 to 3 loop
      A(i) <= A(i-1);
    end loop;
    A(0) <= A(3);
  end P1;
begin
  process
  begin
    wait until B'event;
    P1(B);
    wait for 1 ns;
    P1(B);
  end process;
  C <= B;
end Q1;
```

3.10 The following VHDL code is part of a process. Assume that $A = B =$ '0' before the code is executed. Give the values of the variables $X1$, $X2$, $X3$, and $X4$ immediately after the code is executed.

```
wait until clock'event and clock = '1';
A <= not B;
A <= transport B after 5 ns;
wait for 5 ns;
X1 := A'event;
X2 := A'delayed'event;
X3 := A'last_event;
X4 := A'delayed'last_event;
```

3.11 Write a VHDL function that will take two integer vectors, A and B, and find the dot product $C = \Sigma\, a_i * b_i$. The function call should be of the form `DOT(A,B)`, where A and B are integer vector signals. Use attributes inside the function to determine the length and ranges of the vectors. Make no assumptions about the high and low values of the ranges. For example,

$A(3 \text{ downto } 1) = (1, 2, 3), B(3 \text{ downto } 1) = (4, 5, 6), C = 3 * 6 + 2 * 5 + 1 * 4 = 32$

Output a warning if the ranges are not the same.

3.12 Write a VHDL procedure that will add two $n \times m$ matrices of integers, $C <= A + B$. The procedure call should be of the form `addM(A, B, C)`. The procedure should report an error if the number of rows in A and B are not the same or if the number of columns in A and B are not the same. Make no assumptions about the high and low values or direction of the ranges for either dimension.

3.13 Write a VHDL procedure that will add two bit-vectors that represent signed binary numbers. Negative numbers are represented in 2's complement. If the vectors are of different lengths, the shorter one should be sign-extended during the addition. Make no assumptions about the range for either vector. The procedure call should be of the form `Add2(A, B, Sum, V)`, where $V = 1$ if the addition produces a 2's complement overflow.

3.14 A VHDL entity has inputs A and B, and outputs C and D. A and B are initially high. Whenever A goes low, C will go high 5 ns later, and if A changes again, C will change 5 ns later. D will change if B does not change for 3 ns after A changes.

(a) Write the VHDL architecture with a process that determines the outputs C and D.

(b) Write another process to check that B is stable 2 ns before and 1 ns after A goes high. The process should also report an error if B goes low for a time interval less than 10 ns.

3.15 Write an overloading function for the "<" operator for bit-vectors. Return a boolean TRUE if A is less than B, otherwise return FALSE. Report an error if the bit-vectors are of different lengths.

3.16 Write an overloading function for the unary "−" operator for bit-vectors. If A is a bit-vector `-A` should return the 2's complement of A.

3.17 Consider the following three concurrent statements, where R is a resolved signal of type X01Z:

```
R <= transport '0' after 2 ns, 'Z' after 8 ns;
R <= transport '1' after 10 ns;
R <= transport '1' after 4 ns, '0' after 6 ns;
```

Draw the multiple drivers that will be created and the resolved output signal R from time 0 until time 12 ns.

3.18 Write a VHDL description of an address decoder/address match detector. One input to the address decoder is an 8-bit address, which can have any range with a length of 8 bits; for example, `bit_vector addr(8 to 15)`. The second input is `check: x01z_vector(5 downto 0)`. The address decoder will output *Sel* = '1' if the upper 6 bits of the 8-bit address match the 6-bit check vector. For example, if *addr* = "10001010" and *check* = "1000XX", then *Sel* = '1'. Only the six leftmost bits of *addr* will be compared; the remaining bits are ignored. An 'X' in the check vector is treated as a don't care.

3.19 Write a VHDL model for one flip-flop in a 74HC374 (octal D-type flip-flop with three-state outputs). Use the IEEE-standard nine-valued logic package. Assume that all logic values are 'x', '0', '1' or 'z'. Check setup, hold, and pulse width specs using assert statements. Unless the output is 'z', the output should be 'x' if CLK or OC is 'x', or if an 'x' has been stored in the flip-flop.

3.20 Write a VHDL function to compare two IEEE std_logic_vectors to see if they are equal. Report an error if any bit in either vector is not '0', '1', or '–' (don't care), or if the lengths of the vectors are not the same. The function call should pass only the vectors. The function should return TRUE if the vectors are equal, else FALSE. When comparing the vectors, consider that '0' = '–', and '1' = '–'. Make no assumptions about the index range of the two vectors (for example, one could be `1 to 7` and the other `8 downto 0`).

3.21 Consider the following concurrent statements, where A, B, and C are of type std_logic:

```
A <= transport '1' after 5 ns, '0' after 10 ns, 'Z' after 15 ns;
B <= transport '0' after 4 ns, 'Z' after 10 ns;
C <= A after 6 ns;
C <= transport A after 5 ns;
C <= reject 3 ns inertial B after 4 ns;
```

(a) Draw drivers (see Figure 2-27) for signals A and B.
(b) Draw the three drivers $s(0)$, $s(1)$, and $s(2)$ for C (similar to Figure 3-11). Assume that C is initialized to 'H'.
(c) List the value for C each time it is resolved by the drivers, and draw a timing chart for C.

3.22 Subtype `X01LH` of `std_logic` has values of 'X', '0', '1', 'L', and 'H'. Complete the following table for a resolution function of this subtype.

	'X'	'0'	'1'	'L'	'H'
'X'					
'0'					
'1'					
'L'					
'H'					

3.23 Write an overloading function for "not", where the input and returned value are standard logic vectors. The "not" function should basically simulate a group of inverters. The output bits should be one of the following: 'U', '0', '1', or 'X'. An uninitialized input should give an uninitialized output.

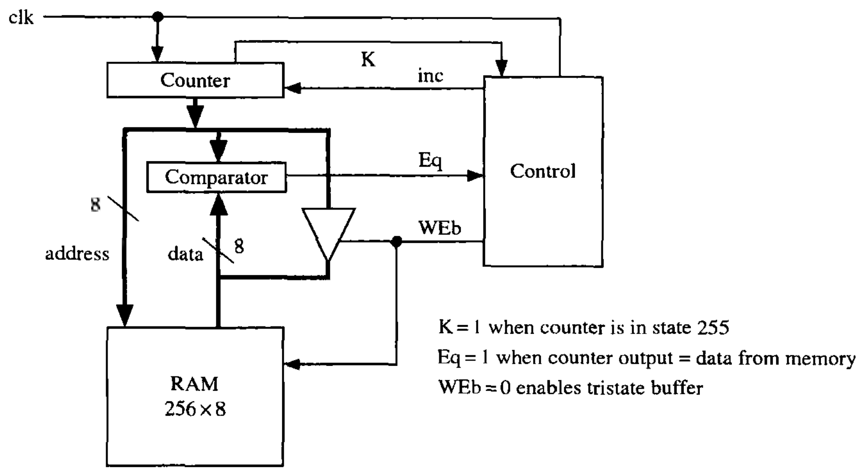
3.24 In the following code, all signals are 1-bit `std_logic`. Draw a logic diagram that corresponds to the code. Assume that a D flip-flop with CE is available.

```
F <= A when EA = '1' else B when EB = '1' else 'Z';
process(CLK)
begin
  if CLK'event and CLK = '1' then
    if Ld = '1' then A <= B; end if;
    if Cm = '1' then A <= not A; end if;
  end if;
end process;
```

3.25 Design a memory-test system to test the first 256 bytes of a static RAM memory. The system consists of simple controller, an 8-bit counter, a comparator, and a memory as shown below. The counter is connected to both the address and data (IO) bus so that 0 will be written to address 0, 1 to address 1, 2 to address 2, . . . , and 255 to address 255. Then the data will be read back from address 0, address 1, . . . , address 255 and compared with the address. If the data does not match, the controller goes to the fail state as soon as a mismatch is detected; otherwise, it goes to a pass state after all 256 locations have been matched. Assume that $OE\_b = 0$ and $CS\_b = 0$.

(a) Draw an SM chart or a state graph for the controller (five states). Assume that the clock period is long enough so that one word can be read every clock period.

(b) Write VHDL code for the memory-test system.



3.26 Design a memory-test system similar to that of Problem 3.25, except write a checkerboard pattern into memory (01010101 into address 0, 10101010 into address 1, etc.). Draw the block diagram and SM chart.

3.27 Design a memory tester that verifies the correct operation of a 6116 static RAM (Figure 3-15). The tester should store a checkerboard pattern (alternating 0's and 1's in the even addresses, and alternating 1's and 0's in the odd addresses) in all memory locations and then read it back. The tester should then repeat the test using the reverse pattern.

(a) Draw a block diagram of the memory tester. Show and explain all control signals.

(b) Draw an SM chart or state graph for the control unit. Use a simple RAM model and disregard timing.

(c) Write VHDL code for the tester and use a test bench to verify its operation.

3.28 A clocked T flip-flop has propagation delays from the rising edge of CLK to the changes in Q and Q' as follows: If Q (or Q') changes to 1, t_{plh} = 8 ns, and if Q (or Q') changes to 0, t_{phl} = 10 ns. The minimum clock pulse width is t_{ck} = 15 ns, the setup time for the T input is t_{su} = 4 ns, and the hold time is t_h = 2 ns. Write a VHDL model for the flip-flop that includes the propagation delay and that reports if any timing specification is violated. Write the model using generic parameters with default values.

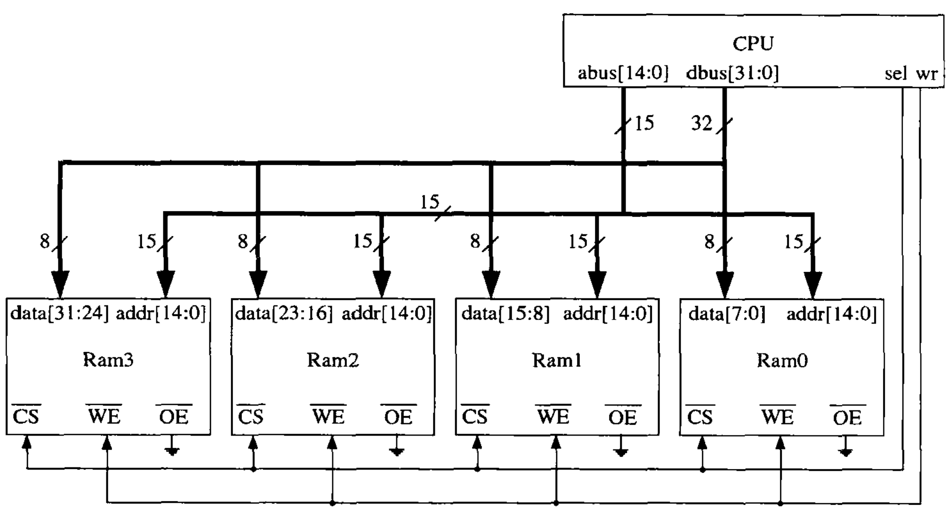
3.29 **(a)** Write a model for a D flip-flop with a direct clear input. Use the following generic timing parameters: t_{plh}, t_{phl}, t_{su}, t_h, and t_{cmin}. The minimum allowable clock period is t_{cmin}. Report appropriate errors if timing violations occur.
(b) Write a test bench to test your model. Include tests for every error condition.

3.30 Write a VHDL model for an N-bit comparator using an iterative circuit. In the entity, use the generic parameter N to define the length of the input bit-vectors A and B. The comparator outputs should be EQ = '1' if $A = B$, and GT = '1' if $A > B$. Use a for loop to do the comparison on a bit-by-bit basis, starting with the high-order bits. Even though the comparison is done on a bit-by-bit basis, the final values of EQ and GT apply to A and B as a whole.

3.31 Four RAM memories are connected to CPU busses as shown below. Assume that the following RAM component is available:

```
component SRAM
  port(cs_b, we_b, oe_b: in bit;
       address: in bit_vector(14 downto 0);
       data: inout std_logic_vector(7 downto 0));
end component;
```

Write a VHDL code segment which will connect the four RAMs to the busses. Use a generate statement and named association.



3.32 Write structural VHDL code for a module that is an N-bit serial-in, serial-out right-shift register. Inputs to the shift register are bit signals: SI (serial input), Sh (shift enable), and CLK. Your module should include a generic in the entity declaration, and a generate statement in the architecture. Assume that a component for a D flip-flop with clock enable (CE) is available.

3.33 Write structural VHDL code for a module that has two inputs: an N-bit vector A, and a control signal B (1 bit). The module has an N-bit output vector, C. When $B = 1$, $C <= A$. When $B = 0$, C is all 0's. Use a generic to specify the value of N (default = 4). To implement the logic, use a generate statement that instantiates N 2-input AND gates.

3.34 Create a 4×4 array multiplier using generate statements. Use full adder, half adder, and AND gate components as in Chapter 4.

3.35 B is an integer array with range 0 to 4. Write a VHDL code segment which will read a line of text from a file named "FILE2" and then read five integers into array B. Assume that TEXTIO libraries are available.

3.36 Write a procedure that has an integer signal and a file name as parameters. Each line of the file contains a delay value and an integer. The procedure reads a line from the file, waits for the delay time, assigns the integer value to the signal, and then reads the next line. The procedure should return when end-of-file is reached.

3.37 Write a procedure that logs the history of values of a bit-vector signal to a text file. Each time the signal changes, write the current time and signal value to the file. VHDL has a built in function called NOW that returns the current simulation time when it is called.

CHAPTER 4

Design Examples

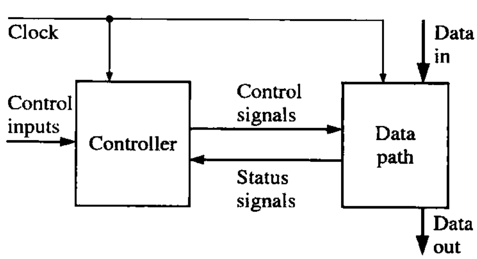
In this chapter, we present several VHDL design examples to illustrate the design of small digital systems. We present the concept of dividing a design into a controller and a data path and using the control circuit to control the sequence of operations in a digital system. We use VHDL to describe a digital system at the behavioral level so that we can simulate the system to test the algorithms used. We also show how designs have to be coded structurally if specific hardware structures are to be generated.

In any design, first you should understand the problem and the design specifications clearly. If the problem has not been stated clearly, try to get the features of the design clarified. In real-world designs, if another team or a client company is providing your team with the specifications, getting the design specifications clarified properly can save you a lot of grief later. Good design starts with a clear specification document.

Once the problem has been stated clearly, often designers start thinking about the basic blocks necessary to accomplish what is specified. Designers often think of standard building blocks, such as adders, shift registers, counters, and so on. Traditional design methodology splits a design into a "data path" and a "controller." The term *data path* refers to the hardware that actually performs the data processing. The controller sends control signals or commands to the data path, as in Figure 4-1. The controller can obtain feedback in the form of status signals from the data path.

In the context of a microprocessor, the data path is the *arithmatic logic unit* (ALU) that performs the core of the processing. The controller is the control logic that sends appropriate control signals to the data path, instructing it to perform addition, multiplication, shifting, or whatever action is called for by the instruction.

FIGURE 4-1: Separation of a Design into Data Path and Controller



Many have a tendency to consider the term *data path* to be synonymous with the data bus, but *data path* in traditional design terminology refers to the actual data processing unit.

Maintaining a distinction between data path and controller helps in debugging (i.e., finding errors in the design). It also helps while modifying the design. Many modifications can be accomplished by changing only the control path because the same data path can support the new requirements. The controller can generate the new sequence of control signals to accomplish the functionality of the modified design. Design often involves refining the data path and controller in iterations.

In this chapter, we will discuss various design examples. Several arithmetic and nonarithmetic examples are presented. Nonarithmetic examples include a seven-segment decoder, a traffic light, a scoreboard, and a keypad scanner. Arithmetic circuits such as adders, multipliers, and dividers are presented.

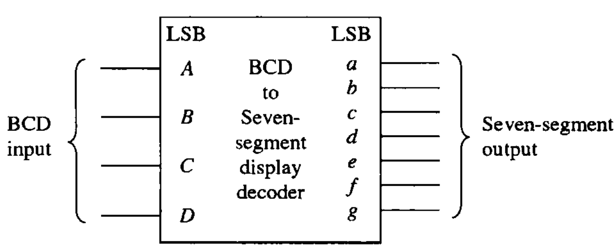
4.1 BCD to Seven-Segment Display Decoder

Seven-segment displays are often used to display digits in digital counters, watches, and clocks. A digital watch displays time by turning on a combination of the segments on a seven-segment display. For this example, the segments are labeled as follows, and the digits have the forms as indicated in Figure 4-2.

FIGURE 4-2: Seven-Segment Display

Let us design a BCD to seven-segment display decoder. BCD stands for binary-coded decimal. In this format, each digit of a decimal number is encoded into 4-bit binary representation. This decoder is a purely combinational circuit, and hence no state machine is involved here. A block diagram of the decoder is shown in Figure 4-3. The decoder for one BCD digit is presented.

FIGURE 4-3: Block Diagram of a BCD to Seven-Segment Display Decoder



We will create a behavioral VHDL architectural description of this BCD to seven-segment decoder by using a single process with a case statement to model this combinational circuit, as in Figure 4-4. The sensitivity list of the process consists of the BCD number (4 bits).

FIGURE 4-4: Behavioral VHDL Code for BCD to Seven-Segment Decoder

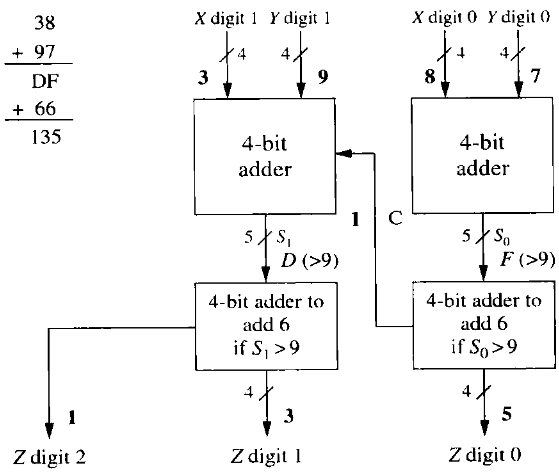
```
entity bcd_seven is
  port(bcd: in bit_vector(3 downto 0);
       seven: out bit_vector(7 downto 1));
                    -- LSB is segment a of the display. MSB is segment g
end bcd_seven;

architecture behavioral of bcd_seven is
begin
  process (bcd)
  begin
    case bcd is
      when "0000" => seven <= "0111111";
      when "0001" => seven <= "0000110";
      when "0010" => seven <= "1011011";
      when "0011" => seven <= "1001111";
      when "0100" => seven <= "1100110";
      when "0101" => seven <= "1101101";
      when "0110" => seven <= "1111101";
      when "0111" => seven <= "0000111";
      when "1000" => seven <= "1111111";
      when "1001" => seven <= "1101111";
      when others => null;
    end case;
  end process;
end behavioral;
```

4.2 A BCD Adder

In this example, we design a two-digit BCD adder, which will add two BCD numbers and produce the sum in BCD format. In BCD representation, each decimal digit is encoded into binary. For instance, decimal number 97 will be represented as 1001 0111 in the BCD format, where the first 4 bits represent digit 9 and the next 4 bits represent digit 7. Note that the BCD representation is different from the binary representation of 97, which is 1100001. It takes 8 bits to represent 97 in BCD, whereas the binary representation of 97 (1100001) only requires 7 bits. The 4-bit binary combinations 1010, 1011, 1100, 1101, 1110, and 1111 corresponding to hexadecimal numbers A to F are not used in the BCD representation. Since 6 out of 16 representations possible with 4 binary bits are skipped, a BCD number will take more bits than the corresponding binary representation.

When BCD numbers are added, each sum digit should be adjusted to skip the six unused codes. For instance, if 6 is added with 8, the sum is 14 in decimal form. A binary adder would yield 1110, but the lowest digit of the BCD sum should read 4. In order to obtain the correct BCD digit, 6 should be added to the sum whenever it is greater than 9. Figure 4-5 illustrates the hardware that will be required to perform the addition of two BCD digits. A binary adder adds the least significant digits. If the sum is greater than 9, an adder adds 6 to yield the correct sum digit and a carry digit to be added with the next digit. The addition of the higher digits is performed in a similar fashion.

FIGURE 4-5: Addition of Two BCD Numbers



The VHDL code for the BCD adder is shown in Figure 4-6. The input BCD numbers are represented by X and Y. The BCD sum of two 2-digit BCD numbers can exceed two digits, and hence three BCD digits are provided for the sum, which is represented by Z. The unsigned type from the IEEE numeric_bit library is used to represent X, Y, and Z. Aliases are defined to denote each digit of each BCD number. For example, the upper digit of X can be denoted by *Xdig1* by using the VHDL statement

```
alias Xdig1: unsigned(3 downto 0) is X(7 downto 4);
```

This statement allows us to use the name *Xdig1* whenever we wish to refer to the upper digit of X. If BCD numbers 97 and 38 are added, the sum is 135, and hence, *Zdig2* equals 1, *Zdig1* equals 3 and *Zdig0* equals 5.

The overloaded '+' operator from the IEEE numeric_bit library is used for adding each BCD digit. Adding two 4-bit vectors can result in a 5-bit sum. The sums are temporarily stored in *S0* and *S1*, which are declared to be 5-bit numbers. Since we want a 5-bit result, we must extend *Xdig0* to 5 bits by concatenating '0' and *Xdig0*. (*Ydig0* will automatically be extended to match.) Hence

```
S0 <= '0' & Xdig0 + Ydig0;
```

FIGURE 4-6: VHDL Code for BCD Adder

```
library IEEE;
use IEEE.numeric_bit.all;

entity BCD_Adder is
  port(X, Y: in unsigned(7 downto 0);
       Z: out unsigned(11 downto 0));
end BCD_Adder;

architecture BCDadd of BCD_Adder is
alias Xdig1: unsigned(3 downto 0) is X(7 downto 4);
alias Xdig0: unsigned(3 downto 0) is X(3 downto 0);
alias Ydig1: unsigned(3 downto 0) is Y(7 downto 4);
alias Ydig0: unsigned(3 downto 0) is Y(3 downto 0);
alias Zdig2: unsigned(3 downto 0) is Z(11 downto 8);
alias Zdig1: unsigned(3 downto 0) is Z(7 downto 4);
alias Zdig0: unsigned(3 downto 0) is Z(3 downto 0);
signal S0, S1: unsigned(4 downto 0);
signal C: bit;
begin
  S0 <= '0' & Xdig0 + Ydig0; -- overloaded +
  Zdig0 <= S0(3 downto 0) + 6 when S0 > 9
      else S0(3 downto 0); -- add 6 if needed
  C <= '1' when S0 > 9 else '0';
  S1 <= '0' & Xdig1 + Ydig1 + unsigned'(0=>C);
                              -- type conversion done on C before adding
  Zdig1 <= S1(3 downto 0) + 6 when S1 > 9
      else S1(3 downto 0);
  Zdig2 <= "0001" when S1 > 9 else "0000";
end BCDadd;
```

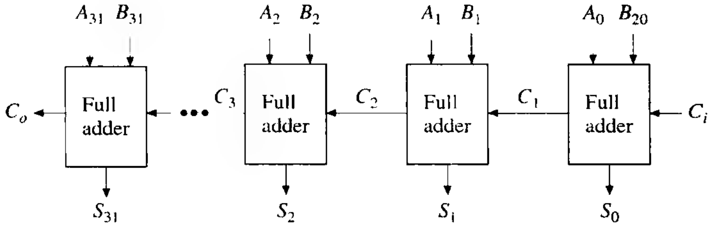
accomplishes the addition of the least significant digits. During the addition of the second digit, the carry digit from the addition of the *XDig0* and *Ydig0* is also added. The carry bit *C* must be converted to the unsigned type before it can be added to *Xdig1* + *Ydig1*. The notation `unsigned'(0=>C)` accomplishes this conversion. Thus, the addition of the second digit is accomplished by the statement

```
S1 <= '0' & Xdig1 + Ydig1 + unsigned'(0=>C);
```

4.3 32-Bit Adders

Let us assume that we have to design a 32-bit adder. A simple manner to construct an adder is to build a **ripple-carry adder**, as in Figure 4-7. In this type of adder, 32 copies of a 1-bit full adder are connected in succession to create the 32-bit adder. The carry "ripples" from the least significant bit to the most significant bit. If gate

FIGURE 4-7: A 32-Bit Ripple-Carry Adder



delays are t_g, a 1-bit adder delay is $2t_g$ (assuming a sum-of-products expression for sum and carry, and ignoring delay for inverters), and a 32-bit ripple-carry adder will take approximately 64 gate delays. For instance, if gate delays are 1 ns, the maximum frequency at which the 32-bit ripple-carry adder can operate is approximately 16 MHz. This is inadequate for many applications. Hence, designers often resort to faster adders.

4.3.1 Carry Look-Ahead Adders

A popular fast-addition technique is carry look-ahead (CLA) addition. In the carry look-ahead adder, the carry signals are calculated in advance, based on the input signals. For any bit position i, we can see that a carry will be generated if the corresponding input bits (i.e., A_i, B_i) are '1' or if there was a carry-in to that bit and at least one of the input bits are '1'. In other words, bit i has carry-out if A_i and B_i are '1' (irrespective of carry-in to bit i); bit i also has a carry-out if C_i = '1' and either A_i or B_i is '1'. Thus, for any stage i, the carry-out is

$$C_{i+1} = A_iB_i + (A_i \oplus B_i) \cdot C_i \tag{4-1}$$

The "$\oplus$" stands for the exclusive OR operation. Equation (4-1) simply expresses that there is a carry out from a bit position if it **generated** a carry by itself (i.e., A_iB_i = '1') or it simply **propagated** the carry from the lower bit forwarded to it (i.e., $(A_i \oplus B_i) \cdot C_i$).

Since A_iB_i = '1' indicates that a stage generated a carry, a general **generate (G_i) function** may be written as

$$G_i = A_iB_i \tag{4-2}$$

Similarly, since $(A_i \oplus B_i)$ indicates whether a stage should propagate the carry it receives from the lower stage, a general **propagate (P_i) function** may be written as

$$P_i = A_i \oplus B_i \tag{4-3}$$

Notice that the propagate and generate functions only depend on the input bits and can be realized with one or two gate delays. Since there will be a carry whether one of A_i or B_i is '1' or both are '1', we can also write the propagate expression as

$$P_i = A_i + B_i \tag{4-4}$$

where the OR operation is substituted for the XOR operation. Logically this propagate function also results in the correct carry-out; however, traditionally it has been customary to define the propagate function as the XOR; that is, the bit

position simply propagates a carry (without generating a carry by itself). Also, typically, the sum signal is expressed as

$$S_i = A_i \oplus B_i \oplus C_i = P_i \oplus C_i \tag{4-5}$$

The expression $P_i \oplus C_i$ can be used for sum only if P_i is defined as $A_i \oplus B_i$.

The carry-out equation can be rewritten by substituting (4-2) and (4-3) in (4-1) for G_i and P_i as

$$C_{i+1} = G_i + P_iC_i \tag{4-6}$$

In a 4-bit adder, the C_i's can be generated by repeatedly applying Equation (4-6) as follows:

$$C_1 = G_0 + P_0C_0 \tag{4-7}$$

$$C_2 = G_1 + P_1C_1 = G_1 + P_1G_0 + P_1P_0C_0 \tag{4-8}$$

$$C_3 = G_2 + P_2C_2 = G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0 \tag{4-9}$$

$$C_4 = G_3 + P_3C_3 = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_0 \tag{4-10}$$

These carry bits are the look-ahead carry bits. They are expressed in terms of P_i's, G_i's, and C_0. Thus, the sum and carry from any stage can be calculated without waiting for the carry to ripple through all the previous stages. Since G_i's and P_i's can be generated with one or two gate delays, the C_i's will be available in three or four gate delays. The advantage is that these delays will be the same independent of the number of bits we need to add, in contrast to the ripple counter. Of course, this is achieved with the extra gates to generate the look-ahead carry bits. A 4-bit carry look-ahead adder can now be built, as illustrated in Figure 4-8.

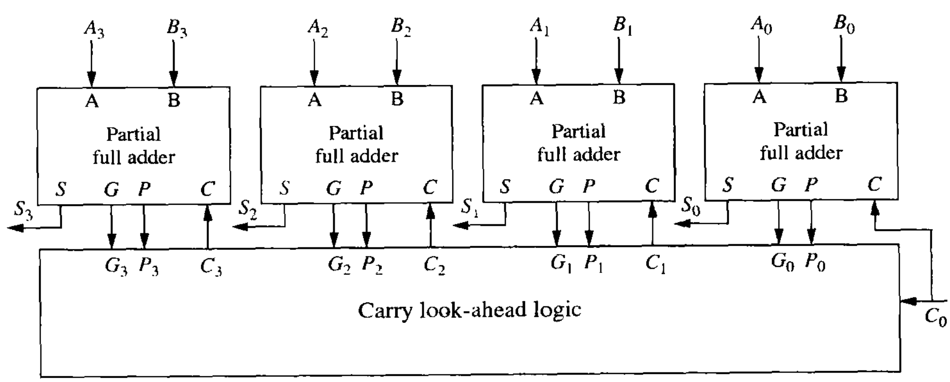


FIGURE 4-8: Block Diagram of a 4-Bit CLA

The disadvantage of the carry look-ahead adder is that the look-ahead carry logic, as in Equations (4-7) through (4-10), is not simple. It gets quite complicated for more than 4 bits. For that reason, carry look-ahead adders are usually implemented as 4-bit modules and are used in a hierarchical structure to realize adders that have multiples of 4 bits. Figure 4-9 shows the block diagram for a 16-bit carry look-ahead adder. Four carry look-ahead adders, similar to the one shown in

FIGURE 4-9: Block Diagram of a 16-Bit CLA

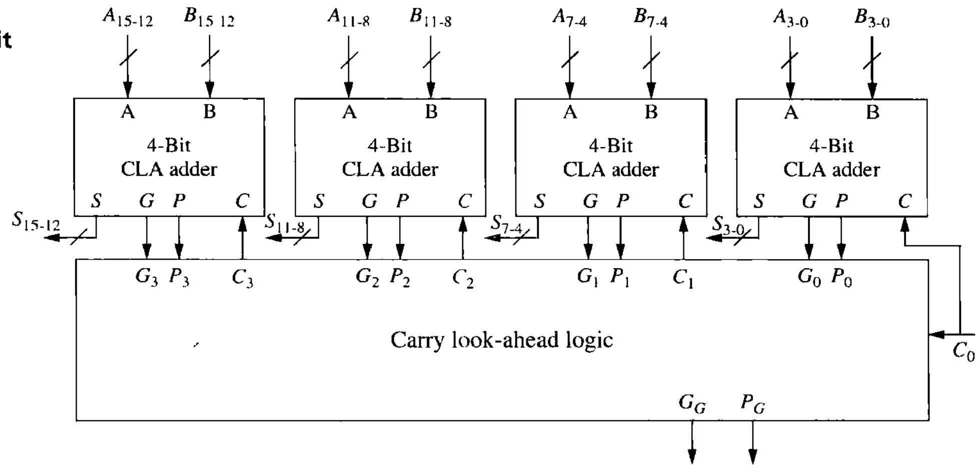


Figure 4-8, are used. Instead of relying on each 4-bit adder to send its carry-out to the next 4-bit adder, the carry look-ahead logic generates input carry bits to be fed to each 4-bit adder. This is accomplished by computing a group propagate (P_G) and group generate (G_G) signal, which is produced by each 4-bit adder. The next level of carry look-ahead logic uses these group propagates/generates and generates the required carry bits in parallel. The propagate for a group is true if all the propagates in that group are true. The generate for a group is true if the MSB generated a carry or if a lower bit generated a carry and every higher bit in the group propagated it. Thus

$$P_G = P_3P_2P_1P_0 \tag{4-11}$$

$$G_G = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 \tag{4-12}$$

The group propagate P_G and generate G_G will be available after three and four gate delays, respectively (one or two additional delays than the P_i and G_i signals, respectively). Figure 4-10 illustrates the VHDL description of a 4-bit carry look-ahead adder.

FIGURE 4-10: VHDL Description of a 4-Bit Carry Look-Ahead Adder

```
entity CLA4 is
  port(A, B: in bit_vector(3 downto 0); Ci: in bit;          -- Inputs
       S: out bit_vector(3 downto 0); Co, PG, GG: out bit);  -- Outputs
end CLA4;

architecture Structure of CLA4 is
component GPFullAdder
  port(X, Y, Cin: in bit;    -- Inputs
       G, P, Sum: out bit);  -- Outputs
end component;
```

```
component CLALogic is
  port(G, P: in bit_vector(3 downto 0); Ci: in bit;          -- Inputs
       C: out bit_vector(3 downto 1); Co, PG, GG: out bit);  -- Outputs
end component;

signal G, P: bit_vector(3 downto 0); -- carry internal signals
signal C: bit_vector(3 downto 1);
begin      --instantiate four copies of the GPFullAdder
  CarryLogic: CLALogic port map (G, P, Ci, C, Co, PG, GG);
  FA0: GPFullAdder port map (A(0), B(0), Ci, G(0), P(0), S(0));
  FA1: GPFullAdder port map (A(1), B(1), C(1), G(1), P(1), S(1));
  FA2: GPFullAdder port map (A(2), B(2), C(2), G(2), P(2), S(2));
  FA3: GPFullAdder port map (A(3), B(3), C(3), G(3), P(3), S(3));
end Structure;

entity CLALogic is
  port(G, P: in bit_vector(3 downto 0); Ci: in bit;          -- Inputs
       C: out bit_vector(3 downto 1); Co, PG, GG: out bit);  -- Outputs
end CLALogic;

architecture Equations of CLALogic is
signal GG_int, PG_int: bit;
begin      -- concurrent assignment statements
  C(1) <= G(0) or (P(0) and Ci);
  C(2) <= G(1) or (P(1) and G(0)) or (P(1) and P(0) and Ci);
  C(3) <= G(2) or (P(2) and G(1)) or (P(2) and P(1) and G(0)) or
          (P(2) and P(1) and P(0) and Ci);
  PG_int <= P(3) and P(2) and P(1) and P(0);
  GG_int <= G(3) or (P(3) and G(2)) or (P(3) and P(2) and G(1)) or
            (P(3) and P(2) and P(1) and G(0));
  Co <= GG_int or (PG_int and Ci);
  PG <= PG_int;
  GG <= GG_int;
end Equations;

entity GPFullAdder is
  port(X, Y, Cin: in bit;     -- Inputs
       G, P, Sum: out bit);   -- Outputs
end GPFullAdder;

architecture Equations of GPFullAdder is
signal P_int: bit;
begin                         -- concurrent assignment statements
  G <= X and Y;
  P <= P_int;
  P_int <= X xor Y;
  Sum <= P_int xor Cin;
end Equations;
```

VHDL code for a 16-bit carry look-ahead adder can be developed by instantiating four copies of the 4-bit carry look-ahead adder and one additional copy of the carry look-ahead logic. A 64-bit adder can be built by one more level of block carry look-ahead logic. The delay increases only by two gate delays when the adder size increases from 16 bits to 64 bits. Developing VHDL code for 16-bit carry look-ahead logic is left as an exercise.

Figure 4-11 illustrates behavioral VHDL code for a 32-bit adder using the overloaded '+' operator from IEEE numeric_bit library. If this code is synthesized, depending on the tools used and the target technology, an adder with characteristics in between a ripple-carry adder and a fast two-level adder will be obtained. The various topologies result in different area, power, and delay characteristics.

FIGURE 4-11: Behavioral Model for a 32-Bit Adder

```
library IEEE;
use IEEE.numeric_bit.all;

entity Adder32 is
  port(A, B: in unsigned(31 downto 0); Ci: in bit;   -- Inputs
       S: out unsigned(31 downto 0); Co: out bit);   -- Outputs
end Adder32;

architecture overload of Adder32 is
signal Sum33: unsigned(32 downto 0);
begin
  Sum33 <= '0' & A + B + unsigned'(0=>Ci);            -- adder
  S <= Sum33(31 downto 0);
  Co <= Sum33(32);
end overload;
```

Example

If gate delays are t_g, what is the delay of the fastest 32-bit adder? Assume that the amount of hardware consumed is not a constraint. Only speed is important.

Answer

We can express each sum bit of a 32-bit adder as a sum of products expression of the input bits. There will be 33 such equations, including one for the carry out bit. These equations will be very long, and some of them could include 60+ variables in the product term. Nevertheless, if gates with any number of inputs are available, theoretically a two-level adder can be made. Although it is not very practical, theoretically, the delay of the fastest adder will be $2t_g$ if gate delays are t_g.

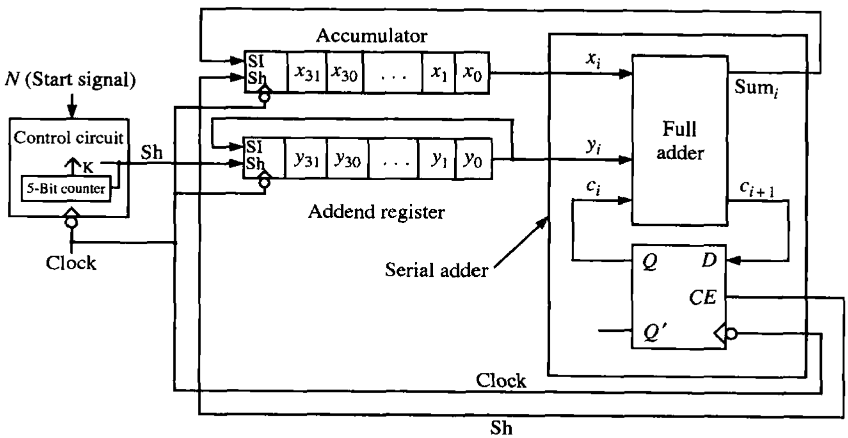
Example

Is ripple-carry adder the smallest 32-bit adder?

Answer

A 32-bit ripple-carry adder uses 32 1-bit adders. We could design a 32-bit serial adder using a single 1-bit full adder. The input numbers are shifted into the adder, one bit at a time, and carry output from addition of each pair of bits is saved in a flip-flop and fed back to the next addition. The hardware illustrated in Figure 4-12 accomplishes this. The delay of adder will be 32 $(2t_g + t_{ff})$, where $2t_g$ is the delay of the 1-bit full adder, and t_{ff} is the delay of the flip-flop (including setup time). If a flip-flop delay is at least two gate delays, the delay of the 32-bit serial adder will be at least $128t_g$. The adder hardware is simple; however, there is also the control circuitry to generate 32 shift signals. The registers storing the operands must have shift capability as well.

FIGURE 4-12: A 32-Bit Serial Adder Built from a Single 1-Bit Adder



Even if you write VHDL code based on dataflow equations, as in Figure 4-10, that does not guarantee that the synthesizer will produce a carry look-ahead adder with the delay characteristics we discussed. The software might optimize the synthesis output depending on the specific hardware components available in the target technology. For instance, if you are using an FPGA with fast adder support, the software may map some of the functions into the fast adder circuitry. Depending on the number of FPGA logic blocks and interconnects used, the delays will be different from the manual calculations. The delays of a ripple-carry, carry look-ahead, and serial adder for a gate-based implementation are presented in Table 4-1 for various adder sizes. We can see that the carry look-ahead adder is very attractive for large adders.

TABLE 4-1: Comparison of Ripple-Carry and Carry Look-Ahead Adders

Adder size	Ripple-Carry Adder Delay	CLA Delay	Serial Adder Delay
4 bit	$8t_g$	$5\text{–}6t_g$	$16t_g$
16 bit	$32t_g$	$7\text{–}8t_g$	$64t_g$
32 bit	$64t_g$	$9\text{–}10t_g$	$128t_g$
64 bit	$128t_g$	$9\text{–}10t_g$	$256t_g$

4.4 Traffic Light Controller

Let us design a sequential traffic light controller for the intersection of street A and street B. Each street has traffic sensors, which detect the presence of vehicles approaching or stopped at the intersection. Sa = '1' means a vehicle is approaching on street A, and Sb = '1' means a vehicle is approaching on street B. Street A is a main street and has a green light until a car approaches on B. Then the lights change, and B has a green light. At the end of 50 seconds, the lights change back unless there is a car on street B and none on A, in which case the B cycle is extended for 10 additional seconds. If cars continue to arrive on street B and no car appears on street A, B continues to have a green light. When A is green, it remains green at least 60 seconds, and then the lights change only when a car approaches on B. Figure 4-13 shows the external connections to the controller. Three of the outputs (*Ga, Ya*, and *Ra*) drive the green, yellow, and red lights on street A. The other three (*Gb, Yb*, and *Rb*) drive the corresponding lights on street B.

FIGURE 4-13: Block Diagram of Traffic Light Controller

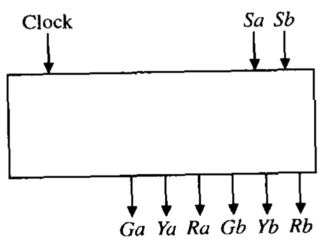
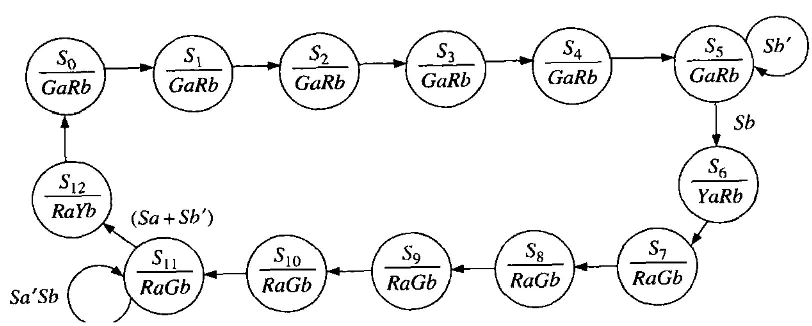


Figure 4-14 shows a Moore state graph for the controller. For timing purposes, the sequential circuit is driven by a clock with a 10-second period. Thus, a state change can occur at most once every 10 seconds. The following notation is used: *GaRb* in a state means that $Ga = Rb = 1$ and all the other output variables are 0. *Sa'Sb* on an arc implies that $Sa = 0$ and $Sb = 1$ will cause a transition along that arc. An arc without a label implies that a state transition will occur when the clock

FIGURE 4-14: State Graph for Traffic Light Controller



occurs, independent of the input variables. Thus, the green A light will stay on for six clock cycles (60 seconds) and then change to yellow if a car is waiting on B street.

The VHDL code for the traffic light controller (Figure 4-15) represents the state machine with two processes. Whenever the state, *Sa*, or *Sb* changes, the first process updates the outputs and *nextstate*. When the rising edge of the clock occurs, the second process updates the state register. The case statement illustrates use of a **when** clause with a range. Since states S_0 through S_4 have the same outputs, and the next states are in numeric sequence, we use a **when** clause with a range instead of five separate **when** clauses:

```
when 0 to 4 => Ga <= '1'; Rb <= '1'; nextstate <= state + 1;
```

FIGURE 4-15: VHDL Code for Traffic Light Controller

```
entity traffic_light is
  port(clk, Sa, Sb: in bit;
       Ra, Rb, Ga, Gb, Ya, Yb: inout bit);
end traffic_light;

architecture behave of traffic_light is
signal state, nextstate: integer range 0 to 12;
type light is (R, Y, G);
signal lightA, lightB: light;  -- define signals for waveform output
begin
  process(state, Sa, Sb)
  begin
    Ra <= '0'; Rb <= '0'; Ga <= '0'; Gb <= '0'; Ya <= '0'; Yb <= '0';
    case state is
      when 0 to 4 => Ga => '1'; Rb => '1'; nextstate => state+1;
      when 5 => Ga <= '1'; Rb <= '1';
        if Sb = '1' then nextstate <= 6; end if;
      when 6 => Ya <= '1'; Rb <= '1'; nextstate <= 7;
      when 7 to 10 => Ra <= '1'; Gb <= '1'; nextstate <= state+1;
      when 11 => Ra <= '1'; Gb <= '1';
        if (Sa='1' or Sb='0') then nextstate <= 12; end if;
      when 12 => Ra <= '1'; Yb <= '1'; nextstate <= 0;
    end case;
  end process;
  process(clk)
  begin
    if clk'event and clk = '1' then
      state <= nextstate;
    end if;
  end process;
  lightA <= R when Ra='1' else Y when Ya='1' else G when Ga='1';
  lightB <= R when Rb='1' else Y when Yb='1' else G when Gb='1';
end behave;
```

For each state, only the signals that are '1' are listed within the case statement. Since in VHDL a signal will hold its value until it is changed, we should turn off each signal when the next state is reached. In state 6 we should set *Ga* to '0', in state 7 we should set *Ya* to '0', and so on. This could be accomplished by inserting appropriate statements in the when clauses. For example, we could insert `Ga <= '0' in the when 6 =>` clause. An easier way to turn off the outputs is to set them all to '0' before the case statement, as shown in Figure 4-15. At first, it seems that a glitch might occur in the output when we set a signal to '0' that should remain '1'. However, this is not a problem because the sequential statements within a process execute instantaneously. For example, suppose that at time = 20 ns a state change from S_2 to S_3 occurs. *Ga* and *Rb* are '1', but as soon as the process starts executing, the first line of code is executed and *Ga* and *Rb* are scheduled to change to '0' at time 20 + Δ. The case statement then executes, and *Ga* and *Rb* are scheduled to change to '1' at time 20 + Δ. Since this is the same time as before, the new value ('1') preempts the previously scheduled value ('0'), and the signals never change to '0'.

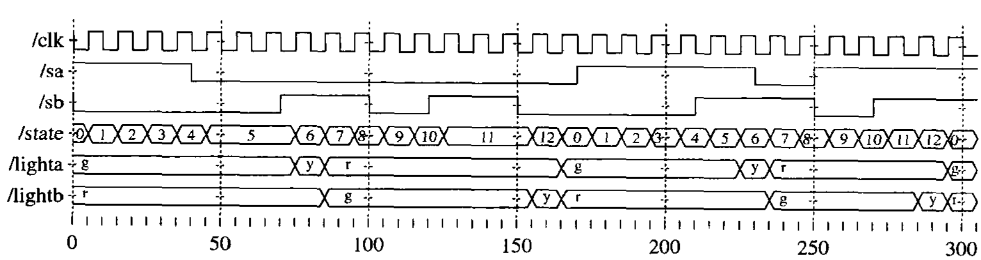
Before completing the design of the traffic controller, we will test the VHDL code to see that it meets specifications. As a minimum, our test sequence should cause all of the arcs on the state graph to be traversed at least once. We may want to perform additional tests to check the timing for various traffic conditions, such as heavy traffic on both A and B, light traffic on both, heavy traffic on A only, heavy traffic on B only, and special cases such as a car failing to move when the light is green, a car going through the intersection when the light is red, and so on.

To make it easier to interpret the simulator output, we define a type named light with the values *R*, *Y*, and *G* and two signals, *lightA* and *lightB*, which can assume these values. Then we add code to set *lightA* to *R* when the light is red, to *Y* when the light is yellow, and to *G* when the light is green. The following simulator command file first tests the case where both self-loops on the graph are traversed and then the case where neither self-loop is traversed:

```
add wave clk SA SB state lightA lightB
force clk 0 0, 1 5 sec -repeat 10 sec
force SA 1 0, 0 40, 1 170, 0 230, 1 250 sec
force SB 0 0, 1 70, 0 100, 1 120, 0 150, 1 210, 0 250, 1 270 sec
```

The test results in Figure 4-16 verify that the traffic lights change at the specified times.

FIGURE 4-16: Test Results for Traffic Light Controller



4.5 State Graphs for Control Circuits

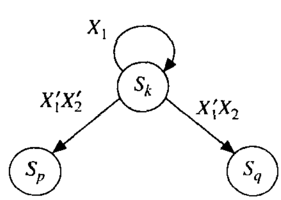
Before continuing with additional examples, we describe the notation we use on control state graphs, and then state the conditions that must be satisfied to have a proper state graph. We usually label control state graphs using variable names instead of 0's and 1's. This makes the graph easier to read, especially when the number of inputs and outputs is large. If we label an arc on a Mealy state graph X_iX_j/Z_pZ_q, this means if inputs X_i and X_j are 1 (we don't care what the other input values are), the outputs Z_p and Z_q are 1 (and the other outputs are 0), and we will traverse this arc to go to the next state. For example, for a circuit with four inputs (X_1, X_2, X_3, X_4) and four outputs (Z_1, Z_2, Z_3, Z_4), the label X_1X_4'/Z_2Z_3 is equivalent to 1--0/0110. In general, if we label an arc with an input expression, I, we will traverse the arc when $I = 1$. For example, if the input label is $AB + C'$, we will traverse the arc when $AB + C' = 1$.

In order to have a completely specified proper state graph in which the next state is always uniquely defined for every input combination, we must place the following constraints on the input labels for every state S_k:

1. If I_i and I_j are any pair of input labels on arcs exiting state S_k, then $I_iI_j = 0$ if $i \neq j$.
2. If n arcs exit state S_k and the n arcs have input labels $I_1, I_2, \ldots, I_n$, respectively, then $I_1 + I_2 + \cdots + I_n = 1$.

Condition 1 assures us that at most one input label can be 1 at any given time, and condition 2 assures us that at least one input label will be 1 at any given time. Therefore, exactly one label will be 1, and the next state will be uniquely defined for every input combination. For example, consider the partial state graph in Figure 4-17, where $I_1 = X_1, I_2 = X_1'X_2'$, and $I_3 = X_1'X_2$:

FIGURE 4-17: Example Partial State Graph



$$(X_1)(X_1'X_2') = 0$$
$$(X_1)(X_1'X_2) = 0$$
$$(X_1'X_2')(X_1'X_2) = 0$$
$$X_1 + X_1'X_2' + X_1'X_2 = 1$$

Conditions 1 and 2 are satisfied for S_k.

An incompletely specified proper state graph must always satisfy condition 2, and it must satisfy condition 1 for all combinations of values of input variables that can occur for each state S_k. Thus, the partial state graph in Figure 4-18 represents part of a proper state graph only if input combination $X_1 = X_2 = 1$ cannot occur in state S_k.

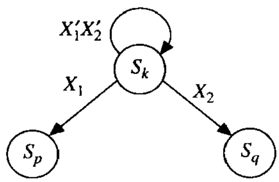


FIGURE 4-18: Example Partial State Graph

If there are three input variables (X_1, X_2, X_3), the preceding partial state graph represents the following state table row:

	000	001	010	011	100	101	110	111
S_k	S_k	S_k	S_q	S_q	S_p	S_p	—	—

4.6 Scoreboard and Controller

In this example, we will design a simple scoreboard, which can display scores from 0 to 99 (decimal). The input to the system should consist of a reset signal and control signals to increment or decrement the score. The two-digit decimal count gets incremented by 1 if increment signal is true and is decremented by 1 if decrement signal is true. If increment and decrement are true simultaneously, no action happens.

The current count is displayed on seven-segment displays. In order to prevent accidental erasure, the reset button must be pressed for five consecutive cycles in order to erase the scoreboard. The scoreboard should allow down counts to correct a mistake (in case of accidentally incrementing more than required).

4.6.1 Data Path

At the core of the design will be a two-digit BCD counter to perform the counting. Two seven-segment displays will be needed to display the current score. We will also require BCD to seven-segment decoders to facilitate the display of each BCD digit. Figure 4-19 illustrates a block diagram of the system. Since true reset should happen only after pressing reset for five clock cycles, we will also use a 3-bit reset counter called *rstcnt*.

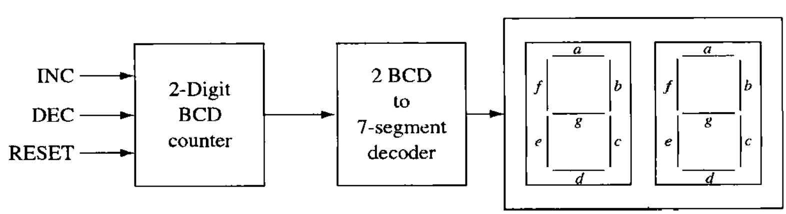
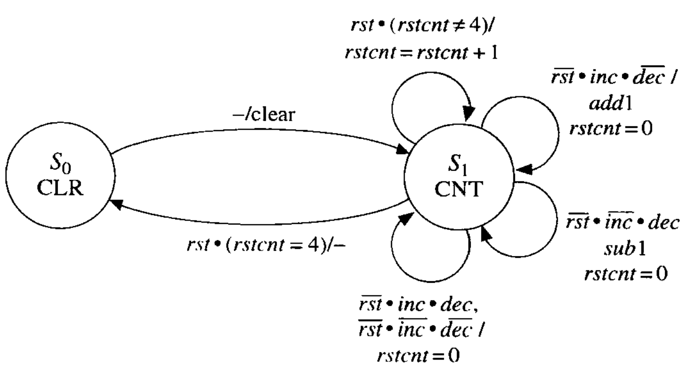


FIGURE 4-19: Overview of Simple Scoreboard

4.6.2 Controller

The controller for this circuit works as follows. There are two states in this finite state machine (FSM), as indicated in Figure 4-20. In the initial state (S_0), the BCD counter is cleared. The reset counter is also made equal to 0. Essentially, S_0 is an initialization state where all the counters are cleared. After the initial start state, the FSM moves to the next state (S_1), which is where counting gets done. In this state, in every clock cycle, incrementing or decrementing is done according to the input signals. If reset signal *rst* arrives, the *rstcnt* is incremented. If reset count has already reached 4, and reset command is still persisting in the fifth clock cycle, a transition to state S_0 is made. If the *inc* signal is present and *dec* is not present, the BCD counter is incremented. The notation *add1* on the arc on the top right is used to indicate that the BCD counter is incremented. If the *dec* signal is present and *inc* is not present, the BCD counter is decremented. The notation *sub1* on the arc on the bottom right is used to indicate that the BCD counter is decremented. In any cycle that the reset signal is not present, the *rstcnt* is cleared. If both the *inc* and *dec* signals are true, or neither are true, the reset counter (*rstcnt*) is cleared and the BCD counter is left unchanged.

FIGURE 4-20: State Graph for Scoreboard



4.6.3 VHDL Model

The VHDL code for the scoreboard is given in Figure 4-21. The two seven-segment displays, *seg7disp1* and *seg7disp2*, are declared as unsigned 7-bit vectors. The segments of the seven-segment display are labeled *a* through *g*, as in Figure 4-19. The unsigned type is used so that the overloaded '+' operator can be used for incrementing the counter by 1. The decoder for the seven-segment display can be implemented as an array or look-up table. The look-up table consists of ten 7-bit vectors. A new datatype called *sevsegarray* is defined for the array of the seven-segment values corresponding to each BCD digit. It is a two-dimensional array with 10 elements, each of which is a 7-bit unsigned vector. The look-up table must be addressed with an integer data type; hence, the conversion function `to_integer` is used to generate the array index. The expression `to_integer(BCD0)` converts *BCD0* to integer type and the statement

```
seg7disp0 <= seg7rom(to_integer(BCD0));
```

accesses the appropriate element from the array *seg7rom* to convert the decimal digit to the seven-segment form. BCD addition is accomplished with the overloaded '+' operator. If the current count is less than 9, it is incremented. If it is 9, adding 1 results in a 0, but the next digit should be incremented. Similarly, decrementing from 0 is performed by borrowing a 1 from the next higher digit.

FIGURE 4-21: VHDL Code for Scoreboard

```
library IEEE;
use IEEE.numeric_bit.all; -- any package with overloaded add and subtract

entity Scoreboard is
  port(clk, rst, inc, dec: in bit;
       seg7disp1, seg7disp0: out unsigned(6 downto 0));
end Scoreboard;

architecture Behavioral of Scoreboard is
signal State: integer range 0 to 1;
signal BCD1, BCD0: unsigned(3 downto 0) := "0000"; -- unsigned bit vector
signal rstcnt: integer range 0 to 4 := 0;
type sevsegarray is array (0 to 9) of unsigned(6 downto 0);
constant seg7Rom: sevsegarray :=
   ("0111111", "0000110", "1011011", "1001111", "1100110", "1101101", "1111100",
    "0000111", "1111111", "1100111"); -- active high with "gfedcba" order
begin
  process(clk)
  begin
    if clk'event and clk = '1' then
      case State is
        when 0 => -- initial state
          BCD1 <= "0000"; BCD0 <= "0000"; -- clear counter
          rstcnt <= 0; -- reset RESETCOUNT
          State <= 1;
        when 1 => -- state in which the scoreboard waits for inc and dec
          if rst = '1' then
            if rstcnt = 4 then -- checking whether 5th reset cycle
              State <= 0;
            else rstcnt <= rstcnt + 1;
            end if;
          elsif inc = '1' and dec = '0' then
            rstcnt <= 0;
            if BCD0 < "1001" then
              BCD0 <= BCD0 + 1; -- library with overloaded "+" required
            elsif BCD1 < "1001" then
              BCD1 <= BCD1 + 1;
              BCD0 <= "0000";
            end if;
```

```
        elsif dec = '1' and inc = '0' then
          rstcnt <= 0;
          if BCD0 > "0000" then
            BCD0 <= BCD0 - 1; -- library with overloaded "-" required
          elsif BCD1 > "0000" then
            BCD1 <= BCD1 - 1;
            BCD0 <= "1001";
          end if;
        elsif (inc = '1' and dec = '1') or (inc = '0' and dec = '0') then
          rstcnt <= 0;
        end if;
      end case;
    end if;
  end process;
  seg7disp0 <= seg7rom(to_integer(BCD0)); -- type conversion function from
  seg7disp1 <= seg7rom(to_integer(BCD1)); -- IEEE numeric_bit package used
end Behavioral;
```

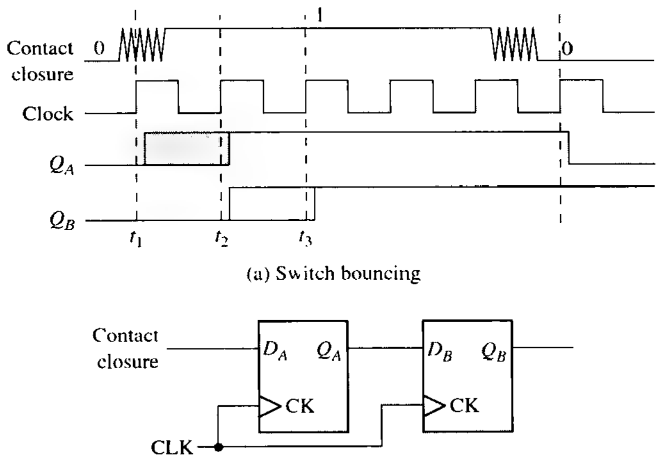
4.7 Synchronization and Debouncing

The *inc, dec,* and *rst* signals to the scoreboard in the previous design are external inputs. An issue in systems involving external inputs is synchronization. Outputs from a keypad or push-button switches are not synchronous to the system clock signal. Since they will be used as inputs to a synchronous sequential circuit, they should be synchronized.

Another issue in systems involving external inputs is switch bounce. When a mechanical switch is closed or opened, the switch contact will bounce, causing noise in the switch output, as shown in Figure 4-22(a). The contact may bounce for several milliseconds before it settles down to its final position. After a switch closure has been detected, we must wait for the bounce to settle before reading the key. In any circuit involving mechanical switches, we should debounce the switches. Debouncing means removing the transients in the switch output.

Flip-flops are very useful devices when contacts need to be synchronized and debounced. Figure 4-22(b) shows a proposed debouncing and synchronizing circuit. In this design, the clock period is greater than the bounce time. If the rising edge of the clock occurs during the bounce, either a 0 or 1 will be clocked into the flip-flop at t_1. If a 0 was clocked in, a 1 will be clocked in at the next active clock edge (t_2). So it appears that Q_A will be a debounced and synchronized version of K. However, a possibility of failure exists if *the switch* changes very close to the clock edge such that the setup or hold time is violated. In this case the flip-flop output Q_A may oscillate or otherwise malfunction. Although this situation will occur very infrequently, it is best to guard against it by adding a second flip-flop. We will choose the clock period so that any oscillation at the output of Q_A will have died out before the next active edge of the clock so that the input D_B will always be stable at the active clock edge. The debounced signal, Q_B, will always be clean and synchronized with the clock, although it may be delayed up to two clock cycles after the switch is pressed.

FIGURE 4-22: Debouncing Mechanical Switches



(a) Switch bouncing

(b) Debouncing and synchronizing circuit

4.7.1 Single Pulser

One assumption in the scoreboard design is that each time the *inc* and *dec* signals are provided, they last only for one clock cycle. Digital systems generally run at speeds higher than actions by humans, and it is very difficult for humans to produce a signal that only lasts for a clock pulse. If the pressing of the button lasted longer than a clock cycle, the counters will continue to get incremented in the aforementioned design. A solution to the problem is to develop a circuit that generates a single pulse for a human action of pressing a button or switch. Such a circuit can be used in a variety of applications involving humans, push buttons, and switches.

Now, let us design a single pulser circuit that delivers a synchronized pulse that is a single clock cycle long, when a button is pressed. The circuit must sense the pressing of a button and assert an output signal for one clock cycle. Then the output stays inactive until the button is released.

Let us create a state diagram for the single pulser. The single pulser circuit must have two states: one in which it will detect the pressing of the key and one in which it will detect the release of the key. Let us call the first state S_0 and the second state S_1. Let us use the symbol SYNCPRESS to denote the synchronized key press. When the circuit is in state S_0 and the button is pressed, the system produces the single pulse and moves to state S_1. The single pulse is a Mealy output as the state changes from S_0 to S_1. Once the system is in state S_1, it waits for the button to be released. As soon as it is released, it moves to the start state S_0 waiting for the next button press. The single pulse output is true only during the transition from S_0 to S_1. The state diagram is illustrated in Figure 4-23.

Since there are only two states for this circuit, it can be implemented using one flip-flop. A single pulser can be implemented as in Figure 4-24. The first block consists of the circuitry in Figure 4-22(b) and generates a synchronized button press, SYNCPRESS. The flip-flop implements the two states of the state machine. Let us assume the state assignments are $S_0 = 0$ and $S_1 = 1$. In such a case, the Q output of

FIGURE 4-23: State Diagram of Single Pulser

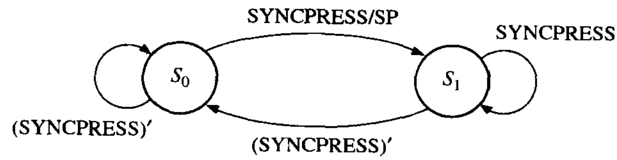
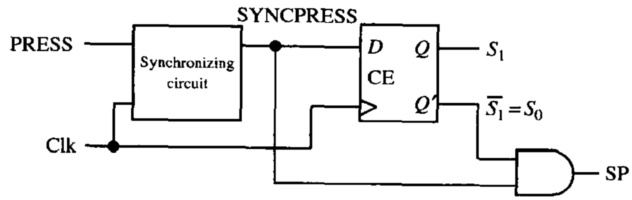


FIGURE 4-24: Single Pulser and Synchronizer Circuit



the flip-flop is synonymous with S_1, and the Q' output of the flip-flop is synonymous with S_0. The equation for the single pulse SP is

$$SP = S0 \cdot SYNCPRESS$$

It may also be noted that $S_0 = S_1'$. Including the two flip-flops inside the synchronizing block, three flip-flops can provide debouncing, synchronization, and single pulsing. If button pushes can be passed through such a circuit, a single pulse that is debounced and synchronized, with respect to the system clock, can be obtained. It is a good practice to feed external push-button signals through such a circuit in order to obtain controlled and predictable operation.

4.8 Add-and-Shift Multiplier

In this section, we will design a multiplier for unsigned binary numbers. When we form the product $A \times B$, the first operand (A) is called the *multiplicand*, and the second operand (B) is called the *multiplier*. As illustrated here, binary multiplication requires only shifting and adding. In the following example, we multiply 13_{10} by 11_{10} in binary:

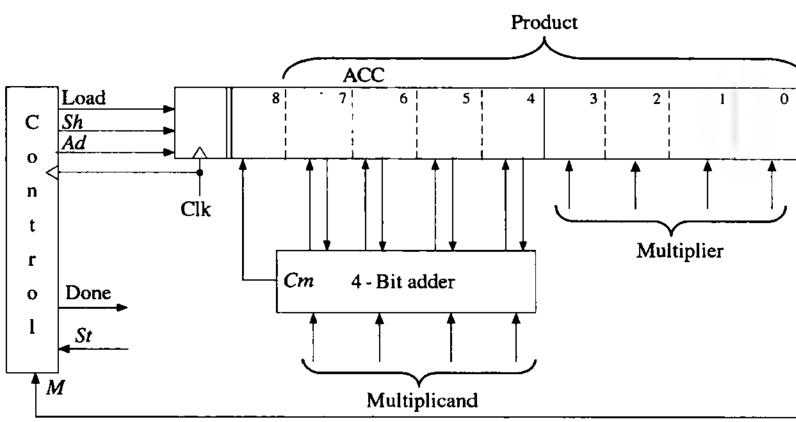
```
Multiplicand ---------> 1 1 0 1  (13)
Mutliplier -----------> 1 0 1 1  (11)
                        -------
                        1 1 0 1
                      1 1 0 1
Partial             -----------
products            1 0 0 1 1 1
                    0 0 0 0
                    -----------
                    1 0 0 1 1 1
                  1 1 0 1
                ---------------
                1 0 0 0 1 1 1 1  (143)
```

Note that each partial product is either the multiplicand (1101) shifted over by the appropriate number of places or zero. Instead of forming all the partial

products first and then adding, each new partial product is added in as soon as it is formed, which eliminates the need for adding more than two binary numbers at a time.

Multiplication of two 4-bit numbers requires a 4-bit multiplicand register, a 4-bit multiplier register, a 4-bit full adder, and an 8-bit register for the product. The product register serves as an accumulator to accumulate the sum of the partial products. If the multiplicand were shifted left each time before it was added to the accumulator, as was done in the previous example, an 8-bit adder would be needed. So it is better to shift the contents of the product register to the right each time, as shown in the block diagram of Figure 4-25. This type of multiplier is sometimes referred to as a serial-parallel multiplier, since the multiplier bits are processed serially, but the addition takes place in parallel. As indicated by the arrows on the diagram, 4 bits from the accumulator (ACC) and 4 bits from the multiplicand register are connected to the adder inputs; the 4 sum bits and the carry output from the adder are connected back to the accumulator. When an add signal (*Ad*) occurs, the adder outputs are transferred to the accumulator by the next clock pulse, thus causing the multiplicand to be added to the accumulator. An extra bit at the left end of the product register temporarily stores any carry that is generated when the multiplicand is added to the accumulator. When a shift signal (*Sh*) occurs, all 9 bits of ACC are shifted right by the next clock pulse.

FIGURE 4-25: Block Diagram for Binary Multiplier



Since the lower 4 bits of the product register are initially unused, we will store the multiplier in this location instead of in a separate register. As each multiplier bit is used, it is shifted out the right end of the register to make room for additional product bits. A shift signal (*Sh*) causes the contents of the product register (including the multiplier) to be shifted right one place when the next clock pulse occurs. The control circuit puts out the proper sequence of add and shift signals after a start signal ($St = 1$) has been received. If the current multiplier bit (M) is 1, the multiplicand is added to the accumulator followed by a right shift; if the multiplier bit is 0, the addition is skipped, and only the right shift occurs. The multiplication example

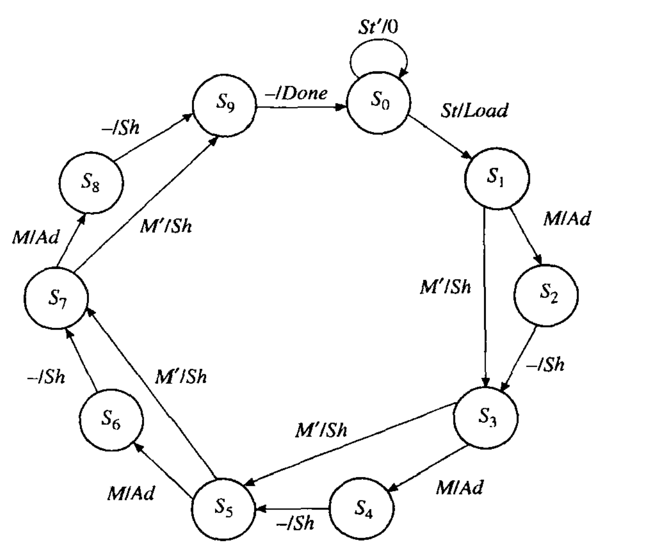
(13 × 11) is reworked below, showing the location of the bits in the registers at each clock time.

initial contents of product register	0 0 0 0 0 | 1 0 1 1 ← M	(11)
(add multiplicand since $M = 1$)	1 1 0 1	(13)
after addition	0 1 1 0 1 | 1 0 1 1	
after shift	0 0 1 1 0 1 | 1 0 1 ← M	
(add multiplicand since $M = 1$)	1 1 0 1	
after addition	1 0 0 1 1 1 | 1 0 1	
after shift	0 1 0 0 1 1 1 | 1 0 ← M	
(skip addition since $M = 0$)		
after shift	0 0 1 0 0 1 1 1 | 1 ← M	
(add multiplicand since $M = 1$)	1 1 0 1	
after addition	1 0 0 0 1 1 1 1 | 1	
after shift (final answer)	0 1 0 0 0 1 1 1 1 |	(143)

dividing line between product and multiplier

The control circuit must be designed to output the proper sequence of add and shift signals. Figure 4-26 shows a state graph for the control circuit. In Figure 4-26, S_0 is the reset state, and the circuit stays in S_0 until a start signal ($St = 1$) is received. This generates a *Load* signal, which causes the multiplier to be loaded into the lower 4 bits of the accumulator (ACC) and the upper 5 bits of the accumulator to be cleared. In state S_1, the low-order bit of the multiplier (M) is tested. If $M = 1$, an add signal is generated, and if $M = 0$, a shift signal is generated. Similarly, in states S_3, S_5, and S_7, the current multiplier bit (M) is tested to determine whether to generate an add or shift signal. A shift signal is always generated at the next clock time following an add signal (states S_2, S_4, S_6, and S_8). After four shifts have been generated, the control network goes to S_9, and a done signal is generated before returning to S_0.

FIGURE 4-26: State Graph for Binary Multiplier Control



The behavioral VHDL model (Figure 4-27) corresponds directly to the state graph. Since there are 10 states, we have declared an integer ranging from 0 to 9 for the state signal. The signal *ACC* represents the 9-bit accumulator output. The statement

```
alias M: bit is ACC(0);
```

allows us to use the name *M* in place of *ACC(0)*. The notation **when** 1|3|5|7 => means that when the state is 1 or 3 or 5 or 7, the action that follows occurs. All register operations and state changes take place on the rising edge of the clock. For example, in state 0, if *St* is '1', the multiplier is loaded into the accumulator at the same time the state changes to 1. The expression `'0' & ACC(7 downto 4) + Mcand` is used to compute the sum of two 4-bit unsigned vectors to give a 5-bit result. This represents the adder output, which is loaded into *ACC* at the same time the state counter is incremented. The right shift on *ACC* is accomplished by loading *ACC* with '0' concatenated with the upper 8 bits of *ACC*. The expression `'0' & ACC(8 downto 1)` could be replaced with `ACC srl 1`.

FIGURE 4-27: Behavioral Model for 4 × 4 Binary Multiplier

```
-- This is a behavioral model of a multiplier for unsigned
-- binary numbers. It multiplies a 4-bit multiplicand
-- by a 4-bit multiplier to give an 8-bit product.

-- The maximum number of clock cycles needed for a
-- multiply is 10.

library IEEE;
use IEEE.numeric_bit.all;

entity mult4X4 is
  port(Clk, St: in bit;
       Mplier, Mcand: in unsigned(3 downto 0);
       Done: out bit);
end mult4X4;

architecture behave1 of mult4X4 is
signal State: integer range 0 to 9;
signal ACC: unsigned(8 downto 0); -- accumulator
alias M: bit is ACC(0);      -- M is bit 0 of ACC
begin
  process(Clk)
  begin
    if Clk'event and Clk = '1' then  -- executes on rising edge of clock
      case State is
        when 0 =>              -- initial State
          if St = '1' then
            ACC(8 downto 4) <= "00000"; -- begin cycle
            ACC(3 downto 0) <= Mplier;  -- load the multiplier
            State <= 1;
          end if;
```

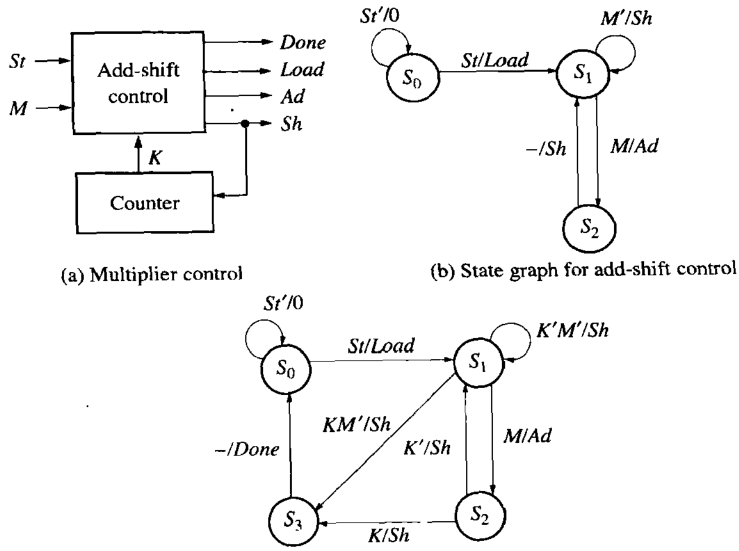
```
      when 1 | 3 | 5 | 7 =>       -- "add/shift" State
        if M = '1' then           -- add multiplicand
          ACC(8 downto 4) <= '0' & ACC(7 downto 4) + Mcand;
          State <= State + 1;
        else
          ACC <= '0' & ACC(8 downto 1);      -- shift accumulator right
          State <= State + 2;
        end if;
      when 2 | 4 | 6 | 8 =>                  -- "shift" State
        ACC <= '0' & ACC(8 downto 1);        -- right shift
        State <= State + 1;
      when 9 =>                              -- end of cycle
        State <= 0;
    end case;
  end if;
 end process;
 Done <= '1' when State = 9 else '0';
end behave1;
```

The *Done* signal needs to be turned on only in state 9. If we had used the statement `when 9 => State <= 0; Done <= '1'`, *Done* would be turned on at the same time *State* changes to 0. This is too late, since we want *Done* to turn on when *State* becomes 9. Therefore, we used a separate concurrent assignment statement. This statement is placed outside the process so that *Done* will be updated whenever *State* changes.

As the state graph for the multiplier (Figure 4-26) indicates, the control performs two functions—generating add or shift signals as needed and counting the number of shifts. If the number of bits is large, it is convenient to divide the control circuit into a counter and an add-shift control, as shown in Figure 4-28(a). First, we will derive a state graph for the add-shift control that tests St and M and outputs the proper sequence of add and shift signals (Figure 4-28(b)). Then we will add a completion signal (K) from the counter that stops the multiplier after the proper number of shifts have been completed. Starting in S_0 in Figure 4-28(b), when a start signal $St = 1$ is received, a load signal is generated and the circuit goes to state S_1. Then if $M = 1$, an add signal is generated and the circuit goes to state S_2; if $M = 0$, a shift signal is generated and the circuit stays in S_1. In S_2, a shift signal is generated since a shift always follows an add. The graph of Figure 4-28(b) will generate the proper sequence of add and shift signals, but it has no provision for stopping the multiplier.

In order to determine when the multiplication is completed, the counter is incremented each time a shift signal is generated. If the multiplier is n bits, n shifts are required. We will design the counter so that a completion signal (K) is generated after $n - 1$ shifts have occurred. When $K = 1$, the circuit should perform one more addition if necessary and then do the final shift. The control operation in Figure 4-28(c) is the same as Figure 4-28(b) as long as $K = 0$. In state S_1, if $K = 1$, we test M as usual. If $M = 0$, we output the final shift signal and go to the done state (S_3); however, if $M = 1$, we add before shifting and go to state S_2. In state S_2, if $K = 1$, we output one more shift signal and then go to S_3. The last shift signal

FIGURE 4-28: Multiplier Control with Counter



(a) Multiplier control

(b) State graph for add-shift control

(c) Final state graph for add-shift control

will increment the counter to 0 at the same time the add-shift control goes to the done state.

As an example, consider the multiplier of Figure 4-25, but replace the control circuit with Figure 4-28(a). Since $n = 4$, a 2-bit counter is needed to count the four shifts, and $K = 1$ when the counter is in state 3 (11_2). Table 4-2 shows the operation of the multiplier when 1101 is multiplied by 1011. S_0, S_1, S_2, and S_3 represent states of the control circuit (Figure 4-28(c)). The contents of the product register at each step are the same as given on page 199.

At time t_0, the control is reset and waiting for a start signal. At time t_1, the start signal St is 1, and a *Load* signal is generated. At time t_2, $M = 1$, so an *Ad* signal is generated. When the next clock occurs, the output of the adder is loaded into the accumulator and the control goes to S_2. At t_3, an *Sh* signal is generated, so at the next clock shifting occurs and the counter is incremented. At t_4, $M = 1$, so $Ad = 1$, and the adder output is loaded into the accumulator at the next clock. At t_5 and t_6, shifting

TABLE 4-2: Operation of Multiplier Using a Counter

Time	State	Counter	Product Register	St	M	K	Load	Ad	Sh	Done
t_0	S_0	00	000000000	0	0	0	0	0	0	0
t_1	S_0	00	000000000	1	0	0	1	0	0	0
t_2	S_1	00	000001011	0	1	0	0	1	0	0
t_3	S_2	00	011011011	0	1	0	0	0	1	0
t_4	S_1	01	001101101	0	1	0	0	1	0	0
t_5	S_2	01	100111101	0	1	0	0	0	1	0
t_6	S_1	10	010011110	0	0	0	0	0	1	0
t_7	S_1	11	001001111	0	1	1	0	1	0	0
t_8	S_2	11	100011111	0	1	1	0	0	1	0
t_9	S_3	00	010001111	0	1	0	0	0	0	1

and counting occur. At t_7, three shifts have occurred and the counter state is 11, so $K = 1$. Since $M = 1$, addition occurs and control goes to S_2. At t_8, $Sh = K = 1$, so at the next clock the final shift occurs and the counter is incremented back to state 00. At t_9, a *Done* signal is generated.

The multiplier design given here can easily be expanded to 8, 16, or more bits simply by increasing the register size and the number of bits in the counter. The add-shift control would remain unchanged.

4.9 Array Multiplier

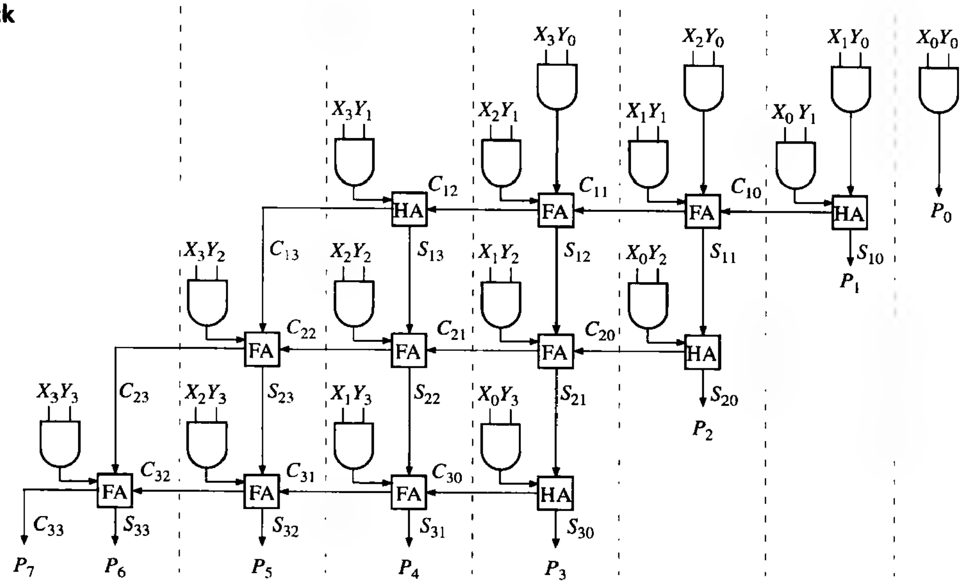
An array multiplier is a parallel multiplier that generates the partial products in a parallel fashion. The various partial products are added as soon as they are available. Consider the process of multiplication as illustrated in Table 4-3. Two 4-bit unsigned numbers, $X_3X_2X_1X_0$ and $Y_3Y_2Y_1Y_0$, are multiplied to generate a product that is possibly 8 bits. Each of the X_iY_j product bits can be generated by an AND gate. Each partial product can be added to the previous sum of partial products using a row of adders. The sum output of the first row of adders, which adds the first two partial products, is $S_{13}S_{12}S_{11}S_{10}$, and the carry output is $C_{13}C_{12}C_{11}C_{10}$. Similar results occur for the other two rows of adders. (We have used the notation S_{ij} and C_{ij} to represent the sums and carries from the ith row of adders.)

TABLE 4-3: Four-bit Multiplier Partial Products

				X_3	X_2	X_1	X_0	Multiplicand
				Y_3	Y_2	Y_1	Y_0	Multiplier
				X_3Y_0	X_2Y_0	X_1Y_0	X_0Y_0	Partial product 0
			X_3Y_1	X_2Y_1	X_1Y_1	X_0Y_1		Partial product 1
			C_{12}	C_{11}	C_{10}			First row carries
		C_{13}	S_{13}	S_{12}	S_{11}	S_{10}		First row sums
		X_3Y_2	X_2Y_2	X_1Y_2	X_0Y_2			Partial product 2
		C_{22}	C_{21}	C_{20}				Second row carries
	C_{23}	S_{23}	S_{22}	S_{21}	S_{20}			Second row sums
	X_3Y_3	X_2Y_3	X_1Y_3	X_0Y_3				Partial product 3
	C_{32}	C_{31}	C_{30}					Third row carries
C_{33}	S_{33}	S_{32}	S_{31}	S_{30}				Third row sums
P_7	P_6	P_5	P_4	P_3	P_2	P_1	P_0	Final product

Figure 4-29 shows the array of AND gates and adders to perform this multiplication. If an adder has three inputs, a full adder (FA) is used, but if an adder has only two inputs, a half-adder (HA) is used. A half-adder is the same as a full adder with one of the inputs set to 0. This multiplier requires 16 AND gates, 8 full adders, and 4 half-adders. After the X and Y inputs have been applied, the carry must propagate along each row of cells, and the sum must propagate from row to row. The time required to complete the multiplication depends primarily on the propagation delay in the adders. The longest path from input to output goes through 8 adders. If t_{ad} is

FIGURE 4-29: Block Diagram of 4 × 4 Array Multiplier



the worst-case (longest possible) delay through an adder, and t_g is the longest AND gate delay, then the worst-case time to complete the multiplication is $8t_{ad} + t_g$.

In general, an n-bit-by-n-bit array multiplier would require n^2 AND gates, $n(n-2)$ full adders, and n half-adders. So the number of components required increases quadratically. For the serial-parallel multiplier previously designed, the amount of hardware required in addition to the control circuit increases linearly with n.

For an $n \times n$ array multiplier, the longest path from input to output goes through n adders in the top row, $n - 1$ adders in the bottom row, and $n - 3$ adders in the middle rows. The corresponding worst-case multiply time is $(3n - 4)t_{ad} + t_g$. The longest delay in a circuit is called critical path. The worst-case delay can be improved to $2nt_{ad} + t_g$ by forwarding carry from each adder to the diagonally lower adder rather than the adder on the left side. When $n = 4$, both expressions are the same; however, for larger values of n, it is beneficial to pass carry diagonally as opposed to rippling it to the left. Note that this multiplier has no sequential logic or registers.

The shift-and-add multiplier that we previously designed requires $2n$ clocks to complete the multiply in the worst case, although this can be reduced to n clocks using a technique discussed in the next section. The minimum clock period depends on the propagation delay through the n-bit adder as well as the propagation delay and setup time for the accumulator flip-flops.

4.9.1 VHDL Coding

If the topology has to be exactly what the designer wants, we need to do structural coding as shown in Figure 4-30. If we made a behavioral model of a multiplier without specifying the topology, the topology generated by the synthesizer would depend on the synthesis tool. Here, we present a structural model for an array

FIGURE 4-30: VHDL Code for 4 × 4 Array Multiplier

```
entity Array_Mult is
  port(X, Y: in bit_vector(3 downto 0);
       P: out bit_vector(7 downto 0));
end Array_Mult;

architecture Behavioral of Array_Mult is
signal C1, C2, C3: bit_vector(3 downto 0);
signal S1, S2, S3: bit_vector(3 downto 0);
signal XY0, XY1, XY2, XY3: bit_vector(3 downto 0);
component FullAdder
  port(X, Y, Cin: in bit;
       Cout, Sum: out bit);
end component;
component HalfAdder
  port(X, Y: in bit;
       Cout, Sum: out bit);
end component;
begin
  XY0(0) <= X(0) and Y(0); XY1(0) <= X(0) and Y(1);
  XY0(1) <= X(1) and Y(0); XY1(1) <= X(1) and Y(1);
  XY0(2) <= X(2) and Y(0); XY1(2) <= X(2) and Y(1);
  XY0(3) <= X(3) and Y(0); XY1(3) <= X(3) and Y(1);

  XY2(0) <= X(0) and Y(2); XY3(0) <= X(0) and Y(3);
  XY2(1) <= X(1) and Y(2); XY3(1) <= X(1) and Y(3);
  XY2(2) <= X(2) and Y(2); XY3(2) <= X(2) and Y(3);
  XY2(3) <= X(3) and Y(2); XY3(3) <= X(3) and Y(3);

  FA1: FullAdder port map (XY0(2), XY1(1), C1(0), C1(1), S1(1));
  FA2: FullAdder port map (XY0(3), XY1(2), C1(1), C1(2), S1(2));
  FA3: FullAdder port map (S1(2), XY2(1), C2(0), C2(1), S2(1));
  FA4: FullAdder port map (S1(3), XY2(2), C2(1), C2(2), S2(2));
  FA5: FullAdder port map (C1(3), XY2(3), C2(2), C2(3), S2(3));
  FA6: FullAdder port map (S2(2), XY3(1), C3(0), C3(1), S3(1));
  FA7: FullAdder port map (S2(3), XY3(2), C3(1), C3(2), S3(2));
  FA8: FullAdder port map (C2(3), XY3(3), C3(2), C3(3), S3(3));
  HA1: HalfAdder port map (XY0(1), XY1(0), C1(0), S1(0));
  HA2: HalfAdder port map (XY1(3), C1(2), C1(3), S1(3));
  HA3: HalfAdder port map (S1(1), XY2(0), C2(0), S2(0));
  HA4: HalfAdder port map (S2(1), XY3(0), C3(0), S3(0));

  P(0) <= XY0(0); P(1) <= S1(0); P(2) <= S2(0);
  P(3) <= S3(0); P(4) <= S3(1); P(5) <= S3(2);
  P(6) <= S3(3); P(7) <= C3(3);
end Behavioral;

-- Full Adder and half adder entity and architecture descriptions
-- should be in the project
```

```
entity FullAdder is
  port(X, Y, Cin: in bit;
       Cout, Sum: out bit);
end FullAdder;

architecture equations of FullAdder is
begin
  Sum <= X xor Y xor Cin;
  Cout <= (X and Y) or (X and Cin) or (Y and Cin);
end equations;

entity HalfAdder is
  port(X, Y: in bit;
       Cout, Sum: out bit);
end HalfAdder;

architecture equations of HalfAdder is
begin
  Sum <= X xor Y;
  Cout <= X and Y;
end equations;
```

multiplier. Full-adder and half-adder modules are created and used as components for the array multiplier. The full adders and half adders are interconnected according to the array multiplier topology. Several instantiation (**port map**) statements are used for this purpose.

4.10 A Signed Integer/Fraction Multiplier

Several algorithms are available for multiplication of signed binary numbers. The following procedure is a straightforward way to carry out the multiplication:

1. Complement the multiplier if negative.
2. Complement the multiplicand if negative.
3. Multiply the two positive binary numbers.
4. Complement the product if it should be negative.

Although this method is conceptually simple, it requires more hardware and computation time than some of the other available methods.

The next method we describe requires only the ability to complement the multiplicand. Complementation of the multiplier or product is not necessary. Although the method works equally well with integers or fractions, we illustrate the method with fractions, since we will later use this multiplier as part of a multiplier for floating-point numbers. Using 2's complement for negative numbers, we will represent signed binary fractions in the following form:

0.101 $+5/8$ 1.011 $-5/8$

The digit to the left of the binary point is the sign bit, which is 0 for positive fractions and 1 for negative fractions. In general, the 2's complement of a binary fraction F is $F^* = 2 - F$. Thus, $-5/8$ is represented by $10.000 - 0.101 = 1.011$. (This method of defining 2's complement fractions is consistent with the integer case ($N^* = 2^n - N$), since moving the binary point $n - 1$ places to the left is equivalent to dividing by 2^{n-1}.) The 2's complement of a fraction can be found by starting at the right end and complementing all the digits to the left of the first 1, the same as for the integer case. The 2's complement fraction 1.000 . . . is a special case. It actually represents the number -1, since the sign bit is negative and the 2's complement of 1.000 . . . is $2 - 1 = 1$. We cannot represent $+1$ in this 2's complement fraction system, since 0.111 . . . is the largest positive fraction.

Binary Fixed-Point Fractions

Fixed-point numbers are number formats in which the decimal or binary point is at a fixed location. We can have a fixed-point 8-bit number format where the binary point is assumed to be after 4 bits (i.e., 4 bits for the fractional part and 4 bits for the integer part). If the binary point is assumed to be located two more bits to the right, there will be 6 bits for the integral part and 2 bits for the fraction. The range and precision of the numbers that can be represented in the different formats depend on the location of the binary point. For instance, if there are 4 bits for the fractional part and 4 bits for the integer, the range, assuming unsigned numbers, is 0.00 to 15.925. If only 2 bits are allowed for the fractional part and 6 bits for the integer, the range increases; however, the precision reduces. Now, the range would be 0.00 to 63.75, but the fractional part can be specified only as a multiple of 0.25.

Let us say we need to represent -13.45 in a 2's complement fixed-point number representation with four fractional bits. To convert any decimal fraction into the binary fraction, one technique is to repeatedly multiply the fractional part (only the fractional part in each intermediate step) with 2. So, starting with 0.45, the repeated multiplication results in

0.90
1.80
1.60
1.20
0.40
0.80
1.60
1.20

Now, the binary representation can be obtained by considering the digits in bold. An appropriate representation can be obtained depending on the number of bits available (e.g., 0111 if 4 bits are available, 01110011 if 8 bits are

available, and so on). The representation for decimal number 13.45 in the fixed-point format with four binary places will be as follows:

13.45: 1101.0111

Note that the represented number is only an approximation of the actual number. The represented number can be converted back to decimal and seen to be 13.4375 (slightly off from the number we started with). The representation approaches the actual number as more and more binary places are added to the representation.

Negative fractions can be represented in 2's complement form. Let us represent −13.45 in 2's complement form. This cannot be done if we have only four places for the integer. We need to have at least 5 bits for the integer in order to handle the sign. Assuming 5 bits are available for the integer, in a 9-bit format,

13.45:	01101.0111
1's complement	10010.1000
2's complement	10010.1001

Hence −13.45 = 10010.1001 in this representation.

When multiplying signed binary numbers, we must consider four cases:

Multiplicand	Multiplier
+	+
−	+
+	−
−	−

When both the multiplicand and the multiplier are positive, standard binary multiplication is used. For example,

0.1 1 1	(+7/8)	←	Multiplicand
× 0.1 0 1	(+5/8)	←	Multiplier
(0. 0 0)0 1 1 1	(+7/64)	←	*Note*: The proper representation of the fractional partial products requires extension of the sign bit past the binary point, as indicated in parentheses. (Such extension is not necessary in the hardware.)
(0.)0 1 1 1	(+7/16)	←	
0. 1 0 0 0 1 1	(+35/64)		

When the multiplicand is negative and the multiplier is positive, the procedure is the same as in the previous case, except that we must extend the sign bit of the

multiplicand so that the partial products and final product will have the proper negative sign. For example,

1.1 0 1	(−3/8)		
× 0.1 0 1	(+5/8)		
(1. 1 1)1 1 0 1	(−3/64)	←	*Note*: The extension of the sign bit provides proper representation of the negative products.
(1.)1 1 0 1	(−3/16)	←	
1. 1 1 0 0 0 1	(−15/64)		

When the multiplier is negative and the multiplicand is positive, we must make a slight change in the multiplication procedure. A negative fraction of the form $1.g$ has a numeric value $-1 + 0.g$; for example, $1.011 = -1 + 0.011 = -(1 - 0.011) = -0.101 = -5/8$. Thus, when multiplying by a negative fraction of the form $1.g$, we treat the fraction part ($.g$) as a positive fraction, but the sign bit is treated as -1. Hence, multiplication proceeds in the normal way as we multiply by each bit of the fraction and accumulate the partial products. However, when we reach the negative sign bit, we must add in the 2's complement of the multiplicand instead of the multiplicand itself. The following example illustrates this:

0.1 0 1	(+5/8)		
× 1.1 0 1	(−3/8)		
(0. 0 0)0 1 0 1	(+5/64)		
(0.)0 1 0 1	(+5/16)		
(0.)0 1 1 0 0 1			
1. 0 1 1	(−5/8)	←	*Note*: The 2's complement of the multiplicand is added at this point.
1. 1 1 0 0 0 1	(−15/64)		

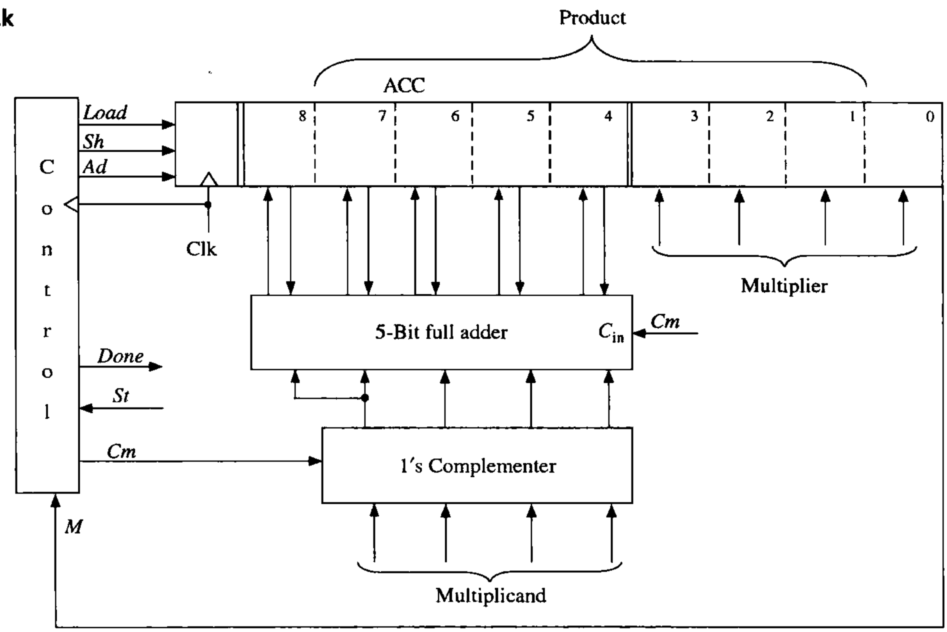
When both the multiplicand and multiplier are negative, the procedure is the same as before. At each step, we must be careful to extend the sign bit of the partial product to preserve the proper negative sign, and at the final step we must add in the 2's complement of the multiplicand, since the sign bit of the multiplier is negative. For example,

1.1 0 1	(−3/8)		
× 1.1 0 1	(−3/8)		
(1. 1 1)1 1 0 1	(−3/64)	←	*Note*: Extend sign bit.
(1.)1 1 0 1	(−3/16)		
1. 1 1 0 0 0 1			
0. 0 1 1	(+3/8)	←	Add the 2's complement of the multiplicand.
0. 0 0 1 0 0 1	(+9/64)		

In summary, the procedure for multiplying signed 2's complement binary fractions is the same as for multiplying positive binary fractions, except that we must be careful to preserve the sign of the partial product at each step, and if the sign of the multiplier is negative, we must complement the multiplicand before adding it in at the last step. The hardware is almost identical to that used for multiplication of positive numbers, except a complementer must be added for the multiplicand.

Figure 4-31 shows the hardware required to multiply two 4-bit fractions (including the sign bit). A 5-bit adder is used so the sign of the sum is not lost due to a carry into the sign bit position. The *M* input to the control circuit is the currently active bit of the multiplier. Control signal *Sh* causes the accumulator to shift right one place with sign extension. *Ad* causes the ADDER output to be loaded into the left 5 bits of the accumulator. The carry-out from the last bit of the adder is discarded,

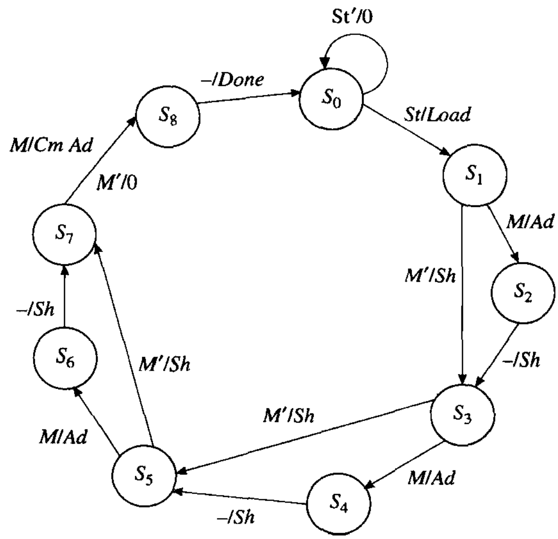
FIGURE 4-31: Block Diagram for 2's Complement Multiplier



since we are doing 2's complement addition. *Cm* causes the multiplicand (Mcand) to be complemented (1's complement) before it enters the adder inputs. *Cm* is also connected to the carry input of the adder so that when $Cm = 1$, the adder adds 1 plus the 1's complement of Mcand to the accumulator, which is equivalent to adding the 2's complement of Mcand. Figure 4-32 shows a state graph for the control circuit. Each multiplier bit (M) is tested to determine whether to add and shift or whether to just shift. In state S_7, M is the sign bit, and if $M = 1$, the complement of the multiplicand is added to the accumulator.

When the hardware in Figure 4-31 is used, the add and shift operations must be done at two separate clock times. We can speed up operation of the multiplier by

FIGURE 4-32: State Graph for 2's Complement Multiplier



moving the wires from the adder output one position to the right (Figure 4-33) so that the adder output is already shifted over one position when it is loaded into the accumulator. With this arrangement, the add and shift operations can occur at the same clock time, which leads to the control state graph of Figure 4-34. When the multiplication is complete, the product (6 bits plus sign) is in the lower 3 bits of A followed by B. The binary point then is in the middle of the A register. If we wanted it between the left 2 bits, we would have to shift A and B left one place.

FIGURE 4-33: Block Diagram for Faster Multiplier

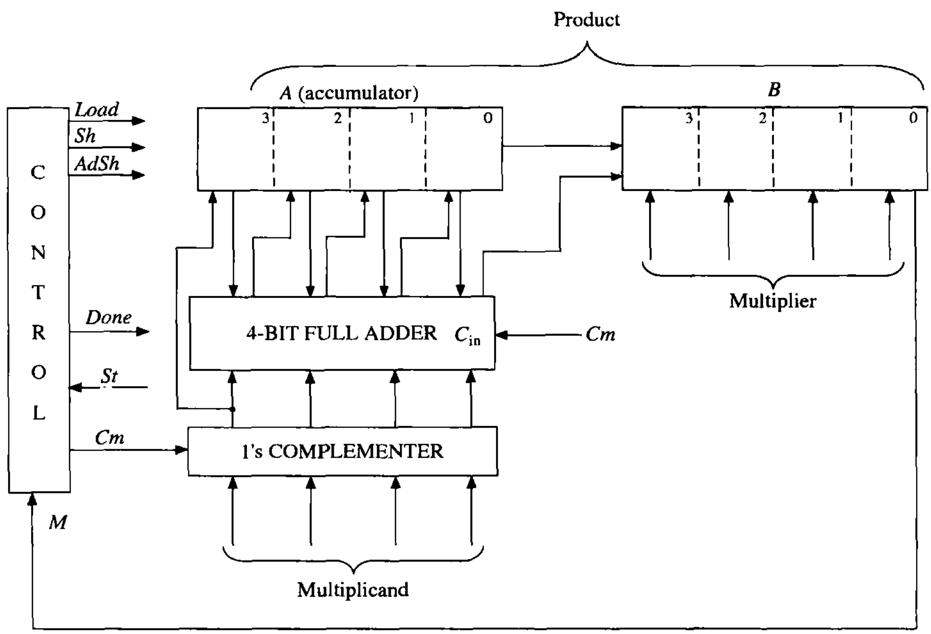
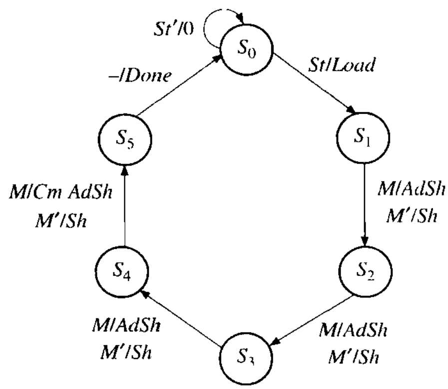


FIGURE 4-34: State Graph for Faster Multiplier



A behavioral VHDL model for this multiplier is shown in Figure 4-35. Shifting the A and B registers together is accomplished by the sequential statements

```
A <= A(3) & A(3 downto 1);
B <= A(0) & B(3 downto 1);
```

Although these statements are executed sequentially, A and B are both scheduled to be updated at the same delta time. Therefore, the old value of $A(0)$ is used when computing the new value of B.

FIGURE 4-35: Behavioral Model for 2's Complement Multiplier

```
library IEEE;
use IEEE.numeric_bit.all;

entity mult2C is
  port(CLK, St: in bit;
       Mplier, Mcand : in unsigned(3 downto 0);
       Product: out unsigned (6 downto 0);
       Done: out bit);
end mult2C;

architecture behave1 of mult2C is
signal State: integer range 0 to 5;
signal A, B: unsigned(3 downto 0);
alias M: bit is B(0);
begin
  process(CLK)
  variable addout: unsigned(3 downto 0);
  begin
    if CLK'event and CLK = '1' then
      case State is
        when 0 =>                -- initial State
          if St = '1' then
            A <= "0000";         -- begin cycle
            B <= Mplier;         -- load the multiplier
```

```
          State <= 1;
        end if;
      when 1 | 2 | 3 =>           -- "add/shift" states
        if M = '1' then
          addout := A + Mcand;    -- add multiplicand to A and shift
          A <= Mcand(3) & addout(3 downto 1);
          B <= addout(0) & B(3 downto 1);
        else
          A <= A(3) & A(3 downto 1);  -- arithmetic right shift
          B <= A(0) & B(3 downto 1);
        end if;
        State <= State + 1;
      when 4 =>
        if M = '1' then
          addout := A + not Mcand + 1;
            -- add 2's complement when sign bit of multiplier is 1
          A <= not Mcand(3) & addout(3 downto 1);
          B <= addout(0) & B(3 downto 1);
        else
          A <= A(3) & A(3 downto 1);  -- arithmetic right shift
          B <= A(0) & B(3 downto 1);
        end if;
        State <= 5;
      when 5 =>
        State <= 0;
    end case;
  end if;
 end process;
 Done <= '1' when State = 5 else '0';
 Product <= A(2 downto 0) & B;    -- output product
end behave1;
```

A variable *addout* has been defined to represent the 5-bit output of the adder. In states 1 through 4, if the current multiplier bit M is '1', then the sign bit of the multiplicand followed by 3 bits of *addout* are loaded into A. At the same time, the low-order bit of *addout* is loaded into B along with the high-order 3 bits of B. The *Done* signal is turned on when control goes to state 5, and then the new value of the product is outputted.

Before continuing with the design, we will test the behavioral level VHDL code to make sure that the algorithm is correct and consistent with the hardware block diagram. At early stages of testing, we will want a step-by-step printout to verify the internal operations of the multiplier and to aid in debugging, if required. When we think that the multiplier is functioning properly, then we will only want to look at the final product output so that we can quickly test a large number of cases.

Figure 4-36 shows the command file and test results for multiplying $+5/8$ by $-3/8$. A clock is defined with a 20-ns period. The *St* signal is turned on at 2 ns and turned off one clock period later. By inspection of the state graph, the multiplication requires six clocks, so the run time is set at 120 ns.

FIGURE 4-36: Command File and Simulation Results for (+5/8 by –3/8)

```
-- command file to test signed multiplier
add list CLK St State A B Done Product
force st 1 2, 0 22
force clk 1 0, 0 10 - repeat 20
-- (5/8 * -3/8)
force Mcand 0101
force Mplier 1101
run 120
```

ns	delta	CLK	St	State	A	B	Done	Product
0	+1	1	0	0	0000	0000	0	0000000
2	+0	1	1	0	0000	0000	0	0000000
10	+0	0	1	0	0000	0000	0	0000000
20	+1	1	1	1	0000	1101	0	0000000
22	+0	1	0	1	0000	1101	0	0000000
30	+0	0	0	1	0000	1101	0	0000000
40	+1	1	0	2	0010	1110	0	0000000
50	+0	0	0	2	0010	1110	0	0000000
60	+1	1	0	3	0001	0111	0	0000000
70	+0	0	0	3	0001	0111	0	0000000
80	+1	1	0	4	0011	0011	0	0000000
90	+0	0	0	4	0011	0011	0	0000000
100	+2	1	0	5	1111	0001	1	1110001
110	+0	0	0	5	1111	0001	1	1110001
120	+1	1	0	0	1111	0001	0	1110001

To thoroughly test the multiplier, we need to test not only the four standard cases (+ +,+ −, − +, and − −) but also special cases and limiting cases. Test values for the multiplicand and multiplier should include 0, the largest positive fraction, the most negative fraction, and all 1's. We will write a VHDL test bench to test the multiplier. The **test bench** will provide a sequence of values for the multiplicand and the multiplier. Thus, it provides stimuli to the system under test, the multiplier. The test bench can also check for the correctness of the multiplier output. The multiplier we are testing will be treated as a component and embedded in the test bench program. The signals generated within the test bench are interfaced to the multiplier as shown in Figure 4-37.

FIGURE 4-37: Interface between Multiplier and Its Test Bench

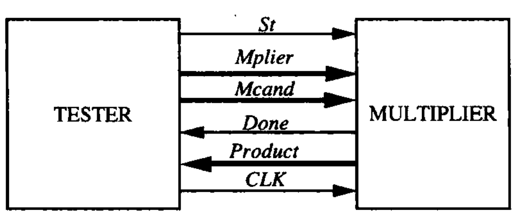


Figure 4-38 shows the VHDL code for the multiplier test bench. The test sequence consists of 11 sets of multiplicands and multipliers, provided in the

FIGURE 4-38: Test Bench for Signed Multiplier

```
library IEEE;
use IEEE.numeric_bit.all;

entity testmult is
end testmult;

architecture test1 of testmult is
component mult2C
  port(CLK, St: in bit;
       Mplier, Mcand: in unsigned(3 downto 0);
       Product: out unsigned(6 downto 0);
       Done: out bit);
end component;

constant N: integer := 11;
type arr is array(1 to N) of unsigned(3 downto 0);
type arr2 is array(1 to N) of unsigned(6 downto 0);
constant Mcandarr: arr := ("0111", "1101", "0101", "1101", "0111",
                  "1000", "0111", "1000", "0000", "1111", "1011");
constant Mplierarr: arr := ("0101", "0101", "1101", "1101", "0111",
                   "0111", "1000", "1000", "1101", "1111", "0000");
constant Productarr: arr2 := ("0100011", "1110001", "1110001",
                              "0001001", "0110001", "1001000",
                              "1001000", "1000000", "0000000",
                              "0000001", "0000000");
signal CLK, St, Done: bit;
signal Mplier, Mcand: unsigned(3 downto 0);
signal Product: unsigned(6 downto 0);
begin
  CLK <= not CLK after 10 ns;
  process
  begin
    for i in 1 to N loop
      Mcand <= Mcandarr(i);
      Mplier <= Mplierarr(i);
      St <= '1';
      wait until CLK = '1' and CLK'event;
      St <= '0';
      wait until Done = '0' and Done'event;
      assert Product = Productarr(i)     -- compare with expected answer
        report "Incorrect Product"
        severity error;
    end loop;
    report "TEST COMPLETED";
  end process;
  mult1: mult2c port map(CLK, St, Mplier, Mcand, Product, Done);
end test1;
```

Mcandarr and *Mplierarr* arrays. The expected outputs from the multiplier are provided in another array, the *Productarr*, in order to test the correctness of the multiplier outputs. The test values and results are placed in constant arrays in the VHDL code. A component declaration is done for the multiplier. A **port map** statement is used to create an instance of the multiplier. The tester also generates the clock and start signal. The for loop reads values from the *Mcandarr* and *Mplierarr* arrays and then sets the start signal to '1'. After the next clock, the start signal is turned off. Then the test bench waits for the *Done* signal. When the trailing edge of *Done* arrives, the multiplier output is compared against the expected output in the array *Productar*. An error is reported if the answers do not match. Since the *Done* signal is turned off at the same time the multiplier control goes back to S_0, the process waits for the falling edge of *Done* before looping back to supply new values of *Mcand* and *Mplier*. Note that the **port map** statement is outside the process that generates the stimulus. The multiplier constantly receives some set of inputs and generates the corresponding set of outputs.

Figure 4-39 shows the command file and simulator output. We have annotated the simulator output to interpret the test results. The –NOtrigger together with

FIGURE 4-39: Command File and Simulation of Signed Multiplier

```
-- Command file to test results of signed multiplier
add list -NOtrigger Mplier Mcand product -Trigger done
run 1320

  ns    delta    mplier   mcand    product    done
   0     +1       0101     0111    0000000     0
  90     +2       0101     0111    0100011     1    5/8 * 7/8 = 35/64
 110     +2       0101     1101    0100011     0
 210     +2       0101     1101    1110001     1    5/8 * -3/8 = -15/64
 230     +2       1101     0101    1110001     0
 330     +2       1101     0101    1110001     1    -3/8 * 5/8 = -15/64
 350     +2       1101     1101    1110001     0
 450     +2       1101     1101    0001001     1    -3/8 * -3/8 = 9/64
 470     +2       0111     0111    0001001     0
 570     +2       0111     0111    0110001     1    7/8 * 7/8 = 49/64
 590     +2       0111     1000    0110001     0
 690     +2       0111     1000    1001000     1    7/8 * -1 = -7/8
 710     +2       1000     0111    1001000     0
 810     +2       1000     0111    1001000     1    -1 * 7/8 = -7/8
 830     +2       1000     1000    1001000     0
 930     +2       1000     1000    1000000     1    -1 * -1 = -1 (error)
 950     +2       1101     0000    1000000     0
1050     +2       1101     0000    0000000     1    -3/8 * 0 = 0
1070     +2       1111     1111    0000000     0
1170     +2       1111     1111    0000001     1    -1/8 * -1/8 = 1/64
1190     +2       0000     1011    0000001     0
1290     +2       0000     1011    0000000     1    0 * -3/8 = 0
1310     +2       0101     0111    0000000     0
```

the –Trigger done in the list statement causes the output to be displayed only when the *Done* signal changes. Without the –NOtrigger and –Trigger, the output would be displayed every time any signal on the list changed. All the product outputs are correct, except for the special case of -1×-1 (1.000×1.000), which gives 1.000000 (-1) instead of $+1$. This occurs because no representation of $+1$ is possible without adding another bit.

Next, we refine the VHDL model for the signed multiplier by explicitly defining the control signals and the actions that occur when each control signal is asserted. The VHDL code (Figure 4-40) is organized in a manner similar to the Mealy machine model of Figure 1-17. In the first process, the *Nextstate* and output control signals are defined for each present *State*. In the second process, after waiting for the rising edge of the clock, the appropriate registers are updated and the *State* is updated. We can test the VHDL code of Figure 4-40 using the same test file we used previously and verify that we get the same product outputs.

FIGURE 4-40: Model for 2's Complement Multiplier with Control Signals

```
-- This VHDL model explicitly defines control signals.

library IEEE;
use IEEE.numeric_bit.all;

entity mult2C is
  port(CLK, St: in bit;
       Mplier, Mcand: in unsigned(3 downto 0);
       Product: out unsigned (6 downto 0);
       Done: out bit);
end mult2C;

-- This architecture of a 4-bit multiplier for 2's complement numbers
-- uses control signals.

architecture behave2 of mult2C is
signal State, Nextstate: integer range 0 to 5;
signal A, B, compout, addout: unsigned(3 downto 0);
signal AdSh, Sh, Load, Cm: bit;
alias M: bit is B(0);
begin
  process(State, St, M)
  begin
    Load <= '0'; AdSh <= '0'; Sh <= '0'; Cm <= '0'; Done <= '0';
    case State is
      when 0 =>                -- initial state
        if St = '1' then Load <= '1'; Nextstate <= 1; end if;
      when 1 | 2 | 3 = >    -- "add/shift" State
        if M = '1' then AdSh < = '1';
        else Sh <= '1';
        end if;
```

```
      Nextstate <= State + 1;
    when 4 =>              -- add complement if sign
      if M = '1' then      -- bit of multiplier is 1
        Cm <= '1'; AdSh <= '1';
      else Sh <= '1';
      end if;
      Nextstate <= 5;
    when 5 =>              -- output product
      Done <= '1';
      Nextstate <= 0;
  end case;
end process;

compout <= not Mcand when Cm = '1' else Mcand; -- complementer
addout <=  A + compout + unsigned'(0=>Cm);     -- 4-bit adder with carry in

process (CLK)
begin
  if CLK'event and CLK = '1' then  -- executes on rising edge
    if Load = '1' then              -- load the multiplier
      A <= "0000";
      B <= Mplier;
    end if;
    if AdSh = '1' then              -- add multiplicand to A and shift
      A <= compout(3) & addout(3 downto 1);
      B <= addout(0) & B(3 downto 1);
    end if;
    if Sh = '1' then
      A <= A(3) & A(3 downto 1);
      B <= A(0) & B(3 downto 1);
    end if;
    State <= Nextstate;
  end if;
end process;
Product <= A(2 downto 0) & B;
end behave2;
```

4.11 Keypad Scanner

In this example, we design a scanner for a keypad with three columns and four rows as in Figure 4-41. The keypad is wired in matrix form with a switch at the intersection of each row and column. Pressing a key establishes a connection between a row and column. The purpose of the scanner is to determine which key has been pressed and output a binary number $N = N_3N_2N_1N_0$, which corresponds to the key number. For example, pressing key 5 must output 0101, pressing the * key must output 1010, and pressing the # key must output 1011. When a valid key has been detected, the scanner should output a signal V for one clock time. Assume that only one key is

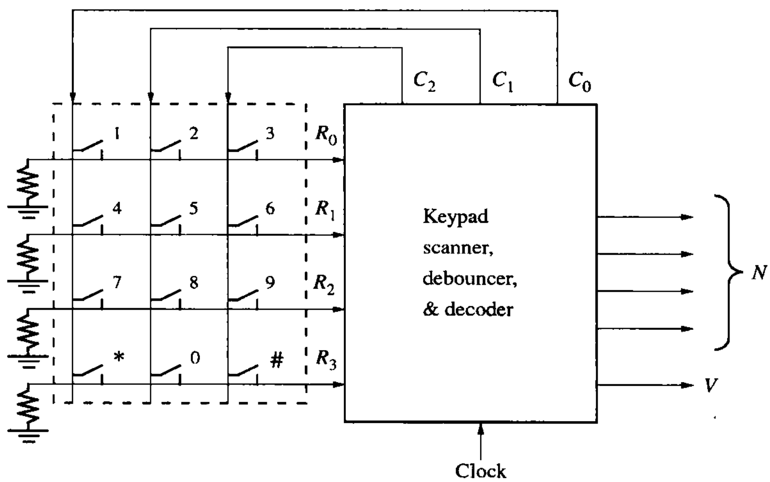
pressed at a time. The design must include hardware to protect the circuitry from malfunction due to keypad bounces.

FIGURE 4-41: Keypad with Three Columns and Four Rows

1	2	3
4	5	6
7	8	9
*	0	#

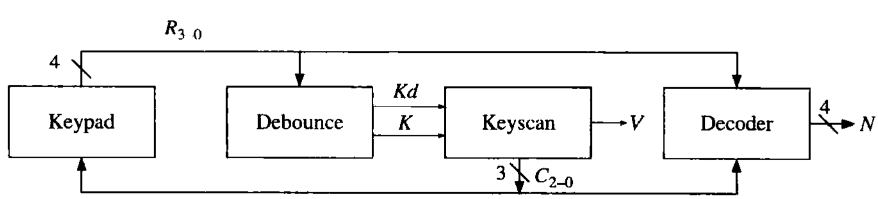
The overall block diagram of the circuit is presented in Figure 4-42. The keypad contains resistors that are connected to ground. When a switch is pressed, a path is established from the corresponding column line to the ground. If a voltage can be applied on the column lines C_0, C_1, and C_2, then the voltage can be obtained on the row line corresponding to the key that is pressed. One among the rows R_0, R_1, R_2, or R_3 will have an active signal.

FIGURE 4-42: Block Diagram for Keypad Scanner



We will divide the design into several modules, as shown in Figure 4-43. The first part of the design will be a scanner that scans the rows and columns of the keypad. The keyscan module generates the column signals to scan the keypad. The debounce module generates a signal K when a key has been pressed and a signal Kd after it has been debounced. When a valid key is detected, the decoder determines the key number from the row and column numbers.

FIGURE 4-43: Scanner Modules



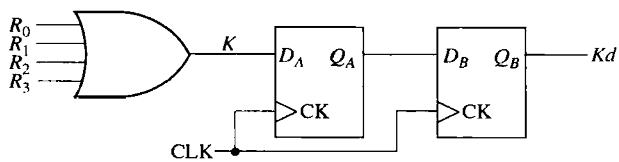
4.11.1 Scanner

We will use the following procedure to scan the keypad: First apply logic 1's to columns C_0, C_1, and C_2 and wait. If any key is pressed, a 1 will appear on R_0, R_1, R_2, or R_3. Then apply a 1 to column C_0 only. If any of the R_i's is 1, a valid key is detected. If R_0 is received, we know that switch 1 was pressed. If R_1, R_2, or R_3 is received, switch 4, 7, or * was pressed. If so, set $V = 1$ and output the corresponding N. If no key is detected in the first column, apply a 1 to C_1 and repeat. If no key is detected in the second column, repeat for C_2. When a valid key is detected, apply 1's to C_0, C_1, and C_2 and wait until no key is pressed. This last step is necessary so that only one valid signal is generated each time a key is pressed.

4.11.2 Debouncer

As discussed in the scoreboard example, we need to debounce the keys to avoid malfunctions due to switch bounce. Figure 4-44 shows a proposed debouncing and synchronizing circuit. The four row signals are connected to an OR gate to from signal K, which turns on when a key is pressed and a column scan signal is applied. The debounced signal Kd will be fed to the sequential circuit.

FIGURE 4-44: Debouncing and Synchronizing Circuit



4.11.3 Decoder

The decoder determines the key number from the row and column numbers using the truth table given in Table 4-4. The truth table has one row for each of the 12 keys. The remaining rows have don't care outputs since we have assumed that only one key is pressed at a time. Since the decoder is a combinational circuit, its output will change

TABLE 4-4: Truth Table for Decoder

R_3	R_2	R_1	R_0	C_0	C_1	C_2	N_3	N_2	N_1	N_0	
0	0	0	1	1	0	0	0	0	0	1	
0	0	0	1	0	1	0	0	0	1	0	
0	0	0	1	0	0	1	0	0	1	1	
0	0	1	0	1	0	0	0	1	0	0	
0	0	1	0	0	1	0	0	1	0	1	
0	0	1	0	0	0	1	0	1	1	0	
0	1	0	0	1	0	0	0	1	1	1	
0	1	0	0	0	1	0	1	0	0	0	
0	1	0	0	0	0	1	1	0	0	1	
1	0	0	0	1	0	0	1	0	1	0	(*)
1	0	0	0	0	1	0	0	0	0	0	
1	0	0	0	0	0	1	1	0	1	1	(#)

Logic Equations for Decoder

$N_3 = R_2C_0' + R_3C_1'$

$N_2 = R_1 + R_2C_0$

$N_1 = R_0C_0' + R_2'C_2 + R_1'R_0'C_0$

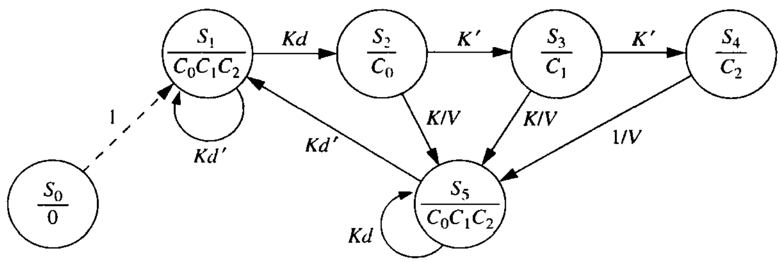
$N_0 = R_1C_1 + R_1'C_2 + R_3'R_1'C_1'$

as the keypad is scanned. At the time a valid key is detected ($K = 1$ and $V = 1$), its output will have the correct value and this value can be saved in a register at the same time the circuit goes to S_5.

4.11.4 Controller

Figure 4-45 shows the state diagram of the controller for the keypad scanner. It waits in S_1 with outputs $C_0 = C_1 = C_2 = 1$ until a key is pressed. In S_2, $C_0 = 1$, so if the key that was pressed is in column 0, $K = 1$, and the circuit outputs a valid signal and goes to S_5. Signal K is used instead of Kd, since the key press is already debounced. If no key press is found in column 0, column 1 is checked in S_3, and if necessary, column 2 is checked in S_4. In S_5, the circuit waits until all keys are released and Kd goes to 0 before resetting.

FIGURE 4-45: State Graph for Keypad Scanner



The state diagram in Figure 4-45 works for many cases; however, it does have some timing problems. Let us analyze the following situations.

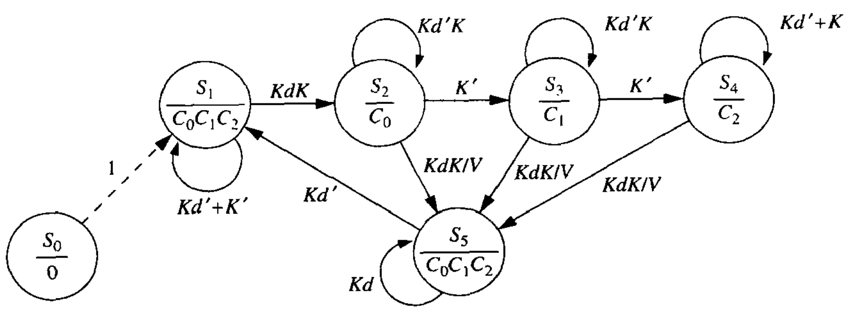
1. Is K true whenever a button is pressed?
   No. Although K is true if any one of the row signals R_1, R_2, R_3, or R_4 is true, if the column scan signals are not active, none of R_1–R_4 can be true, although the button is pressed.
2. Can Kd be false when a button is continuing to be pressed?
   Yes. Signal Kd is nothing but K delayed by two clock cycles. K can go to 0 during the scan process even when the button is being pressed. For instance, consider the case when a key in the rightmost column is pressed. During scan of the first two columns, K goes to 0. If K goes to 0 at any time, Kd will go to zero two cycles later. Hence, neither K nor Kd is synonymous to pressing the button.
3. Can you go from S_5 to S_1 when a button is still pressed?
   In the state diagram in Figure 4-45, the S_4-to-S_5 transition could happen when Kd is false. Kd might have become false while scanning C_0 and C_1. Hence, it is possible that we reach back to S_1 when the key is still being pressed. As an example, let us assume that a button is pressed in column C_2. This is to be detected in S_4. However, during the scanning process in S_2 and S_3, K is 0;

hence, two cycles later Kd will be 0 even if the button stays pressed. During the scan in S_4, the correct key can be found; however, the system can reach S_5 when Kd is still 0 and a malfunction can happen. S_5 is intended to sense the release of the key. However, Kd is not synonymous to pressing the button and Kd' does not truly indicate that the button got released. Since Kd' can appear when the button is still pressed, if you reach S_5 when Kd' is true due to scanning activity in a previous state, the system can go from S_5 to S_1 without a key release. In such a case, the same key may be read multiple times.

4. What if a key is pressed for only one or two clock cycles?
   If the key is pressed and released very quickly, there would be problems especially if the key is in the third column. By the time the scanner reaches state S_4, the key might have been released already. The key should be pressed long enough for the scanner to go through the longest path in the state graph from S_0 to S_5. This may not be a serious problem because usually the digital system clock is much faster than any mechanical switch.

These problems can be fixed by assuring that we can reach S_5 only if Kd is true. A modified state diagram is presented in Figure 4-46. Before transitioning to state S_5, this circuit waits in state S_2, S_3, and S_4 until Kd also becomes 1.

FIGURE 4-46: Modified State Graph for Keypad Scanner



4.11.5 VHDL Code

The VHDL code used to implement the design is shown in Figure 4-47. The decoder equations as well as the equations for K and V are implemented by concurrent statements. The process implements the next state equations for the keyscan and debounce flip-flops.

FIGURE 4-47: VHDL Code for Scanner

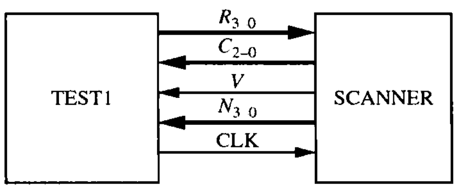
```
entity scanner is
  port(R0, R1, R2, R3, CLK: in bit;
       C0, C1, C2: inout bit;
       N0, N1, N2, N3, V: out bit);
end scanner;
```

```
architecture behavior of scanner is
signal QA, K,Kd: bit;
signal state, nextstate: integer range 0 to 5;
begin
  K <= R0 or R1 or R2 or R3;   -- this is the decoder section
  N3 <= (R2 and not C0) or (R3 and not C1);
  N2 <= R1 or (R2 and C0);
  N1 <= (R0 and not C0) or (not R2 and C2) or (not R1 and not R0 and C0);
  N0 <= (R1 and C1) or (not R1 and C2) or (not R3 and not R1 and not C1);

  process(state, R0, R1, R2, R3, C0, C1, C2, K, Kd, QA)
  begin
    C0 <= '0'; C1 <= '0'; C2 <= '0'; V <= '0';
    case state is
      when 0 => nextstate < = 1;
      when 1 => C0 <= '1'; C1 <= '1'; C2 <= '1';
        if (Kd and K) = '1' then nextstate <= 2;
        else nextstate <= 1;
        end if;
      when 2 => C0 <= '1';
        if (Kd and K) = '1' then V <= '1'; nextstate <= 5;
        elsif K = '0' then nextstate <= 3;
        else nextstate <= 2;
        end if;
      when 3 => C1 <= '1';
        if (Kd and K) = '1' then V <= '1'; nextstate <= 5;
        elsif K = '0' then nextstate <= 4;
        else nextstate <= 3;
        end if;
      when 4 => C2 <= '1';
        if (Kd and K) = '1' then V <= '1'; nextstate <= 5;
        else nextstate <= 4;
        end if;
      when 5 => C0 <= '1'; C1 <= '1'; C2 <= '1';
        if Kd = '0' then nextstate <= 1;
        else nextstate <= 5;
        end if;
    end case;
  end process;

  process(CLK)
  begin
    if CLK = '1' and CLK'EVENT then
      state <= nextstate;
      QA <= K;
      Kd <= QA;
    end if;
  end process;
end behavior;
```

4.11.6 Test Bench for Keypad Scanner

This VHDL code would be very difficult to test by supplying waveforms for the inputs R_0, R_1, R_2, and R_3, since these inputs depend on the column outputs (C_0, C_1, C_2). A much better way to test the scanner is by using a test bench in VHDL. The scanner we are testing will be treated as a component and embedded in the test bench program. The signals generated within the test bench are interfaced to the scanner as shown in Figure 4-48. The test bench simulates a key press by supplying the appropriate R signals in response to the C signals from the scanner. When test bench receives $V = 1$ from the scanner, it checks to see if the value of N corresponds to the key that was pressed.

FIGURE 4-48: Interface for Test Bench



The VHDL code for the keypad test bench is shown in Figure 4-49. A copy of the scanner is instantiated within the *test1* architecture, and connections to the scanner are made by the port map. The sequence of key numbers used for testing is stored in the array *KARRAY*. The tester simulates the keypad operation using

FIGURE 4-49: VHDL for Scanner Test Bench

```
library IEEE;
use IEEE.numeric_bit.all;

entity scantest is
end scantest;

architecture test1 of scantest is
component scanner
  port(R0, R1, R2, R3, CLK: in bit;
       C0, C1, C2: inout bit;
       N0, N1, N2, N3, V: out bit);
end component;

type arr is array (0 to 23) of integer;          -- array of keys to test
constant KARRAY: arr := (2,5,8,0,3,6,9,11,1,4,7,10,1,2,3,4,5,6,7,8,9,10,11,0);
signal C0, C1, C2, V, CLK, R0, R1, R2, R3: bit;   -- interface signals
signal N: unsigned(3 downto 0);
signal KN: integer;                              -- key number to test
begin
  CLK <= not CLK after 20 ns;                    -- generate clock signal
```

```
  -- this section emulates the keypad
  R0 <= '1' when (C0='1' and KN=1) or (C1='1' and KN=2) or (C2='1' and KN=3)
        else '0';
  R1 <= '1' when (C0='1' and KN=4) or (C1='1' and KN=5) or (C2='1' and KN=6)
        else '0';
  R2 <= '1' when (C0='1' and KN=7) or (C1='1' and KN=8) or (C2='1' and KN=9)
        else '0';
  R3 <= '1' when (C0='1' and KN=10) or (C1='1' and KN=0) or (C2='1' and KN=11)
        else '0';

  process                               -- this section tests scanner
  begin
    for i in 0 to 23 loop               -- test every number in key array
      KN <= KARRAY(i);                  -- simulates keypress
      wait until (V = '1' and rising_edge(CLK));
      assert (to_integer(N) = KN)       -- check if output matches
        report "Numbers don't match"
        severity error;
      KN <= 15;                         -- equivalent to no key pressed
      wait until rising_edge(CLK); - wait for scanner to reset
      wait until rising_edge(CLK);
      wait until rising_edge(CLK);
    end loop;
    report "Test Complete.";
  end process;
  scanner1: scanner port map(R0,R1,R2,R3,CLK,C0,C1,C2,N(0),N(1),N(2),N(3),V);
                                        -- connect test1 to scanner
end test1;
```

concurrent statements for R_0, R_1, R_2, and R_3. Whenever C_0, C_1, C_2, or the key number (KN) changes, new values for the Rs are computed. For example, if $KN = 5$ (to simulate pressing key 5), then $R_0R_1R_2R_3 = 0100$ is sent to the scanner when $C_0C_1C_2 = 010$. The test process is as follows:

1. Read a key number from the array to simulate pressing a key.
2. Wait until $V = 1$ and the rising edge of the clock occurs.
3. Verify that the N output from the scanner matches the key number.
4. Set $KN = 15$ to simulate no key pressed. (Since 15 is not a valid key number, all R's will go to 0.)
5. Wait until $Kd = 0$ before selecting a new key.

Key presses in row order and column order are tried using the various numbers in $KARRAY$. The test bench uses **assert** statements to test whether the reported number matches the key pressed. The **report** statement is used to report an error if the scanner generates the wrong key number, and it will report "Testing Complete." when all keys have been tested.

4.12 Binary Dividers

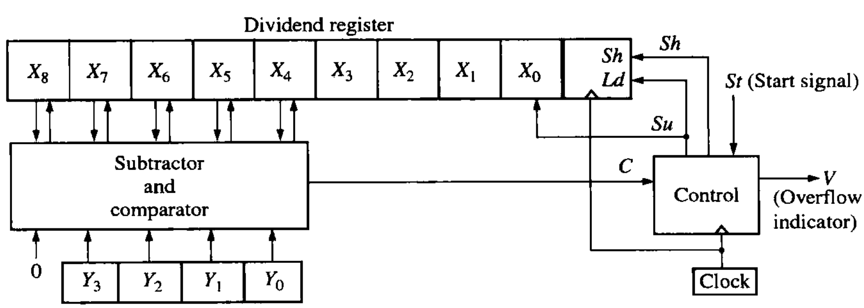
4.12.1 Unsigned Divider

We will consider the design of a parallel divider for positive binary numbers. As an example, we will design a circuit to divide an 8-bit dividend by a 4-bit divisor to obtain a 4-bit quotient. The following example illustrates the division process:

```
                       1010      Quotient
Divisor  1101 / 10000111      Dividend
                  1101
(135÷13=10 with    0111
 a remainder of 5)  0000
                     1111
                     1101
                      0101
                      0000
                      0101      Remainder
```

Just as binary multiplication can be carried out as a series of add and shift operations, division can be carried out by a series of subtract and shift operations. To construct the divider, we will use a 9-bit dividend register and a 4-bit divisor register, as shown in Figure 4-50. During the division process, instead of shifting the divisor right before each subtraction, we will shift the dividend to the left. Note that an extra bit is required on the left end of the dividend register so that a bit is not lost when the dividend is shifted left. Instead of using a separate register to store the quotient, we will enter the quotient bit-by-bit into the right end of the dividend register as the dividend is shifted left.

FIGURE 4-50: Block Diagram for Parallel Binary Divider



The preceding division example (135 divided by 13) is reworked next, showing the location of the bits in the registers at each clock time. Initially, the dividend and divisor are entered as follows:

0	1	0	0	0	0	1	1	1
	1	1	0	1				

Subtraction cannot be carried out without a negative result, so we will shift before we subtract. Instead of shifting the divisor one place to the right, we will shift the dividend one place to the left:

```
                        | ←— Dividing line between dividend and quotient
1 0 0 0 0 1 1 1 | 0
  1 1 0 1       |      Note that after the shift, the rightmost position
                       in the dividend register is "empty."
```

Subtraction is now carried out and the first quotient digit of 1 is stored in the unused position of the dividend register:

```
0 0 0 1 1 1 1 1 | 1 ←——— first quotient digit
```

Next we shift the dividend one place to the left:

```
  0 0 1 1 1 1 1 | 1 0
    1 1 0 1     |
```

Since subtraction would yield a negative result, we shift the dividend to the left again, and the second quotient bit remains zero:

```
    0 1 1 1 1 1 | 1 0 0
      1 1 0 1   |
```

Subtraction is now carried out, and the third quotient digit of 1 is stored in the unused position of the dividend register:

```
    0 0 0 1 0 1 | 1 0 1 ←——— third quotient digit
```

A final shift is carried out and the fourth quotient bit is set to 0:

$$\underbrace{0\ 0\ 1\ 0\ 1}_{\text{remainder}} \;\Big|\; \underbrace{1\ 0\ 1\ 0}_{\text{quotient}}$$

The final result agrees with that obtained in the first example.

If, as a result of a division operation, the quotient contains more bits than are available for storing the quotient, we say that an *overflow* has occurred. For the divider of Figure 4-50, an overflow would occur if the quotient is greater than 15, since only 4 bits are provided to store the quotient. It is not actually necessary to carry out the division to determine if an overflow condition exists, since an initial comparison of the dividend and divisor will tell if the quotient will be too large. For example, if we attempt to divide 135 by 7, the initial contents of the registers are

```
0 1 0 0 0 0 1 1 1
  0 1 1 1
```

Since subtraction can be carried out with a nonnegative result, we should subtract the divisor from the dividend and enter a quotient bit of 1 in the rightmost

place in the dividend register. However, we cannot do this because the rightmost place contains the least significant bit of the dividend, and entering a quotient bit here would destroy that dividend bit. Therefore, the quotient would be too large to store in the 4 bits we have allocated for it, and we have detected an overflow condition. In general, for Figure 4-50, if initially $X_8X_7X_6X_5X_4 \geq Y_3Y_2Y_1Y_0$ (i.e., if the left 5 bits of the dividend register exceed or equal the divisor), the quotient will be greater than 15 and an overflow occurs. Note that if $X_8X_7X_6X_5X_4 \geq Y_3Y_2Y_1Y_0$, the quotient is

$$\frac{X_8X_7X_6X_5X_4X_3X_2X_1X_0}{Y_3Y_2Y_1Y_0} \geq \frac{X_8X_7X_6X_5X_40000}{Y_3Y_2Y_1Y_0} = \frac{X_8X_7X_6X_5X_4 \times 16}{Y_3Y_2Y_1Y_0} \geq 16$$

The operation of the divider can be explained in terms of the block diagram of Figure 4-50. A shift signal (*Sh*) will shift the dividend one place to the left. A subtract signal (*Su*) will subtract the divisor from the five leftmost bits in the dividend register and set the quotient bit (the rightmost bit in the dividend register) to 1. If the divisor is greater than the five leftmost dividend bits, the comparator output is $C = 0$; otherwise, $C = 1$. Whenever $C = 0$, subtraction cannot occur without a negative result, so a shift signal is generated. Whenever $C = 1$, a subtract signal is generated, and the quotient bit is set to 1. The control circuit generates the required sequence of shift and subtract signals.

Figure 4-51 shows the state diagram for the control circuit. When a start signal (*St*) occurs, the 8-bit dividend and 4-bit divisor are loaded into the appropriate registers. If *C* is 1, the quotient would require five or more bits. Since space is only provided for a 4-bit quotient, this condition constitutes an overflow, so the divider is stopped and the overflow indicator is set by the *V* output. Normally, the initial value of *C* is 0, so a shift will occur first, and the control circuit will go to state S_2. Then, if $C = 1$, subtraction occurs. After the subtraction is completed, *C* will always be 0, so the next clock pulse will produce a shift. This process continues until four shifts have occurred and the control is in state S_5. Then a final subtraction occurs, if necessary, and the control returns to the stop state. For this example, we will assume that when the start signal (*St*) occurs, it will be 1 for one clock time, and then it will remain 0 until the control circuit is back in state S_0. Therefore, *St* will always be 0 in states S_1 through S_5.

FIGURE 4-51: State Diagram for Divider Control Circuit

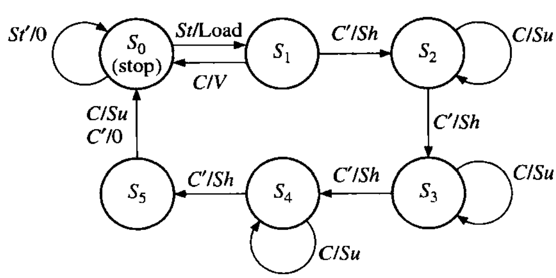


Table 4-5 gives the state table for the control circuit. Since we assumed that $St = 0$ in states S_1, S_2, S_3, and S_4, the next states and outputs are "don't cares" for these states

when $St = 1$. The entries in the output table indicate which outputs are 1. For example, the entry Sh means $Sh = 1$ and the other outputs are 0.

TABLE 4-5: State Table for Divider Control Circuit

State	StC 00	01	11	10	StC 00	01	11	10
S_0	S_0	S_0	S_1	S_1	0	0	Load	Load
S_1	S_2	S_0	—	—	Sh	V	—	—
S_2	S_3	S_2	—	—	Sh	Su	—	—
S_3	S_4	S_3	—	—	Sh	Su	—	—
S_4	S_5	S_4	—	—	Sh	Su	—	—
S_5	S_0	S_0	—	—	0	Su	—	—

This example illustrates a general method for designing a divider for unsigned binary numbers, and the design can easily be extended to larger numbers such as 16 bits divided by 8 bits or 32 bits divided by 16 bits. Using a separate counter to count the number of shifts is recommended if more than four shifts are required.

4.12.2 Signed Divider

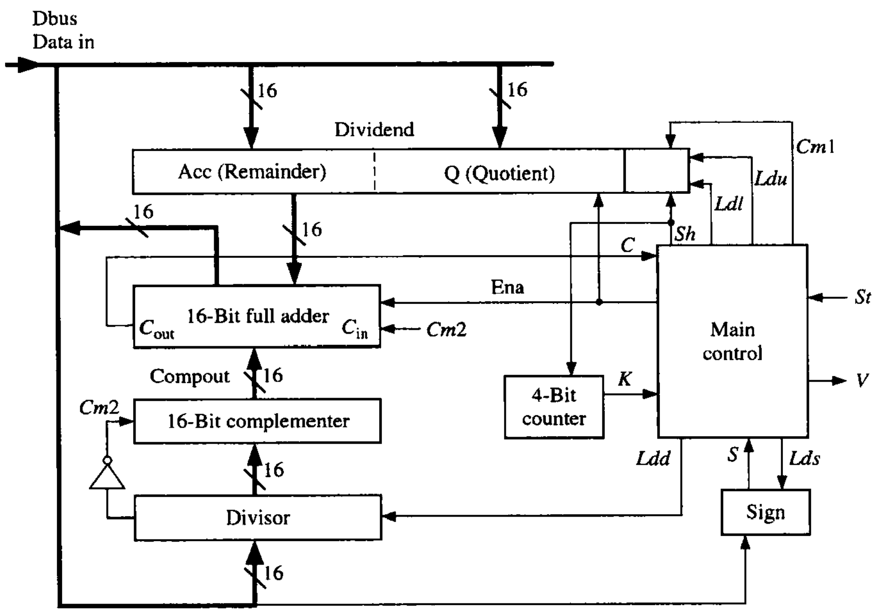
We now design a divider for signed (2's complement) binary numbers that divides a 32-bit dividend by a 16-bit divisor to give a 16-bit quotient. Although algorithms exist to divide the signed numbers directly, such algorithms are rather complex. So we take the easy way out and complement the dividend and divisor if they are negative; when division is complete, we complement the quotient if it should be negative.

Figure 4-52 shows a block diagram for the divider. We use a 16-bit bus to load the registers. Since the dividend is 32 bits, two clocks are required to load the upper and lower halves of the dividend register, and one clock is needed to load the divisor. An extra sign flip-flop is used to store the sign of the dividend. We will use a dividend register with a built-in 2's complementer. The subtracter consists of an adder and a complementer, so subtraction can be accomplished by adding the 2's complement of the divisor to the dividend register. If the divisor is negative, using a separate step to complement it is unnecessary; we can simply disable the complementer and add the negative divisor instead of subtracting its complement. The control circuit is divided into two parts—a main control, which determines the sequence of shifts and subtracts, and a counter, which counts the number of shifts. The counter outputs a signal $K = 1$ when 15 shifts have occurred. Control signals are defined as follows:

LdU	Load upper half of dividend from bus.
LdL	Load lower half of dividend from bus.
Lds	Load sign of dividend into sign flip-flop.
S	Sign of dividend.
$Cm1$	Complement dividend register (2's complement).
Ldd	Load divisor from bus.
Su	Enable adder output onto bus (Ena) and load upper half of dividend from bus.

Cm2 Enable complementer. (*Cm2* equals the complement of the sign bit of the divisor, so a positive divisor is complemented and a negative divisor is not.)
Sh Shift the dividend register left one place and increment the counter.
C Carry output from adder. (If $C = 1$, the divisor can be subtracted from the upper dividend.)
St Start.
V Overflow.
Qneg Quotient will be negative. (*Qneg* = 1 when the sign of the dividend and divisor are different.)

FIGURE 4-52: Block Diagram for Signed Divider



The procedure for carrying out the signed division is as follows:

1. Load the upper half of the dividend from the bus, and copy the sign of the dividend into the sign flip-flop.
2. Load the lower half of the dividend from the bus.
3. Load the divisor from the bus.
4. Complement the dividend if it is negative.
5. If an overflow condition is present, go to the done state.
6. Else carry out the division by a series of shifts and subtracts.
7. When division is complete, complement the quotient if necessary, and go to the done state.

Testing for overflow is slightly more complicated than for the case of unsigned division. First, consider the case of all positive numbers. Since the divisor and quotient

are each 15 bits plus sign, their maximum value is 7FFFh. Since the remainder must be less than the divisor, its maximum value is 7FFEh. Therefore, the maximum dividend for no overflow is

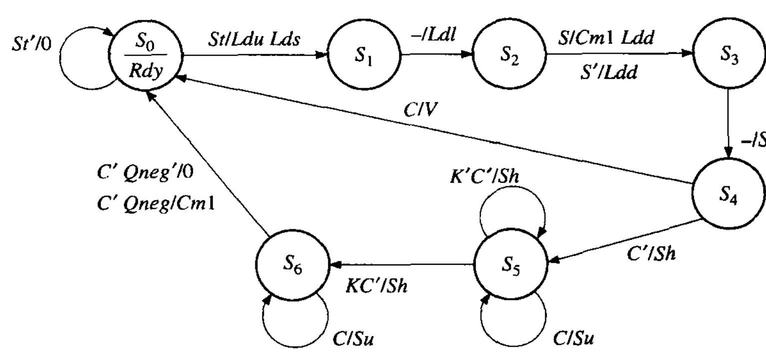
$$\text{divisor} \times \text{quotient} + \text{remainder} = 7FFFh \times 7FFFh + 7FFEh = 3FFF7FFFh$$

If the dividend is 1 larger (3FFF8000h), division by 7FFFh (or anything smaller) will give an overflow. We can test for the overflow condition by shifting the dividend left one place and then comparing the upper half of the dividend (divu) with the divisor. If divu $\geq$ divisor, the quotient would be greater than the maximum value, which is an overflow condition. For the preceding example, shifting 3FFF8000h left once gives 7FFF0000h. Since 7FFFh equals the divisor, there is an overflow. On the other hand, shifting 3FFF7FFFh left gives 7FFEFFFEh, and since 7FFEh < 7FFFh, no overflow occurs when dividing by 7FFFh.

Another way of verifying that we must shift the dividend left before testing for overflow is as follows. If we shift the dividend left one place and then divu $\geq$ divisor, we could subtract and generate a quotient bit of 1. However, this bit would have to go in the sign bit position of the quotient. This would make the quotient negative, which is incorrect. After testing for overflow, we must shift the dividend left again, which gives a place to store the first quotient bit after the sign bit. Since we work with the complement of a negative dividend or a negative divisor, this method for detecting overflow will work for negative numbers, except for the special case where the dividend is 80000000h (the largest negative value). Modifying the design to detect overflow in this case is left as an exercise.

Figure 4-53 shows the state graph for the control circuit. When $St = 1$, the registers are loaded. In S_2, if the sign of the dividend (S) is 1, the dividend is complemented. In S_3, we shift the dividend left one place and then we test for overflow in S_4. If $C = 1$, subtraction is possible, which implies an overflow, and the circuit goes to the done state. Otherwise, the dividend is shifted left. In S_5, C is tested. If $C = 1$, then $Su = 1$, which implies Ldu and Ena, so the adder output is enabled onto the bus and loaded into the upper dividend register to accomplish the subtraction. Otherwise, $Sh = 1$ and the dividend register is shifted. This continues until $K = 1$, at which time the last shift occurs if $C = 0$, and the circuit goes to S_6. Then if the sign of the divisor and the saved sign of the dividend are different, the dividend register is complemented so that the quotient will have the correct sign.

FIGURE 4-53: State Graph for Signed Divider Control Circuit



The VHDL code for the signed divider is shown in Figure 4-54. Since the 1's complementer and adder are combinational circuits, we have represented their operation by concurrent statements. All the signals that represent register outputs are updated on the rising edge of the clock, so these signals are updated in the process after waiting for *CLK* to change to '1'. The counter is simulated by a signal, *count*. For convenience in listing the simulator output, we have added a ready signal (*Rdy*), which is turned on in S_0 to indicate that the division is completed.

FIGURE 4-54: VHDL Model of 32-Bit Signed Divider

```
library IEEE;
use IEEE.numeric_bit.all;

entity sdiv is
  port(CLK, St: in bit;
       Dbus: in unsigned(15 downto 0);
       Quotient: out unsigned(15 downto 0);
       V, Rdy: out bit);
end sdiv;

architecture Signdiv of Sdiv is
signal State: integer range 0 to 6;
signal Count: unsigned(3 downto 0); -- integer range 0 to 15
signal Sign, C, Cm2: bit;
signal Divisor, Sum, Compout: unsigned(15 downto 0);
signal Dividend: unsigned(31 downto 0);
alias Acc: unsigned(15 downto 0) is Dividend(31 downto 16);
begin                                         -- concurrent statements
  Cm2 <= not divisor(15);
  compout <= divisor when Cm2 = '0'           -- 1's complementer
             else not divisor;
  Sum <= Acc + compout + unsigned'(0=>Cm2);  -- adder output
  C <= not Sum(15);
  Quotient <= Dividend(15 downto 0);
  Rdy <= '1' when State = 0 else '0';
  process(CLK)
  begin
    if CLK'event and CLK = '1' then  -- wait for rising edge of clock
      case State is
        when 0 =>
          if St = '1' then
            Acc <= Dbus;                            -- load upper dividend
            Sign <= Dbus(15);
            State <= 1;
            V <= '0';                               -- initialize overflow
            Count <= "0000";                        -- initialize counter
          end if;
```

```
        when 1 =>
          Dividend (15 downto 0) <= Dbus;            -- load lower dividend
          State <= 2;
        when 2 =>
          Divisor <= Dbus;
          if Sign = '1' then  -- two's complement Dividend if necessary
            dividend <= not dividend + 1;
          end if;
          State <= 3;
        when 3 =>
          Dividend <= Dividend(30 downto 0) & '0';   -- left shift
          Count <= Count+1; State <= 4;
        when 4 =>
          if C = '1' then                            -- C
            v <= '1'; State <= 0;
          else                                       -- C'
            Dividend <= Dividend(30 downto 0) & '0'; -- left shift
            Count <= Count+1; State <= 5;
          end if;
        when 5 =>
          if C = '1' then                            -- C
            ACC <= Sum;                              -- subtract
            dividend(0) <= '1';
          else
            Dividend <= Dividend(30 downto 0) & '0'; -- left shift
            if Count = 15 then State <= 6; end if;   -- KC'
            Count <= Count+1;
          end if;
        when 6 =>
          state <= 0;
          if C = '1' then                            -- C
            Acc <= Sum;                              -- subtract
            dividend(0) <= '1'; State <= 6;
          elsif (Sign xor Divisor(15)) = '1' then    -- C'Qneg
            Dividend <= not Dividend + 1;
          end if;                                    -- 2's complement Dividend
      end case;
    end if;
  end process;
end signdiv;
```

We are now ready to test the divider design by using the VHDL simulator. We will need a comprehensive set of test examples that will test all the different special cases that can arise in the division process. To start with, we need to test the basic operation of the divider for all the different combinations of signs for the divisor and dividend (++, +−, −+, and −−). We also need to test the overflow detection for these four cases. Limiting cases must also be tested, including largest quotient, zero quotient, and so on. Use of a VHDL test bench is

convenient because the test data must be supplied in sequence at certain times, and the length of time to complete the division is dependent on the test data. Figure 4-55 shows a test bench for the divisor. The test bench contains a dividend array and a divisor array for the test data. The notation X"07FF00BB" is the hexadecimal representation of a bit string. The process in testsdiv first puts the upper dividend on *Dbus* and supplies a start signal. After waiting for the clock, it puts the lower dividend on *Dbus*. After the next clock, it puts the divisor on *Dbus*. It then waits until the *Rdy* signal indicates that division is complete before continuing. *Count* is set equal to the loop-index, so that the change in *Count* can be used to trigger the listing output.

FIGURE 4-55: Test Bench for Signed Divider

```
library IEEE;
use IEEE.numeric_bit.all;

entity testsdiv is
end testsdiv;

architecture test1 of testsdiv is
component sdiv
  port(CLK, St: in bit;
       Dbus: in unsigned(15 downto 0);
       Quotient: out unsigned(15 downto 0);
       V, Rdy: out bit);
end component;

constant N: integer : = 12;                       -- test sdiv1 N times
type arr1 is array(1 to N) of unsigned(31 downto 0);
type arr2 is array(1 to N) of unsigned(15 downto 0);
constant dividendarr: arr1 := (X"0000006F", X"07FF00BB", X"FFFFFE08",
     X"FF80030A", X"3FFF8000", X"3FFF7FFF", X"C0008000", X"C0008000",
     X"C0008001", X"00000000", X"FFFFFFFF", X"FFFFFFFF");
constant divisorarr: arr2 := (X"0007", X"E005", X"001E", X"EFFA", X"7FFF",
   X"7FFF", X"7FFF", X"8000", X"7FFF", X"0001", X"7FFF", X"0000");
signal CLK, St, V, Rdy: bit;
signal Dbus, Quotient, divisor: unsigned(15 downto 0);
signal Dividend: unsigned(31 downto 0);
signal Count: integer range 0 to N;

begin
  CLK <= not CLK after 10 ns;
  process
  begin
    for i in 1 to N loop
      St <= '1';
      Dbus <= dividendarr(i) (31 downto 16);
      wait until (CLK'event and CLK = '1');
```

```
      Dbus <= dividendarr(i) (15 downto 0);
      wait until (CLK'event and CLK = '1');
      Dbus <= divisorarr(i);
      St <= '0';
      dividend <= dividendarr(i) (31 downto 0);   -- save dividend for listing
      divisor <= divisorarr(i);                   -- save divisor for listing
      wait until (Rdy = '1');
      count <= i;                                  -- save index for triggering
    end loop;
  end process;
  sdiv1: sdiv port map(CLK, St, Dbus, Quotient, V, Rdy);
end test1;
```

Figure 4-56 shows the simulator command file and output. The `-NOtrigger`, together with the `-Trigger count` in the list statement, causes the output to be displayed only when the *count* signal changes. Examination of the simulator output shows that the divider operation is correct for all of the test cases, except for the following case:

$$\text{C0008000h} \div \text{7FFFh} = -\text{3FFF8000} \div \text{7FFFh} = -\text{8000h} = \text{8000h}$$

In this case, the overflow is turned on, and division never occurs. In general, the divider will indicate an overflow whenever the quotient should be 8000h (the most negative value). This occurs because the divider basically divides positive numbers, and the largest positive quotient is 7FFFh. If it is important to be able to generate the quotient 8000h, the overflow detection can be modified so it does not generate an overflow in this special case.

FIGURE 4-56: Simulation Test Results for Signed Divider

```
-- Command file to test results of signed divider
add list -hex -NOtrigger dividend divisor Quotient V -Trigger count
run 5300

  ns  delta  dividend  divisor  quotient  v  count
   0    +0   00000000    0000      0000   0      0
 470    +3   0000006F    0007      000F   0      1
 910    +3   07FF00BB    E005      BFFE   0      2
1330    +3   FFFFFE08    001E      FFF0   0      3
1910    +3   FF80030A    EFFA      07FC   0      4
2010    +3   3FFF8000    7FFF      0000   1      5
2710    +3   3FFF7FFF    7FFF      7FFF   0      6
2810    +3   C0008000    7FFF      0000   1      7
3510    +3   C0008000    8000      7FFF   0      8
4210    +3   C0008001    7FFF      8001   0      9
4610    +3   00000000    0001      0000   0      A
5010    +3   FFFFFFFF    7FFF      0000   0      B
5110    +3   FFFFFFFF    0000      0002   1      C
```

In this chapter, we presented several design examples. The examples included several arithmetic and nonarithmetic circuits. A seven-segment display, a BCD adder, a traffic light controller, a scoreboard, and a keypad scanner are examples of nonarithmetic circuits presented in the chapter. We also described algorithms for addition, multiplication, and division of unsigned and signed binary numbers. Specific designs such as the carry look-ahead adder and the array multiplier were presented. We designed digital systems to implement these algorithms. After developing a block diagram for such a system and defining the required control signals, we used state graphs to define a sequential machine that generates control signals in the proper sequence. We used VHDL to describe the systems at several different levels so that we can simulate and test for correct operation of the systems we have designed.

4.13 Problems

4.1 Design the correction circuit for a BCD adder that computes Z digit 0 and C for S_0 (see Figures 4-5 and 4-6). This correction circuit adds "0110" to S_0 if $S_0 > 9$. This is the same as adding "0AA0" to S_0, where A = '1' if $S_0 > 9$. Draw a block diagram for the correction circuit using one full adder, three half-adders, and a logic circuit to compute A. Design a circuit for A using a minimum number of gates. Note that the maximum possible value of S_0 is 10010.

4.2 **(a)** If gate delays are 5 ns, what is the delay of the fastest 4-bit ripple carry adder? Explain your calculation.
(b) If gate delays are 5 ns, what is the delay of the fastest 4-bit adder? What kind of an adder will it be? Explain your calculation.

4.3 Develop a VHDL model for a 16-bit carry look-ahead adder utilizing the 4-bit adder from Figure 4-10 as a component.

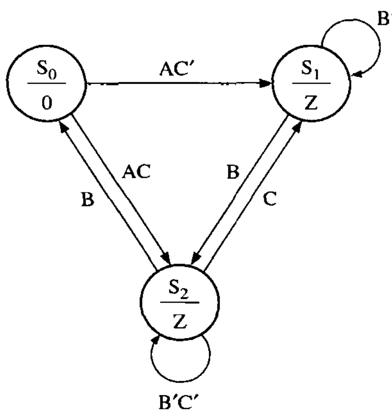
4.4 Derive generates, propagates, group generates, group propagates, and the final sum and carry out for the 16-bit carry look ahead adder of Figure 4-9, while adding 0101 1010 1111 1000 and 0011 1100 1100 0011.

4.5 **(a)** Write a VHDL module that describes one bit of a full adder with accumulator. The module should have two control inputs, Ad and L. If $Ad = 1$, the Y input (and carry input) are added to the accumulator. If $L = 1$, the Y input is loaded into the accumulator.
(b) Using the module defined in (a), write a VHDL description of a 4-bit subtracter with accumulator. Assume negative numbers are represented in 1's complement. The subtracter should have control inputs Su (subtract) and Ld (load).

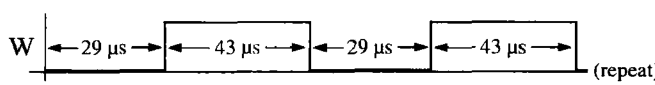
4.6 **(a)** Implement the traffic-light controller of Figure 4-14 using a modulo 13 counter with added logic. The counter should increment every clock, with two exceptions. Use a ROM to generate the outputs.

(b) Write a VHDL description of your answer to (a).
(c) Write a test bench for part (b) and verify that your controller works correctly. Use concurrent statements to generate test inputs for *Sa* and *Sb*.

4.7 Make the necessary additions to the following state graph so that it is a proper, completely specified state graph. Demonstrate that your answer is correct. Convert the graph to a state table using 0's and 1's for inputs and outputs.



4.8 Write synthesizable VHDL code that will generate the given waveform (W). Use a single process. Assume that a clock with a 1 μs period is available as an input.



4.9 A BCD adder adds two BCD numbers (each of range 0 to 9) and produces the sum in BCD form. For example, if it adds 9 (1001) and 8 (1000) the result would be 17 (1 0111). Implement such a BCD adder using a 4-bit binary adder and appropriate control circuitry. Assume that the two BCD numbers are already loaded into two 4-bit registers (A and B), and there is a 5-bit sum register (S) available. You need some kind of correction to get the sum in the BCD form, because the binary adder produces results in the range 0000 to 1111 (plus a carry in some cases). If any addition is required for this correction, use the same adder (i.e., you can use only one adder). Use multiplexers at the adder inputs to steer the appropriate numbers to the adder in each cycle. Assume a start signal to initiate the addition and a done signal to indicate completion.

(a) Draw a block diagram of the system. Label each component appropriately to indicate its functionality and size.
(b) Describe step-by-step the algorithm that you would use to perform the addition. Explain and illustrate the correction step.
(c) Draw a state graph for the controller.

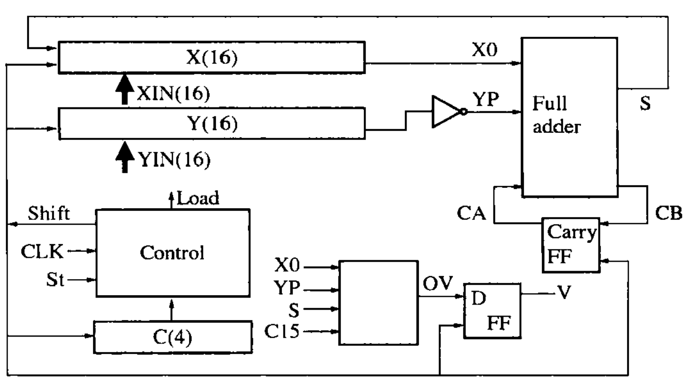
4.10 Write VHDL code for a shift register module that includes a 16-bit shift register, a controller, and a 4-bit down counter. The shifter can shift a variable number of bits depending on a count provided to the shifter module. Inputs to the module are a number N (indicating shift count) in the range 1 to 15, a 16-bit vector *par_in*, a clock, and a start signal, St. When St = '1', N is loaded into the down counter, and *par_in* is loaded into the shift register. Then the shift register does a cycle left shift N times, and the controller returns to the start state. Assume that St is only '1' for one clock time. All operations are synchronous on the falling edge of the clock.

(a) Draw a block diagram of the system and define any necessary control signals.
(b) Draw a state graph for the controller (two states).
(c) Write VHDL code for the shift-register module. Use two processes (one for the combinational part of the circuit, and one for updating the registers).

4.11 **(a)** Figure 4-12 shows the block diagram for a 32-bit serial adder with accumulator. The control circuit uses a 5-bit counter, which outputs a signal $K = 1$ when it is in state 11111. When a start signal (St) is received, the registers should be loaded. Assume that St will remain 1 until the addition is complete. When the addition is complete, the control circuit should go to a stop state and remain there until St is changed back to 0. Draw a state diagram for the control circuit (excluding the counter).
(b) Write the VHDL for the complete system, and verify its correct operation.

4.12 A block diagram for a 16-bit 2's complement serial subtracter is given here. When $St = 1$, the registers are loaded and then subtraction occurs. The shift counter, C, produces a signal $C15 = 1$ after 15 shifts. V should be set to 1 if an overflow occurs. Set the carry flip-flop to 1 during load in order to form the 2's complement. Assume that St remains 1 for one clock time.

(a) Draw a state diagram for the control (two states).
(b) Write VHDL code for the system. Use two processes. The first process should determine the next state and control signals; the second process should update the registers on the rising edge of the clock.



4.13 This problem involves the design of a BCD to binary converter. Initially a three-digit BCD number is placed in the *A* register. When a *St* signal is received, conversion to binary takes place, and the resulting binary number is stored in the *B* register. At each step of the conversion, the entire BCD number (along with the binary number) is shifted one place to the right. If the result in a given decade is greater than or equal 1000, the correction circuit subtracts 0011 from that decade. (If the result is less than 1000, the correction circuit leaves the contents of the decade unchanged.) A shift counter is provided to count the number of shifts. When conversion is complete, the maximum value of *B* will be 999 (in binary). *Note: B* is 10 bits.

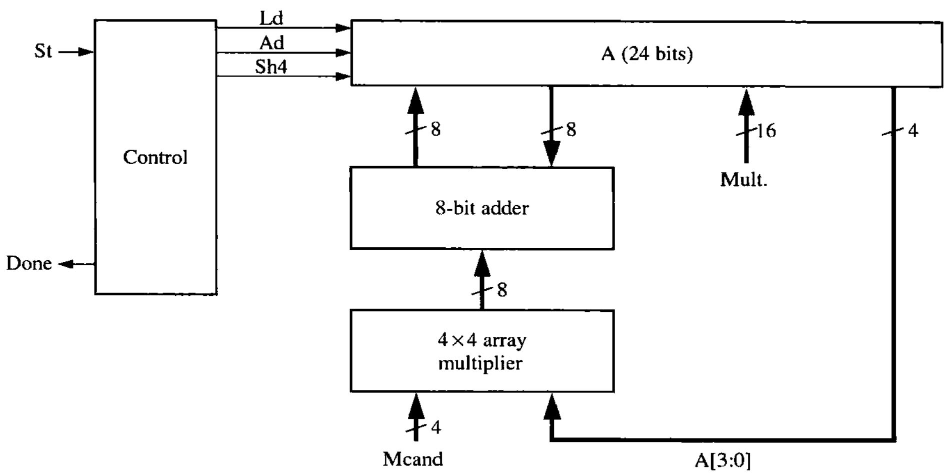
(a) Illustrate the algorithm starting with the BCD number 857, showing *A* and *B* at each step.
(b) Draw the block diagram of the BCD-to-binary converter.
(c) Draw a state diagram of the control circuit (three states). Use the following control signals: *St*: start conversion; *Sh*: shift right; *Co*: subtract correction if necessary; and *C9*: counter is in state 9, or *C10*: counter is in state 10. (Use either *C9* or *C10* but not both.)
(d) Write a VHDL description of the system.

4.14 This problem involves the design of a circuit that finds the square root of an 8-bit unsigned binary number *N* using the method of subtracting out odd integers. To find the square root of *N*, we subtract 1, then 3, then 5, and so on, until we can no longer subtract without the result going negative. The number of times we subtract is equal to the square root of *N*. For example, to find $\sqrt{27}$: $27 - 1 = 26$; $26 - 3 = 23$; $23 - 5 = 18$; $18 - 7 = 11$; $11 - 9 = 2$; $2 - 11$ (can't subtract). Since we subtracted five times, $\sqrt{27} = 5$. Note that the final odd integer is $11_{10} = 1011_2$, and this consists of the square root ($101_2 = 5_{10}$) followed by a 1.

(a) Draw a block diagram of the square rooter that includes a register to hold *N*, a subtracter, a register to hold the odd integers, and a control circuit. Indicate where to read the final square root. Define the control signals used on the diagram.
(b) Draw a state graph for the control circuit using a minimum number of states. The *N* register should be loaded when $St = 1$. When the square root is complete, the control circuit should output a done signal and wait until $St = 0$ before resetting.

4.15 This problem concerns the design of a multiplier for unsigned binary numbers that multiplies a 4-bit number by a 16-bit number to give a 20-bit product. To speed up the multiplication, a 4-by-4 array multiplier is used so that we can multiply by 4 bits in one clock time instead of only by 1 bit at each clock time. The hardware includes a 24-bit accumulator register that can be shifted right 4 bits at a time using a control signal *Sh4*. The array multiplier multiplies 4 bits by 4 bits to give an 8-bit product. This product is added to the accumulator using an *Ad* control signal. When a *St* signal occurs, the 16-bit multiplier is loaded into the lower part of the *A* register. A done signal should be turned on when the multiplication is complete. Since both the array multiplier and adder are combinational circuits, the 4-bit multiply and the 8-bit add can both be completed in the same clock cycle. Do NOT include the array

multiplier logic in your code, just use the overloaded "*" operator. If D and E are 4-bit unsigned numbers, $D * E$ will compute an 8-bit product.



(a) Draw a state graph for the controller (10 states)

(b) Write VHDL code for the multiplier. Use two processes (a combinational process and a clocked process). All signals should be of type unsigned or bit.

4.16 **(a)** Estimate how many AND gates and adders will be required for a 16-bit × 16-bit array multiplier.

(b) What is the longest delay in a 16 × 16 array multiplier, assuming an AND gate delay is t_g, and adder delay (full adder and half adder) is t_{ad}?

4.17 **(a)** Draw the organization of an 8 × 8 array multiplier and calculate how many full adders, half-adders, and AND gates are required.

(b) Highlight the critical path in your answer to (a) (If there are many equivalent ones, highlight any one of them.)

(c) What is the longest delay in an 8 × 8 array multiplier, assuming an AND gate delay is t_g = 1 ns, and adder delay (full adder and half adder) is t_{ad} = 2 ns?

(d) For an 8-bit × 8-bit add-and-shift multiplier (similar to Figure 4-25), how fast must the clock be in order to complete the multiplication in the same time as in part (c)?

4.18 An $n \times n$ array multiplier, as in Figure 4-29, takes $3n - 4$ adder delays + 1 gate delay to calculate a product. Design an array multiplier which is faster than this for $n > 4$. (*Hint*: Instead of passing carry output to the left adder, pass it to the diagonally lower one, speeding up the critical path. This topology is called "multiplier using carry-save adder.")

4.19 The block diagram for a multiplier for signed (2's complement) binary numbers is shown in Figure 4-33. Give the contents of the A and B registers after each clock pulse when multiplicand = $-1/8$ and multiplier = $-3/8$.

4.20 In Section 4.10 we developed an algorithm for multiplying signed binary fractions, with negative fractions represented in 2's complement.

(a) Illustrate this algorithm by multiplying 1.0111 by 1.101.

(b) Draw a block diagram of the hardware necessary to implement this algorithm for the case where the multiplier is 4 bits, including sign, and the multiplicand is 5 bits, including sign.

4.21 The objective of this problem is to use VHDL to describe and simulate a multiplier for signed binary numbers using Booth's algorithm. Negative numbers should be represented by their 2's complement. Booth's algorithm works as follows, assuming each number is n bits including sign: Use an $(n + 1)$-bit register for the accumulator (A) so the sign bit will not be lost if an overflow occurs. Also, use an $(n + 1)$-bit register (B) to hold the multiplier and an n-bit register (C) to hold the multiplicand.

1. Clear A (the accumulator), load the multiplier into the upper n bits of B, clear B_0, and load the multiplicand into C.
2. Test the lower two bits of B (B_1B_0).
   If $B_1B_0 = 01$, then add C to A (C should be sign-extended to $n + 1$ bits and added to A using an $(n + 1)$-bit adder).
   If $B_1B_0 = 10$, then add the 2's complement of C to A.
   If $B_1B_0 = 00$ or 11, skip this step.
3. Shift A and B together right one place with sign extended.
4. Repeat steps 2 and 3, $n - 1$ more times.
5. The product will be in A and B, except ignore B_0.

Example for $n = 5$: Multiply -9 by -13.

	A	B	B_1B_0	
1. Load registers.	000000	100110	10	$C = 10111$
2. Add 2's comp. of C to A.	$\underline{001001}$			
	001001	100110		
3. Shift $A\&B$.	000100	110011	11	
3. Shift $A\&B$.	000010	011001	01	
2. Add C to A.	$\underline{110111}$			
	111001	011001		
3. Shift $A\&B$.	111100	101100	00	
3. Shift $A\&B$.	111110	010110	10	
2. Add 2's comp. of C to A.	$\underline{001001}$			
	000111	010110		
3. Shift $A\&B$.	000011	101011		

Final result: 0001110101 = +117

(a) Draw a block diagram of the system for $n = 8$. Use 9-bit registers for A and B, a 9-bit full adder, an 8-bit complementer, a 3-bit counter, and a control circuit. Use the counter to count the number of shifts.

(b) Draw a state graph for the control circuit. When the counter is in state 111, return to the start state at the time the last shift occurs (three states should be sufficient).

(c) Write behavioral VHDL code for the multiplier.

(d) Simulate your VHDL design using the following test cases (in each pair, the second number is the multiplier):

$$01100110 \times 00110011$$
$$10100110 \times 01100110$$
$$01101011 \times 10001110$$
$$11001100 \times 10011001$$

Verify that your results are correct.

4.22 Design a multiplier that will multiply two 16-bit signed binary integers to give a 32-bit product. Negative numbers should be represented in 2's complement form. Use the following method: First complement the multiplier and multiplicand if they are negative, multiply the positive numbers, and then complement the product if necessary. Design the multiplier so that after the registers are loaded, the multiplication can be completed in 16 clocks.

(a) Draw a block diagram of the multiplier. Use a 4-bit counter to count the number of shifts. (The counter will output a signal $K = 1$ when it is in state 15.) Define all condition and control signals used on your diagram.

(b) Draw a state diagram for the multiplier control using a minimum number of states (five states). When the multiplication is complete, the control circuit should output a done signal and then wait for $ST = 0$ before returning to state S_0.

(c) Write a VHDL behavioral description of the multiplier without using control signals (for example, see Figure 4-35) and test it.

(d) Write a VHDL behavioral description using control signals (for example, see Figure 4-40) and test it.

4.23 This problem involves the design of a parallel adder-subtracter for 8-bit numbers expressed in sign and magnitude notation. The inputs X and Y are in sign and magnitude, and the output Z must be in sign and magnitude. Internal computation may be done in either 2's complement or 1's complement (specify which you use), but no credit will be given if you assume the inputs X and Y are in 1's or 2's complement. If the input signal $Sub = 1$, then $Z = X - Y$, else $Z = X + Y$. Your circuit must work for all combinations of positive and negative inputs for both add and subtract. You may use only the following components: an 8-bit adder, a 1's complementer (for the input Y), a second complementer (which may be either 1's complement or 2's complement—specify which you use), and a combinational logic circuit to generate control signals. (*Hint*: $-X + Y = -(X - Y)$. Also generate an overflow signal that is 1 if the result cannot be represented in 8-bit sign and magnitude.)

(a) Draw the block diagram. No registers, multiplexers, or tristate busses are allowed.

(b) Give a truth table for the logic circuit that generates the necessary control signals. Inputs for the table should be Sub, Xs, and Ys in that order, where Xs is the sign of X and Ys is the sign of Y.

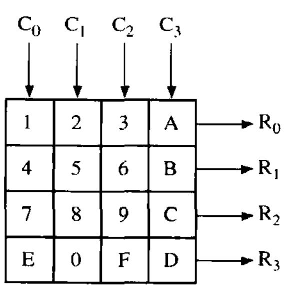
(c) Explain how you would determine the overflow and give an appropriate equation.

4.24 Four push buttons (B_0, B_1, B_2, and B_3) are used as inputs to a logic circuit. Whenever a button is pushed, it is debounced and then the circuit loads the button number in binary into a 2-bit register (N). For example, if B_2 is pushed, the register output becomes $N = 10_2$. The register holds this value until another button is pushed. Use a total of two flip-flops for debouncing. Use a 10-bit counter as a clock divider to provide a slow clock for debouncing. Kd is a signal which is 1 when any button has been pushed and debounced.

(a) Draw a state graph (two states) to generate the signal that loads the register when $Kd = 1$.

(b) Draw a logic circuit diagram showing the 10-bit counter, the 2-bit register N, and all necessary gates and flip-flops.

4.25 Design a 4×4 keypad scanner for the following keypad layout.



(a) Assuming only one key can be pressed at a time, find the equations for a number decoder given R_{3-0} and C_{3-0}, whose output corresponds to the binary value of the key. For example, the F key will return $N_{3-0} = 1111$ in binary, or 15.

(b) Design a debouncing circuit that detects when a key has been pressed or depressed. Assume switch bounce will die out in one or two clock cycles. When a key has been pressed, $K = 1$ and Kd is the debounced signal.

(c) Design and draw a state graph that performs the keyscan and issues a valid pulse when a valid key has been pressed using inputs from part (b).

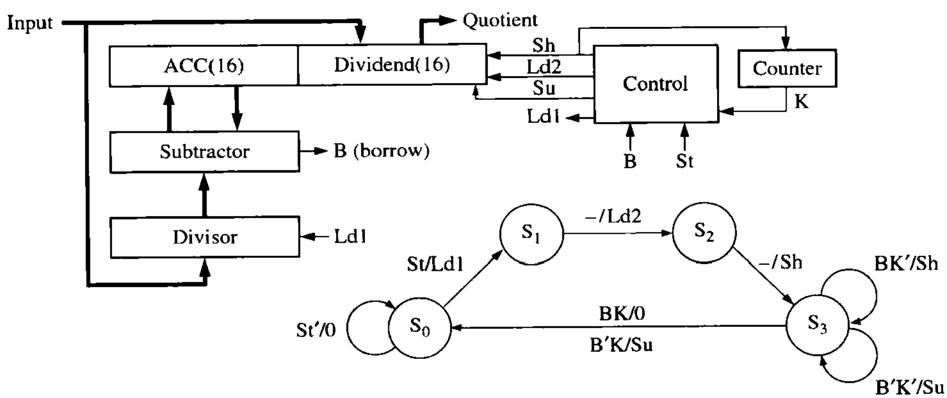
(d) Write a VHDL description of your keypad scanner and include the decoder, debouncing circuit, and scanner.

4.26 This problem concerns the design of a divider for unsigned binary numbers that will divide a 16-bit dividend by an 8-bit divisor to give an 8-bit quotient. Assume that the start signal ($ST = 1$) is 1 for exactly one clock time. If the quotient would require more than 8 bits, the divider should stop immediately and output $V = 1$ to indicate an overflow. Use a 17-bit dividend register and store the quotient in the lower 8 bits

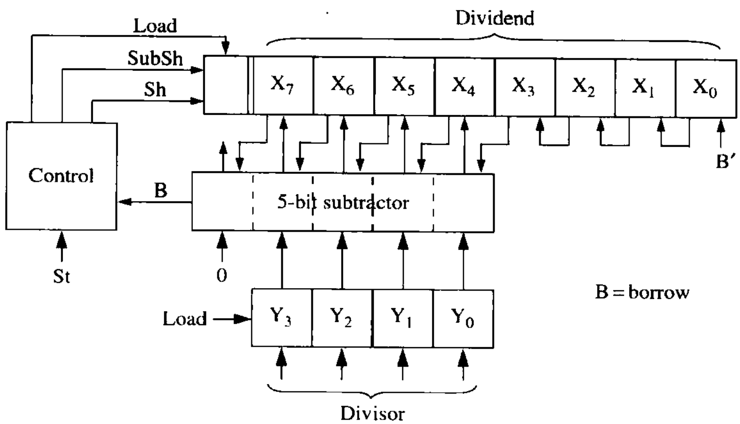
of this register. Use a 4-bit counter to count the number of shifts, together with a subtract-shift controller.

(a) Draw a block diagram of the divider.
(b) Draw a state graph for the subtract-shift controller (three states).
(c) Write a VHDL description of the divider. Use two processes, similar to Figure 4-40.
(d) Write a test bench for your divider (similar to Figure 4-55).

4.27 A block diagram and state graph for a divider for unsigned binary numbers is shown below. This divider divides a 16-bit dividend by a 16-bit divisor to give a 16-bit quotient. The divisor can be any number in the range 1 to $2^{16} - 1$. The only case where an overflow can occur is when the divisor is 0. Control signals are defined as follows: *Ld*1: load the divisor from the input bus; *Ld*2: load the dividend from the input bus and clear ACC; *Sh*: left shift ACC & Dividend; *Su*: load the subtractor output into ACC and set the lower quotient bit to 1; $K = 1$ when 16 shifts have been made. Write complete VHDL code for the divider. All signals must be of type unsigned or bit. Use two processes.



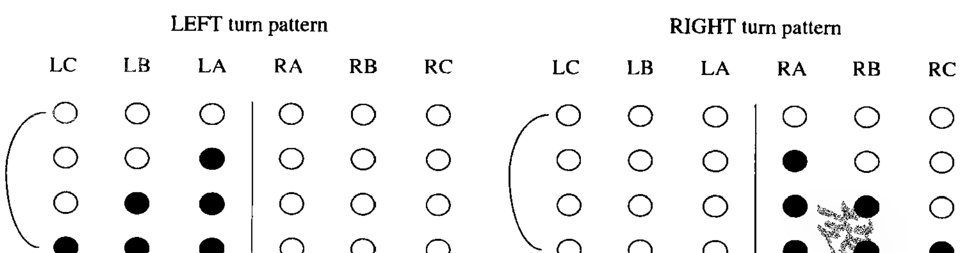
4.28 A block diagram for a divider that divides an 8-bit unsigned number by a 4-bit unsigned number to give a 4-bit quotient is given below. Note that the X_i inputs to the subtractors are shifted over one position to the left. This means that the shift-and-subtract operation can be completed in one clock time instead of two. Depending on the borrow from the subtractor, a shift or shift-and-subtract operation occurs at each clock time, and the division can always be completed in four clock times after the registers are loaded. Ignore overflow. When the start signal (*St*) is 1, the *X* and *Y* registers are loaded. Assume that the start signal (*St*) is 1 for only one clock time. *Sh* causes *X* to shift left with 0 fill. *SubSh* causes the subtractor output to be loaded into the left part of *X*, and at the same time the rest of *X* is shifted left.



(a) Draw a state graph for the controller (5 states).
(b) Complete the VHDL code given below. Registers and signals should be of type unsigned so that overloaded operators may be used. Write behavioral code that uses a single process.

```
library IEEE;
use IEEE.numeric_bit.all;

entity divu is
  port(dividend: in unsigned(7 downto 0);
       divisor: in unsigned(3 downto 0);
       St, clk: in bit;
       quotient: out unsigned(3 downto 0));
end entity divu;

architecture div of divu is
```

4.29 An older model Thunderbird car has three left (LA, LB, LC) and three right (RA, RB, RC) tail lights which flash in unique patterns to indicate left and right turns.



Design a Moore sequential circuit to control these lights. The circuit has three inputs *LEFT*, *RIGHT*, and *HAZ*. *LEFT* and *RIGHT* come from the driver's turn signal switch and cannot be 1 at the same time. As indicated above, when *LEFT* = 1

the lights flash in a pattern *LA* on; *LA* and *LB* on; *LA*, *LB*, and *LC* on; all off; and then the sequence repeats. When *RIGHT* = 1, a similar sequence appears on lights RA, RB, and RC, as indicated on the right side of the picture. If a switch from *LEFT* to *RIGHT* (or vice versa) occurs in the middle of a flashing sequence, the circuit should immediately go to the IDLE (lights off) state and then start the new sequence. *HAZ* comes from the hazard switch, and when *HAZ* = 1, all six lights flash on and off in unison. *HAZ* takes precedence if *LEFT* or *RIGHT* is also on.

Assume that a clock signal is available with a frequency equal to the desired flashing rate.

(a) Draw the state graph (eight states).
(b) Realize the circuit using six D flip-flops, and make a one-hot state assignment such that each flip-flop output drives one of the six lights directly. (You may use *LogicAid*.)
(c) Realize the circuit using three D flip-flops, using the guidelines from Section 1.7 to determine a suitable encoded state assignment. Note the tradeoff between more flip-flops and more gates in (b) and (c).

4.30 Design a sequential circuit to control the motor of a tape player. The logic circuit will have five inputs and three outputs. Four of the inputs are the control buttons on the tape player. The input *PL* is 1 if the play button is pressed, the input *RE* is 1 if the rewind button is pressed, the input *FF* is 1 if the fast forward button is pressed, and the input *ST* is 1 if the stop button is pressed. The fifth input to the control circuit is *M*, which is 1 if the special "music sensor" detects music at the current tape position. The three outputs of the control circuit are *P*, *R*, and *F*, which make the tape play, rewind, and fast forward, respectively, when 1. No more than one output should ever be on at a time; all outputs off causes the motor to stop. The buttons control the tape as follows: If the play button is pressed, the tape player will start playing the tape (output $P = 1$). If the play button is held down and the rewind button is pressed and released, the tape player will rewind to the beginning of the current song (output $R = 1$ until $M = 0$) and then start playing. If the play button is held down and the fast forward button is pressed and released, the tape player will fast forward to the end of the current song (output $F = 1$ until $M = 0$) and then start playing. If rewind or fast forward is pressed while play is released, the tape player will rewind or fast forward the tape. Pressing the stop button at any time should stop the tape player motor.

(a) Construct a state graph chart for the tape player controller. You may assume that only one of the four buttons can be pressed at any given time.
(b) Write VHDL code for the controller.

CHAPTER 5

SM Charts and Microprogramming

A state machine is often used to control a digital system that carries out a step-by-step procedure or algorithm. State diagrams or state graphs with circles representing states and arcs representing transitions have traditionally been used to specify the operation of the controller state machine. As an alternative to using state graphs, a special type of flow chart, called a *state machine chart*, or ***SM chart***, may be used to describe the behavior of a state machine. These charts are also called *algorithmic state machine charts*, or ***ASM charts***. SM charts are often used to design control units for digital systems.

In this chapter, we first describe the properties of SM charts and how they are used in the design of state machines. Then we show examples of SM charts for a multiplier and a dice game controller. We construct VHDL descriptions of these systems from the SM charts, and we simulate the VHDL code to verify correct operation. We then proceed with the design and show how the SM chart can be realized with hardware. We then introduce **microprogramming** as a technique to implement the SM chart.

5.1 State Machine Charts

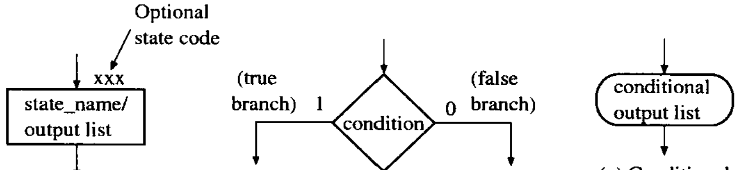
SM charts resemble software flow charts. Flow charts have been very useful in software design for decades, and in a similar fashion, SM charts have been useful in hardware design. This is especially true in behavioral-level design entry.

SM charts offer several advantages over state graphs. It is often easier to understand the operation of a digital system by inspection of the SM chart instead of the equivalent state graph. A proper state graph has to obey some conditions: (1) One and exactly one transition from a state must be true at any time, and (2) the next state must be uniquely defined for every input combination. These conditions are automatically satisfied for an SM chart. An SM chart also directly leads to a hardware realization. A given SM chart can be converted into several equivalent forms, and different forms might naturally result in different implementations. Hence, a designer may optimize and transform SM charts to suit the implementation style/technology that he or she is looking for.

An SM chart differs from an ordinary flow chart in that certain specific rules must be followed in constructing the SM chart. When these rules are followed, the SM chart is equivalent to a state graph, and it leads directly to a hardware realization.

Figure 5-1 shows the three principal components of an SM chart. The state of the system is represented by a *state box*. The state box contains a *state name*, followed by a slash (/) and an optional *output list*. After a state assignment has been made, a *state code* may be placed outside the box at the top. A *decision box* is represented by a diamond-shaped symbol with true and false branches. The *condition* placed in the box is a Boolean expression that is evaluated to determine which branch to take. The *conditional output box*, which has curved ends, contains a *conditional output list*. The conditional outputs depend on both the state of the system and the inputs.

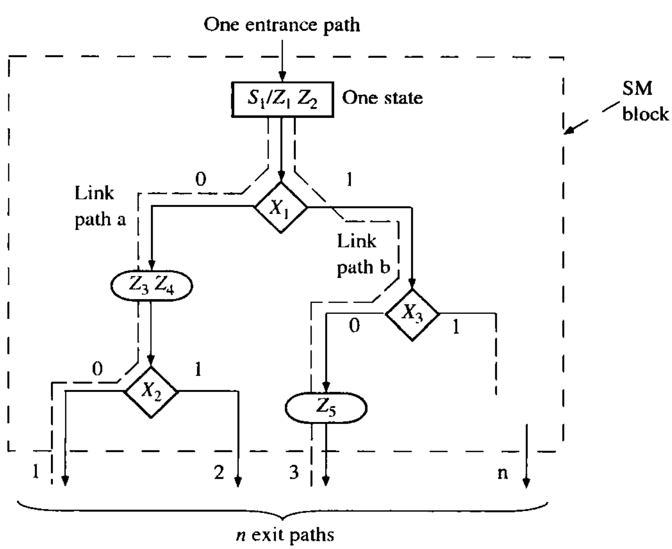
FIGURE 5-1: Principal Components of an SM Chart



(a) State box (b) Decision box (c) Conditional output box

An SM chart is constructed from *SM blocks*. Each SM block (Figure 5-2) contains exactly one state box, together with the decision boxes and conditional output boxes associated with that state. An SM block has one *entrance path* and one or more *exit paths*. Each SM block describes the machine operation during the time that the machine is in one state. When a digital system enters the state associated with a given SM block, the outputs on the output list in the state box become true. The conditions in the decision boxes are evaluated to determine which paths are followed through the SM block. When a conditional output box is encountered along such a path, the corresponding conditional outputs become true. If an output is not encountered along a path, that output is false by default. A path through an SM block from entrance to exit is referred to as a *link path*.

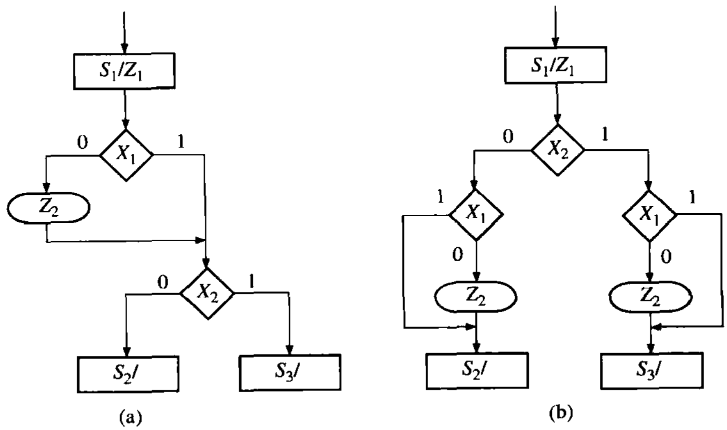
FIGURE 5-2: Example of an SM Block



For the example of Figure 5-2, when state S_1 is entered, outputs Z_1 and Z_2 become 1. If input $X_1 = 0$, Z_3 and Z_4 also become 1. If $X_1 = X_2 = 0$, at the end of the state time, the machine goes to the next state via exit path 1. On the other hand, if $X_1 = 1$ and $X_3 = 0$, the output Z_5 is 1, and exiting to the next state will occur via exit path 3. Since Z_3 and Z_4 are not encountered along this link path, $Z_3 = Z_4 = 0$ by default.

A given SM block can generally be drawn in several different forms. Figure 5-3 shows two equivalent SM blocks. In both (a) and (b), the output $Z_2 = 1$ if $X_1 = 0$; the next state is S_2 if $X_2 = 0$ and S_3 if $X_2 = 1$. As illustrated in this example, the order in which the inputs are tested may affect the complexity of the SM chart.

FIGURE 5-3: Equivalent SM Blocks

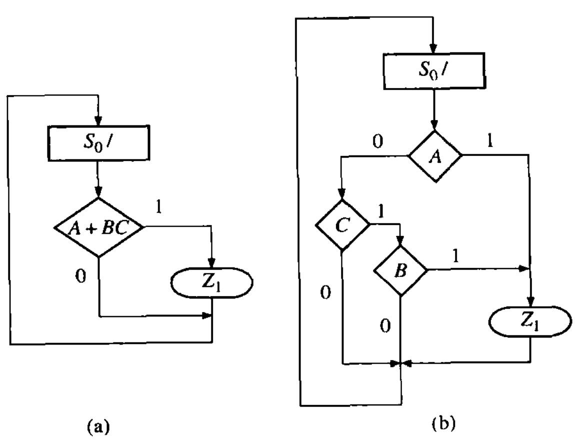


The SM charts of Figures 5-4(a) and (b) each represent a combinational circuit, since there is only one state and no state change occurs. The output is $Z_1 = 1$ if $A + BC = 1$; otherwise $Z_1 = 0$. Figure 5-4(b) shows an equivalent SM chart in which the input variables are tested individually. The output is $Z_1 = 1$ if $A = 1$ or if $A = 0$, $B = 1$, and $C = 1$. Hence

$$Z_1 = A + A'BC = A + BC$$

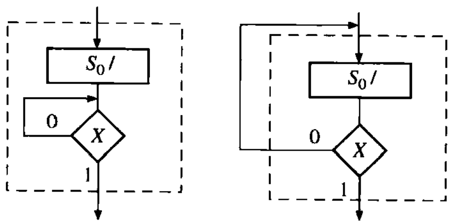
which is the same output function realized by the SM chart of Figure 5-4(a).

FIGURE 5-4: Equivalent SM Charts for a Combinational Circuit



Certain rules must be followed when constructing an SM block. First, for every valid combination of input variables, there must be exactly one exit path defined. This is necessary since each allowable input combination must lead to a single next state. Second, no internal feedback within an SM block is allowed. Figure 5-5 shows incorrect and correct ways of drawing an SM block with feedback.

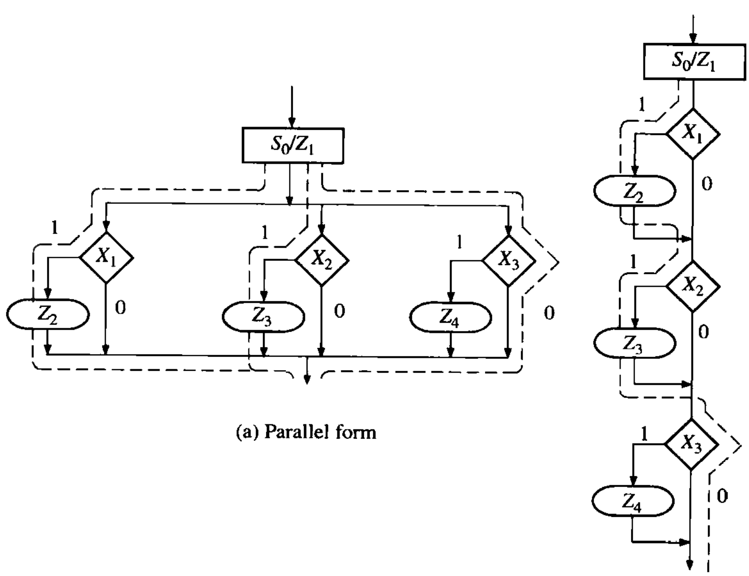
FIGURE 5-5: SM Block with Feedback



(a) Incorrect (b) Correct

As shown in Figure 5-6(a), an SM block can have several parallel paths that lead to the same exit path, and more than one of these paths can be active at the same time. For example, if $X_1 = X_2 = 1$ and $X_3 = 0$, the link paths marked with dashed lines are active, and the outputs Z_1, Z_2, and Z_3 are 1. Although Figure 5-6(a) would not be a valid flow chart for a program for a serial computer, it presents no problems for a state machine implementation. The state machine can have a multiple-output circuit that generates Z_1, Z_2, and Z_3 at the same time. Figure 5-6(b) shows a serial SM block, which is equivalent to Figure 5-6(a). In the serial block, only one active link path between entrance and exit is possible. For any combination of input values, the outputs will be the same as in the equivalent parallel form. The link path for $X_1 = X_2 = 1$ and $X_3 = 0$

FIGURE 5-6: Equivalent SM Blocks



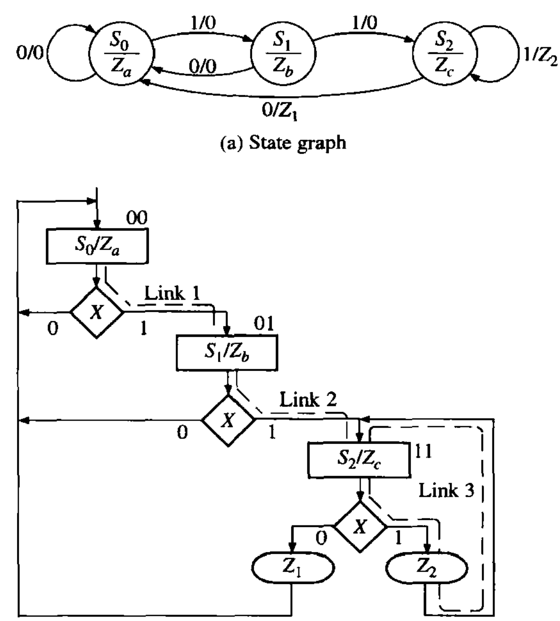
(a) Parallel form

(b) Serial form

is shown with a dashed line, and the outputs encountered on this path are Z_1, Z_2, and Z_3. Regardless of whether the SM block is drawn in serial or parallel form, all the tests take place within one clock time. In the rest of this text, we use only the serial form for SM charts.

It is easy to convert a state graph for a sequential machine to an equivalent SM chart. The state graph of Figure 5-7(a) has both Moore and Mealy outputs. The equivalent SM chart has three blocks—one for each state. The Moore outputs (Z_a, Z_b, Z_c) are placed in the state boxes, since they do not depend on the input. The Mealy outputs (Z_1, Z_2) appear in conditional output boxes, since they depend on both the state and input. In this example, each SM block has only one decision box, since only one input variable must be tested. For both the state graph and SM chart, Z_c is always 1 in state S_2. If $X = 0$ in state S_2, $Z_1 = 1$ and the next state is S_0. If $X = 1$, $Z_2 = 1$ and the next state is S_2. We have added a state assignment ($S_0 = 00$, $S_1 = 01$, $S_2 = 11$) next to the state boxes.

FIGURE 5-7: Conversion of a State Graph to an SM Chart

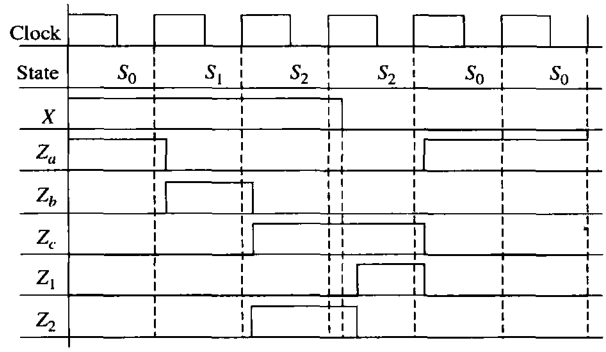


(a) State graph

(b) Equivalent SM chart

Figure 5-8 shows a timing chart for the SM chart of Figure 5-7 with an input sequence $X = 1, 1, 1, 0, 0, 0$. In this example, all state changes occur immediately after the rising edge of the clock. Since the Moore outputs (Z_a, Z_b, Z_c) depend on the state, they can change only immediately following a state change. The Mealy outputs (Z_1, Z_2) can change immediately after a state change or an input change. In any case, all outputs will have their correct values at the time of the active clock edge.

FIGURE 5-8: Timing Chart for Figure 5-7



5.2 Derivation of SM Charts

The method used to derive an SM chart for a sequential control circuit is similar to that used to derive the state graph. First, we should draw a block diagram of the system we are controlling. Next, we should define the required input and output signals to the control circuit. Then we can construct an SM chart that tests the input signals and generates the proper sequence of output signals. In this section, we give two examples of derivation of SM charts.

5.2.1 Binary Multiplier

The first example is an SM chart for control of the binary multiplier shown in Figures 4-25 and 4-28(a). The add-shift control generates the required sequence of add and shift signals. The counter counts the number of shifts and outputs $K = 1$ just before the last shift occurs. The SM chart for the multiplier control (Figure 5-9) corresponds closely to the state graph of Figure 4-28(c). In state S_0, when the start signal *St* is 1, the registers are loaded. In S_1, the multiplier bit M is tested. If $M = 1$, an add signal is generated and the next state is S_2. If $M = 0$, a shift signal is generated and K is tested. If $K = 1$, this will be the last shift and the next state is S_3. In S_2, a shift signal is generated, since a shift must always follow an add. If $K = 1$, the circuit goes to S3 at the time of the last shift; otherwise, the next state is S_1. In S_3, the done signal is turned on.

Conversion of an SM chart to a VHDL process is straightforward. A **case** statement can be used to specify what happens in each state. Each condition box corresponds directly to an **if** statement (or an **elsif**). Figure 5-10 shows the VHDL code for the SM chart in Figure 5-9. Two processes are used. The first process represents the combinational part of the circuit, and the second process updates the state register on the rising edge of the clock. The signals *Load*, *Sh*, and *Ad* are turned on in the appropriate states, and they must be turned off when the state changes. A convenient way to do this is to set them all to 0 at the start of the process. This VHDL code only models the controller. It assumes the presence of adders and shifters (shift registers) in the architecture and generates the appropriate signals to load the registers, to add and/or to shift.

FIGURE 5-9: SM Chart for Binary Multiplier

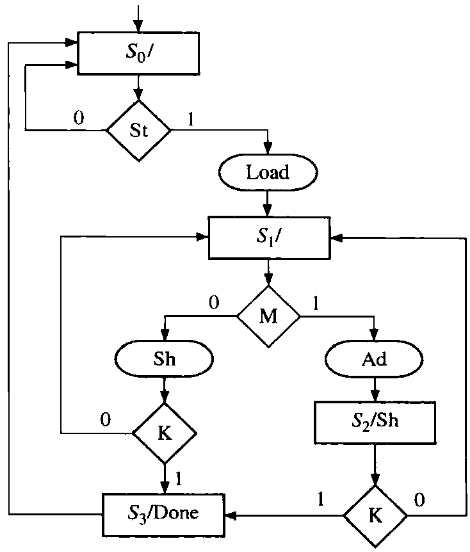


FIGURE 5-10: Behavioral VHDL for Multiplier Controller (SM Chart of Figure 5-9)

```
entity Mult is
  port(CLK, St, K, M: in bit;
      Load, Sh, Ad, Done: out bit);
end Mult;

architecture SMbehave of Mult is
signal State, Nextstate: integer range 0 to 3;
begin
  process(St, K, M, State)                -- start if state or inputs change
  begin
    Load <= '0'; Sh <= '0'; Ad <= '0'; Done <= '0';
    case State is
      when 0 =>
        if St = '1' then                  -- St (state 0)
          Load <= '1';
          Nextstate <= 1;
        else Nextstate <= 0;              -- St'
        end if;
      when 1 =>
        if M = '1' then                   -- M (state 1)
          Ad <= '1';
          Nextstate <= 2;
        else                              -- M'
          Sh <= '1';
          if K = '1' then Nextstate <= 3;   -- K
          else Nextstate <= 1;              -- K'
          end if;
        end if;
```

```
      when 2 =>
        Sh <= '1';                              -- (state 2)
        if K = '1' then Nextstate <= 3;         -- K
        else Nextstate <= 1;                    -- K'
        end if;
      when 3 =>
        Done <= '1';                            -- (state 3)
        Nextstate <= 0;
    end case;
  end process;
  process(CLK)
  begin
    if CLK = '1' and CLK'event then
      State <= Nextstate;                       -- update state on rising edge
    end if;
  end process;
end SMbehave;
```

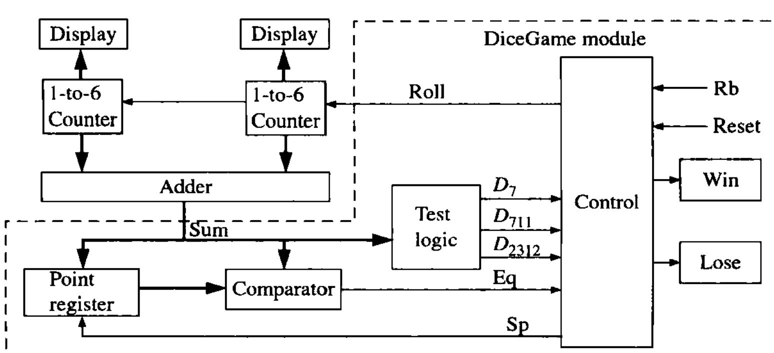
5.2.2 A Dice Game

As a second example of SM chart construction, we will design an electronic dice game. This game is popularly known as craps in the United States. The game involves two dice, each of which can have a value between 1 and 6. Two counters are used to simulate the roll of the dice. Each counter counts in the sequence 1, 2, 3, 4, 5, 6, 1, 2, ... Thus, after the "roll" of the dice, the sum of the values in the two counters will be in the range 2 through 12. The rules of the game are as follows:

1. After the first roll of the dice, the player wins if the sum is 7 or 11. The player loses if the sum is 2, 3, or 12. Otherwise, the sum the player obtained on the first roll is referred to as a point, and he or she must roll the dice again.
2. On the second or subsequent roll of the dice, the player wins if the sum equals the point, and he or she loses if the sum is 7. Otherwise, the player must roll again until he or she finally wins or loses.

Figure 5-11 shows the block diagram for the dice game. The inputs to the dice game come from two push buttons, *Rb* (roll button) and *Reset*. *Reset* is used to

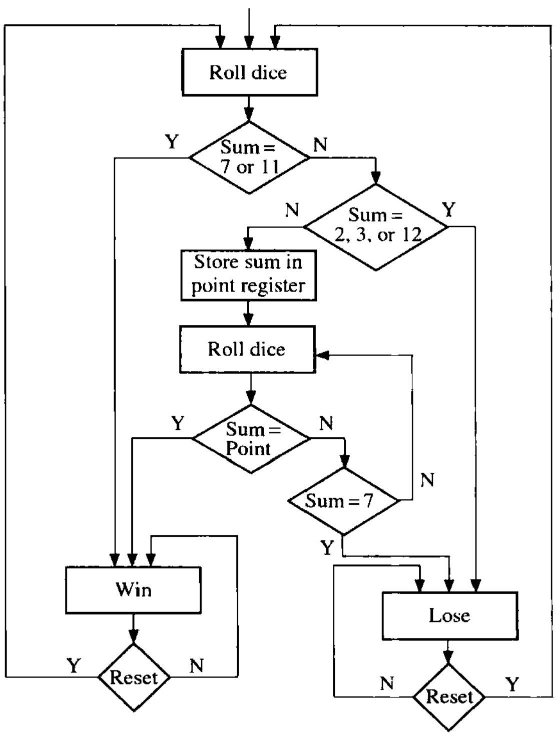
FIGURE 5-11: Block Diagram for Dice Game



initiate a new game. When the roll button is pushed, the dice counters count at a high speed, so the values cannot be read on the display. When the roll button is released, the values in the two counters are displayed.

Figure 5-12 shows a flow chart for the dice game. After rolling the dice, the sum is tested. If it is 7 or 11, the player wins; if it is 2, 3, or 12, he or she loses. Otherwise the sum is saved in the point register, and the player rolls again. If the new sum equals the point, the player wins; if it is 7, he or she loses. Otherwise, the player rolls again. If the *Win* light or *Lose* light is not on, the player must push the roll button again. After winning or losing, he or she must push *Reset* to begin a new game. We will assume at this point that the push buttons are properly debounced and that changes in *Rb* are properly synchronized with the clock. A method for debouncing and synchronization was discussed in Chapter 4.

FIGURE 5-12: Flow Chart for Dice Game



The components for the dice game shown in the block diagram (Figure 5-11) include an adder, which adds the two counter outputs, a register to store the point, test logic to determine conditions for win or lose, and a control circuit. Input signals to the control circuit are defined as follows:

D_7 = 1 if the sum of the dice is 7

D_{711} = 1 if the sum of the dice is 7 or 11

D_{2312} = 1 if the sum of the dice is 2, 3, or 12
Eq = 1 if the sum of the dice equals the number stored in the point register
Rb = 1 when the roll button is pressed
Reset = 1 when the reset button is pressed

Outputs from the control circuit are defined as follows:

Roll = 1 enables the dice counters
Sp = 1 causes the sum to be stored in the point register
Win = 1 turns on the win light
Lose = 1 turns on the lose light

The *Rb* and *Roll* signals may look synonymous; however, they are different. We are using electronic dice counters, and *Roll* is the signal to let the counters continue to count. *Rb* is a push-button signal requesting that the dice be rolled. Thus, *Rb* is an input to the control circuit, while *Roll* is an output from the control circuit. When the control circuit is in a state looking for a new roll of the dice, whenever the push button is pressed (i.e., *Rb* is activated), the control circuit will generate the *Roll* signal to the electronic dice.

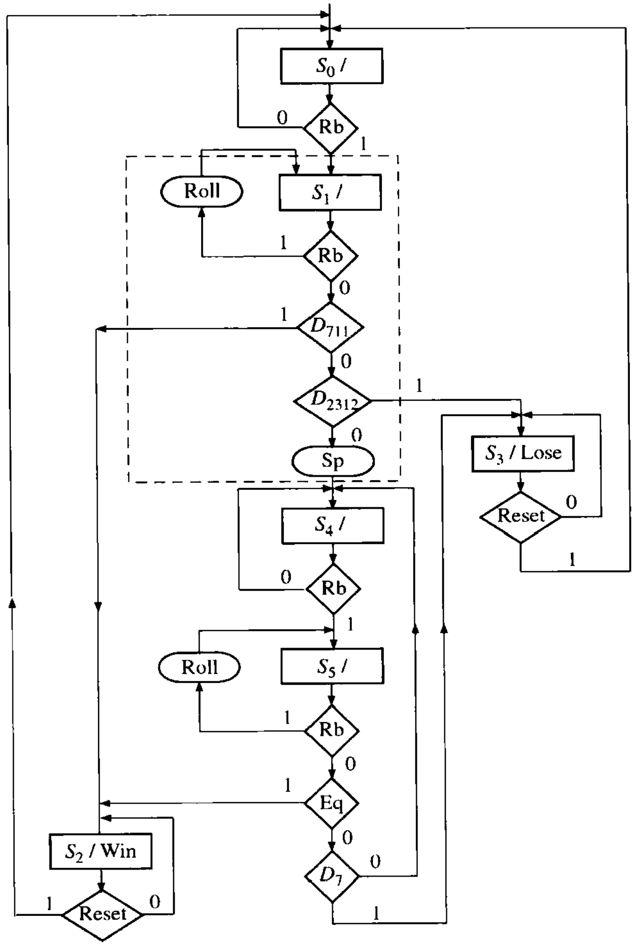
We now convert the flow chart for the dice game to an SM chart for the control circuit using the control signals defined above. Figure 5-13 shows the resulting SM chart.

The control circuit waits in state S_0 until the roll button is pressed ($Rb = 1$). Then, it goes to state S_1, and the roll counters are enabled as long as $Rb = 1$. As soon as the roll button is released ($Rb = 0$), D_{711} is tested. If the sum is 7 or 11, the circuit goes to state S_2 and turns on the *Win* light; otherwise, D_{2312} is tested. If the sum is 2, 3, or 12, the circuit goes to state S_3 and turns on the *Lose* light; otherwise, the signal *Sp* becomes 1 and the sum is stored in the point register. It then enters S_4 and waits for the player to "roll the dice" again. In S_5, after the roll button is released, if $Eq = 1$, the sum equals the point and state S_2 is entered to indicate a win. If $D_7 = 1$, the sum is 7 and S_3 is entered to indicate a loss. Otherwise, control returns to S_4 so that the player can roll again. When in S_2 or S_3, the game is reset to S_0 when the *Reset* button is pressed.

Instead of using an SM chart, we could construct an equivalent state graph from the flow chart. Figure 5-14 shows a state graph for the dice game controller. The state graph has the same states, inputs, and outputs as the SM chart. The arcs have been labeled consistently with the rules for proper state graphs given in Section 4.5. Thus, the arcs leaving state S1 are labeled Rb, $Rb'D_{711}$, $Rb'D'_{711}D_{2312}$, and $Rb'D'_{711}D'_{2312}$.

Before proceeding with the design, it is important to verify that the SM chart (or state graph) is correct. We will write a behavioral VHDL description based on the SM chart and then write a test bench to simulate the roll of the dice. Initially, we will write a dice game module that contains the control circuit, point register, and comparator (see Figure 5-11). Later, we will add the counters and adder so that we can simulate the complete dice game.

FIGURE 5-13: SM Chart for Dice Game



The VHDL code for the dice game in Figure 5-15 corresponds directly to the SM chart of Figure 5-13. The **case** statement in the first **process** tests the state, and in each state nested **if-then-else** (or **elsif**) statements are used to implement the conditional tests. In State 1 the *Roll* signal is turned on when *Rb* is 1. If all conditions test false, *Sp* is set to 1 and the next state is 4. In the second **process**, the state is updated after the rising edge of the clock, and if *Sp* is 1, the sum is stored in the point register.

We are now ready to test the behavioral model of the dice game. It is not convenient to include the counters that generate random numbers in the initial test, since we want to specify a sequence of dice rolls that will test all paths on the SM chart. We could prepare a simulator command file that would generate a sequence of data for *Rb*, *Sum*, and *Reset*. This would require careful analysis of the timing to make sure that the input signals change at the proper time. A better approach for testing the dice game is to design a VHDL test bench module to monitor the output signals from the dice game module and supply a sequence of inputs in response.

FIGURE 5-14: State Graph for Dice Game Controller

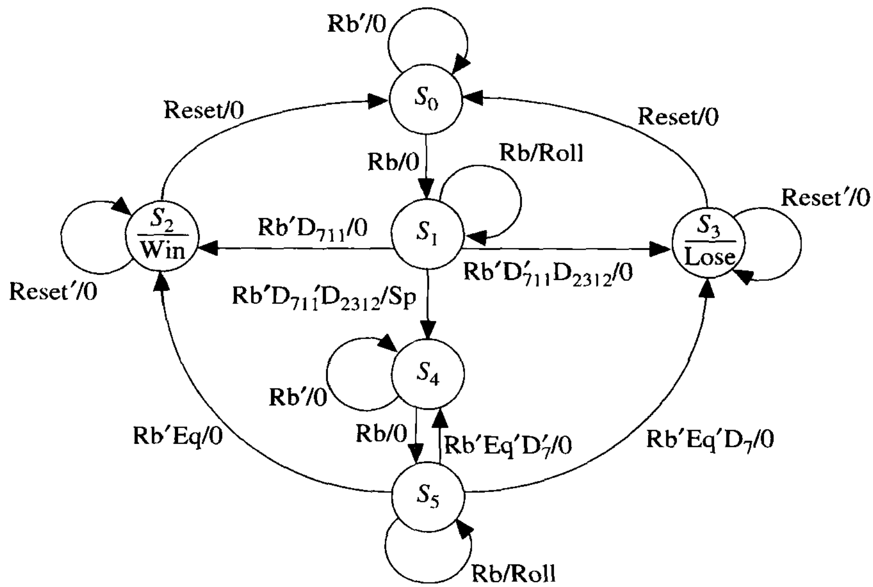


FIGURE 5-15: Behavioral Model for Dice Game Controller

```
entity DiceGame is
  port(Rb, Reset, CLK: in bit;
       Sum: in integer range 2 to 12;
       Roll, Win, Lose: out bit);
end DiceGame;

architecture DiceBehave of DiceGame is
signal State, Nextstate: integer range 0 to 5;
signal Point: integer range 2 to 12;
signal Sp: bit;
begin
  process(Rb, Reset, Sum, State)
  begin
    Sp <= '0'; Roll <= '0'; Win <= '0'; Lose <= '0';
    case State is
      when 0 => if Rb = '1' then Nextstate <= 1; end if;
      when 1 =>
        if Rb = '1' then Roll <= '1';
        elsif Sum = 7 or Sum = 11 then Nextstate <= 2;
        elsif Sum = 2 or Sum = 3 or Sum = 12 then Nextstate <= 3;
        else Sp <= '1'; Nextstate <= 4;
        end if;
      when 2 => Win <= '1';
        if Reset = '1' then Nextstate <= 0; end if;
      when 3 => Lose <= '1';
        if Reset = '1' then Nextstate <= 0; end if;
```

```
        when 4 => if Rb = '1' then Nextstate <= 5; end if;
        when 5 =>
          if Rb = '1' then Roll <= '1';
          elsif Sum = Point then Nextstate <= 2;
          elsif Sum = 7 then Nextstate <= 3;
          else Nextstate <= 4;
          end if;
      end case;
    end process;

    process(CLK)
    begin
      if CLK'event and CLK = '1' then
        State <= Nextstate;
        if Sp = '1' then Point <= Sum; end if;
      end if;
    end process;
  end DiceBehave;
```

Figure 5-16 shows the DiceGame connected to a module called GameTest. GameTest needs to perform the following functions:

1. Initially supply the *Rb* signal.
2. When the DiceGame responds with a *Roll* signal, supply a *Sum* signal, which represents the sum of the two dice.
3. If no *Win* or *Lose* signal is generated by the DiceGame, repeat steps 1 and 2 to roll again.
4. When a *Win* or *Lose* signal is detected, generate a *Reset* signal and start again.

FIGURE 5-16: Dice Game with Test Bench

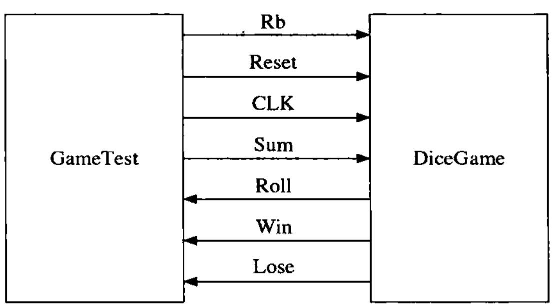


Figure 5-17 shows an SM chart for the GameTest module. *Rb* is generated in state T_0. When DiceGame detects *Rb*, it goes to S_1 and generates *Roll*. When GameTest detects *Roll*, the *Sum* that represents the next roll of the dice is read from *Sumarray(i)* and *i* is incremented. When the state goes to T_1, *Rb* goes to 0. The DiceGame goes to S_2, S_3, or S_4 and GameTest goes to T_2. The *Win* and *Lose* outputs are tested in state T_2. If *Win* or *Lose* is detected, a *Reset* signal is generated before the next roll of the dice. After *N* rolls of the dice, GameTest goes to state T_3, and no further action occurs.

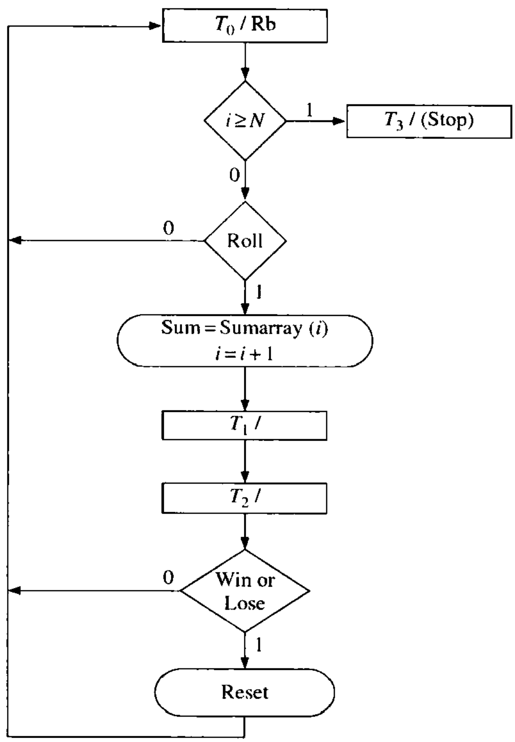


FIGURE 5-17: SM Chart for Dice Game Test

`GameTest` (Figure 5-18) implements the SM chart for the `GameTest` module. It contains an array of test data, a concurrent statement that generates the clock, and two processes. The first **process** generates *Rb, Reset*, and *Tnext* (the next state) whenever *Roll, Win, Lose*, or *Tstate* changes. The second **process** updates *Tstate* (the state of `GameTest`). When running the simulator, we want to display only one line of output for each roll of the dice. To facilitate this, we have added a signal *Trig1*, which changes every time state T_2 is entered.

Tester (Figure 5-19) connects the `DiceGame` and `GameTest` components so that the game can be tested. Figure 5-20 shows the simulator command file and output. The listing is triggered by *Trig1* once for every roll of the dice. The `run 2000` command runs for more than enough time to process all the test data.

FIGURE 5-18: Dice Game Test Module

```
entity GameTest is
  port(Rb, Reset: out bit;
       Sum: out integer range 2 to 12;
       CLK: inout bit;
       Roll, Win, Lose: in bit);
end GameTest;
```

```
architecture dicetest of GameTest is
signal Tstate, Tnext: integer range 0 to 3;
signal Trig1: bit;
type arr is array(0 to 11) of integer;
constant Sumarray:arr := (7, 11, 2, 4, 7, 5, 6, 7, 6, 8, 9, 6);
begin
  CLK <= not CLK after 20 ns;
  process(Roll, Win, Lose, Tstate)
  variable i: natural;                          -- i is initialized to 0
  begin
    case Tstate is
      when 0 => Rb <= '1';                      -- wait for Roll
        Reset <= '0';
        if i >= 12 then Tnext <= 3;
        elsif Roll = '1' then
          Sum <= Sumarray(i);
          i := i + 1;
          Tnext <= 1;
        end if;
      when 1 => Rb <= '0'; Tnext <= 2;
      when 2 => Tnext <= 0;
        Trig1 <= not Trig1;                     -- toggle Trig1
        if (Win or Lose) = '1' then
          Reset <= '1';
        end if;
      when 3 => null;                           -- Stop state
    end case;
  end process;

  process(CLK)
  begin
    if CLK = '1' and CLK'event then
      Tstate <= Tnext;
    end if;
  end process;
end dicetest;
```

FIGURE 5-19: Tester for DiceGame

```
entity tester is
end tester;

architecture test of tester is
component GameTest
  port(Rb, Reset: out bit;
       Sum: out integer range 2 to 12;
       CLK: inout bit;
       Roll, Win, Lose: in bit);
end component;
```

```
component DiceGame
  port(Rb, Reset, CLK: in bit;
       Sum: in integer range 2 to 12;
       Roll, Win, Lose: out bit);
end component;

signal rb1, reset1, clk1, roll1, win1, lose1: bit;
signal sum1: integer range 2 to 12;
begin
  Dice: Dicegame port map (rb1, reset1, clk1, sum1, roll1, win1, lose1);
  Dicetest: GameTest port map (rb1, reset1, sum1, clk1, roll1, win1, lose1);
end test;
```

FIGURE 5-20: Simulation and Command File for Dice Game Tester

```
add list /dicetest/trig1 -NOTrigger sum1 win1 lose1 /dice/point
run 2000

  ns   delta  trig1  sum1  win1  lose1  point
   0     +0     0      2     0     0      2
 100     +3     0      7     1     0      2
 260     +3     0     11     1     0      2
 420     +3     0      2     0     1      2
 580     +2     1      4     0     0      4
 740     +3     1      7     0     1      4
 900     +2     0      5     0     0      5
1060     +2     1      6     0     0      5
1220     +3     1      7     0     1      5
1380     +2     0      6     0     0      6
1540     +2     1      8     0     0      6
1700     +2     0      9     0     0      6
1860     +3     0      6     1     0      6
```

5.3 Realization of SM Charts

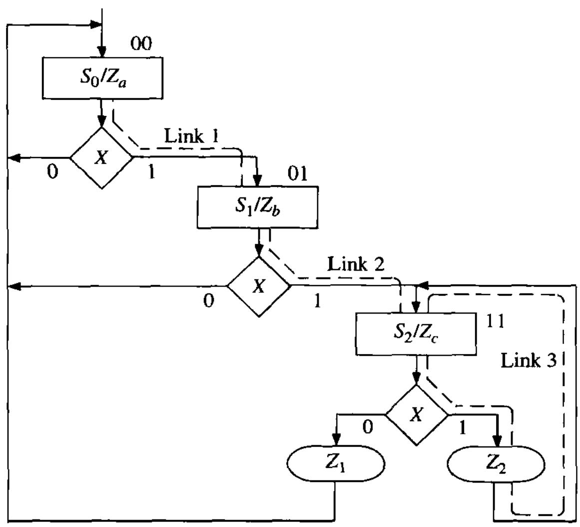
Methods used to realize SM charts are similar to the methods used to realize state graphs. As with any sequential circuit, the realization will consist of a combinational subcircuit, together with flip-flops for storing the state of the circuit. In some cases, it may be possible to identify equivalent states in an SM chart and eliminate redundant states using the same method as was used for reducing state tables. However, an SM chart is usually incompletely specified in the sense that all inputs are not tested in every state, which makes the reduction procedure more

difficult. Even if the number of states in an SM chart can be reduced, it is not always desirable to do so, since combining states may make the SM chart more difficult to interpret.

Before deriving next state and output equations from an SM chart, a state assignment must be made. The best way of making the assignment depends on how the SM chart is realized. If gates and flip-flops (or the equivalent PLD realization) are used, the guidelines for state assignment given in Section 1.7 may be useful. If programmable gate arrays are used, a one-hot assignment may be best.

As an example of realizing an SM chart, consider the SM chart in Figure 5-21.

FIGURE 5-21: Example SM Chart for Implementation



We have made the state assignment $AB = 00$ for S0, $AB = 01$ for S_1, and $AB = 11$ for S_2. After a state assignment has been made, output and next-state equations can be read directly from the SM chart. Since the Moore output Z_a is 1 only in state 00, $Z_a = A'B'$. Similarly, $Z_b = A'B$ and $Z_c = AB$. The conditional output $Z_1 = ABX'$, since the only link path through Z_1 starts with $AB = 11$ and takes the $X = 0$ branch. Similarly, $Z_2 = ABX$. There are three link paths (labeled link 1, link 2, and link 3 in Figure 5-21), which terminate in a state that has $B = 1$. Link 1 starts with a present state $AB = 00$, takes the $X = 1$ branch, and terminates on a state in which $B = 1$. Therefore, the next state of B (B^+) equals 1 when $A'B'X = 1$. Link 2 starts in state 01, takes the $X = 1$ branch, and ends in state 11, so B^+ has a term $A'BX$. Similarly, B^+ has a term ABX from link 3. The next state equation for B thus has three terms corresponding to the three link paths:

$$B^+ = \underbrace{A'B'X}_{\text{link 1}} + \underbrace{A'BX}_{\text{link 2}} + \underbrace{ABX}_{\text{link 3}}$$

Similarly, two link paths terminate in a state with $A = 1$, so

$$A^+ = A'BX + ABX$$

These output and next state equations can be simplified with Karnaugh maps using the unused state assignment ($AB = 10$) as a "don't care" condition.

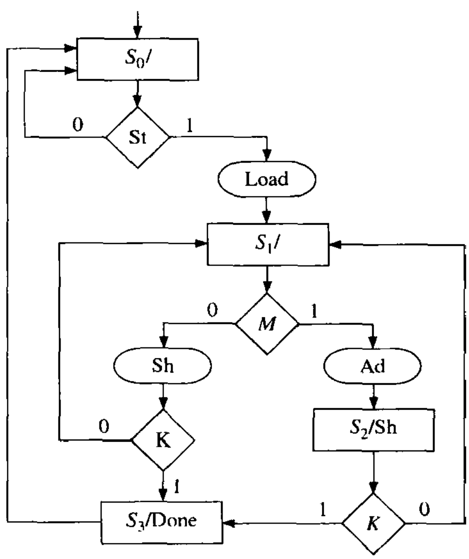
As illustrated above for flip-flops A and B, the procedure for deriving the next state equation for a flip-flop Q from the SM chart is as follows:

1. Identify all of the states in which $Q = 1$.
2. For each of these states, find all the link paths that lead *into* the state.
3. For each of these link paths, find a term that is 1 when the link path is followed. That is, for a link path from S_i to S_j, the term will be 1 if the machine is in state S_i and the conditions for exiting to S_j are satisfied.
4. The expression for Q^+ (the next state of Q) is formed by OR'ing together the terms found in step 3.

5.3.1 Implementation of Binary Multiplier Controller

Next, consider the SM chart for the multiplier control repeated here, in Figure 5-22.

FIGURE 5-22: SM Chart for Multiplier Controller



We can realize this SM chart with two D flip-flops and a combinational circuit. Let us assume that the state assignments are $AB = 00$ for S_0, $AB = 01$ for S_1, $AB = 10$ for S_2, and $AB = 11$ for S_3.

The logic equations for the multiplier control and the next state equations can be derived by tracing link paths on the SM chart and then simplifying the resulting equations. First, let us consider the control signals. *Load* is true only in S_0 and only if *St* is true. Hence, $Load = S_0St = A'B'St$. Similarly, *Ad* is true only in S_1 and only

if M is true. Hence, $Ad = A'BM$. *Done* is a Moore output in S_3, and hence $Done = S_3 = AB$. In summary, the logic equations for the multiplier control are

$$Load = A'B'St$$
$$Sh = A'BM'(K' + K) + AB'(K' + K) = A'BM' + AB'$$
$$Ad = A'BM$$
$$Done = AB$$

The next state equations can be derived by inspection of the SM chart and considering the state assignments. A is true in states S_2 and S_3. State S_2 is the next state when current state is S_1 and M is true ($A'BM$). State S_3 is the next state when current state is S_1, M is false, and K is true ($A'BM'K$) and when current state is S_2 and K is true ($AB'K$). Hence, we can write that

$$A^+ = A'BM'K + A'BM + AB'K = A'B(M + K) + AB'K$$

Similarly, we can derive the next state equation for B by inspection of the ASM diagram:

$$B^+ = A'B'St + A'BM'(K' + K) + AB'(K' + K) = A'B'St + A'BM' + AB'$$

The multiplier controller can be implemented in a hardwired fashion by two flip-flops and a few logic gates. The logic gates implement the next state equations and control signal equations. The circuit can be implemented with discrete gates or in a PLA, CPLD, or FPGA.

Table 5-1 illustrates a state transition table for the multiplier control. Each row in the table corresponds to one of the link paths in the SM chart. Since S_0 has two exit paths, the table has two rows for present state S_0. The first row corresponds to the $St = 0$ exit path, so the next state and outputs are 0. In the second row, $St = 1$, so the next state is 01 and the other outputs are 1000. Since St is not tested in states S_1, S_2, and S_3, St is a "don't care" in the corresponding rows. The outputs for each row can be filled in by tracing the corresponding link paths on the SM chart. For example, the link path from S_1 to S_2 passes through conditional output Ad, so $Ad = 1$ in this row. Since S_2 has a Moore output Sh, $Sh = 1$ in both of the rows for which $AB = 10$.

TABLE 5-1: State Transition Table for Multiplier Control

	A	B	St	M	K	A^+	B^+	Load	Sh	Ad	Done
S_0	0	0	0	—	—	0	0	0	0	0	0
	0	0	1	—	—	0	1	1	0	0	0
S_1	0	1	—	0	0	0	1	0	1	0	0
	0	1	—	0	1	1	1	0	1	0	0
	0	1	—	1	—	1	0	0	0	1	0
S_2	1	0	—	—	0	0	1	0	1	0	0
	1	0	—	—	1	1	1	0	1	0	0
S_3	1	1	—	—	—	0	0	0	0	0	1

The design may also be implemented with ROM. If it has to be implemented using the ROM method, we can calculate the size of the ROM as follows. There are

five different inputs to the combinational circuit here (A, B, St, M, and K). Hence, the ROM will have 32 entries. The combinational circuit should generate six signals (four control signals plus two next states). Hence, each entry has to be 6 bits wide. Thus, this design can be implemented using a 32 × 6 ROM and two D flip-flops. If the combinational logic is implemented with a PLA instead of a ROM, the PLA table is the same as the state transition table. The PLA would have 5 inputs, 6 outputs, and 8 product terms.

If a ROM is used, the table must be expanded to $2^5 = 32$ rows since there are five inputs. To expand the table, the dashes in each row must be replaced with all possible combinations of 0's and 1's. If a row has n dashes, it must be replaced with 2^n rows. For example, the fifth row in Table 5-1 would be replaced with the following 4 rows:

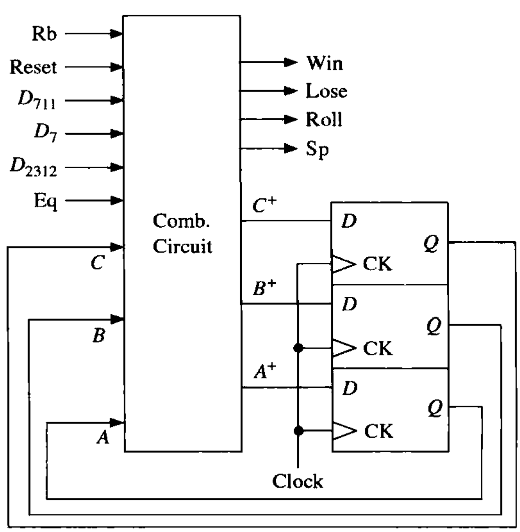
0	1	**0**	1	**0**	1	0	0	0	1	0
0	1	**0**	1	**1**	1	0	0	0	1	0
0	1	**1**	1	**0**	1	0	0	0	1	0
0	1	**1**	1	**1**	1	0	0	0	1	0

The added entries are printed in boldface.

5.4 Implementation of the Dice Game

We can realize the SM chart for the dice game (Figure 5-13) using combinational circuitry and three D flip-flops, as shown in Figure 5-23. We use a straight binary state assignment. The combinational circuit has nine inputs and seven outputs. Three of the inputs correspond to current state, and three of the outputs provide the next state information. All inputs and outputs are listed at the top of Table 5-2. The state

FIGURE 5-23: Realization of Dice Game Controller



transition table has one row for each link path on the SM chart. In state $ABC = 000$, the next state is $A^+B^+C^+ = 000$ or 001, depending on the value of Rb. Since state 001 has four exit paths, the table has four corresponding rows. When Rb is 1, *Roll* is 1 and there is no state change. When $Rb = 0$ and D_{711} is 1, the next state is 010. When $Rb = 0$ and $D_{2312} = 1$, the next state is 011. For the link path from state 001 to 100, Rb, D_{711}, and D_{2312} are all 0, and Sp is a conditional output. This path corresponds to row 4 of the state transition table, which has $Sp = 1$ and $A^+B^+C^+ = 100$. In state 010, the *Win* signal is always on, and the next state is 010 or 000, depending on the value of *Reset*. Similarly, *Lose* is always on in state 011. In state 101, $A^+B^+C^+ = 010$ if $Eq = 1$; otherwise, $A^+B^+C^+ = 011$ or 100, depending on the value of D_7. Since states 110 and 111 are not used, the next states and outputs are don't cares when $ABC = 110$ or 111.

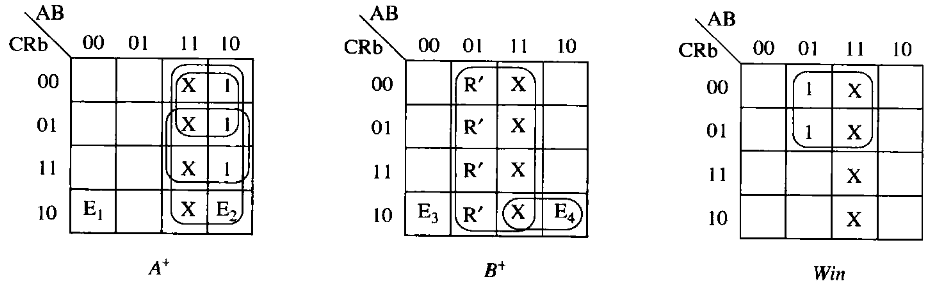
We can use Table 5-2 and derive equations for the control signals and the next state equations. The required equations can be derived from Table 5-2 using the method of map-entered variables (see Chapter 1) or using a CAD program such as *LogicAid*. These equations can also be derived by tracing link paths on the SM chart and then simplifying the resulting equations using the "don't care" next states.

Figure 5-24 shows K-maps for A^+, B^+, and *Win*, which were plotted directly from the table. Since A, B, C, and Rb have assigned values in most of the rows of the table, these four variables are used on the map edges, and the remaining variables are entered within the map. (Chapter 1 described the K-map technique that uses map-entered variables.) E_1, E_2, E_3, and E_4 on the maps represent the expressions given below the maps. From the A^+ column in the table, A^+ is 1 in row 4, so we should enter $D'_{711}D'_{2312}$ in the $ABCRb = 0010$ square of the map. To save space, we define $E_1 = D'_{711}D'_{2312}$ and place E_1 in the square. Since A^+ is 1 in rows 11, 12, and 16, 1's are placed on the map squares $ABCRb = 1000$, 1001, and 1011. From row 13, we place

TABLE 5-2: State Transition Table (PLA Table) for Dice Game

	ABC	Rb	Reset	D_7	D_{711}	D_{2312}	Eq	A^+	B^+	C^+	Win	Lose	Roll	Sp
1	000	0	—	—	—	—	—	0	0	0	0	0	0	0
2	000	1	—	—	—	—	—	0	0	1	0	0	0	0
3	001	1	—	—	—	—	—	0	0	1	0	0	1	0
4	001	0	—	—	0	0	—	1	0	0	0	0	0	1
5	001	0	—	—	0	1	—	0	1	1	0	0	0	0
6	001	0	—	—	1	—	—	0	1	0	0	0	0	0
7	010	—	0	—	—	—	—	0	1	0	1	0	0	0
8	010	—	1	—	—	—	—	0	0	0	1	0	0	0
9	011	—	1	—	—	—	—	0	0	0	0	1	0	0
10	011	—	0	—	—	—	—	0	1	1	0	1	0	0
11	100	0	—	—	—	—	—	1	0	0	0	0	0	0
12	100	1	—	—	—	—	—	1	0	1	0	0	0	0
13	101	0	—	0	—	—	0	1	0	0	0	0	0	0
14	101	0	—	1	—	—	0	0	1	1	0	0	0	0
15	101	0	—	—	—	—	1	0	1	0	0	0	0	0
16	101	1	—	—	—	—	—	1	0	1	0	0	1	0
17	110	—	—	—	—	—	—	—	—	—	—	—	—	—
18	111	—	—	—	—	—	—	—	—	—	—	—	—	—

FIGURE 5-24: Maps Derived from Table 5-2



$E_1 = D'_{711}D'_{2312}$
$E_2 = D'_7Eq'$
$E_4 = Eq + Eq'D_7 = Eq + D_7$

$R = Reset$
$E_3 = D_{711} + D'_{711}D_{2312} = D_{711} + D_{2312}$

$E_2 = D'_7Eq'$ in the 1010 square. In rows 7 and 8, *Win* is always 1 when $ABC = 010$, so 1's are plotted in the corresponding squares of the *Win* map.

The resulting equations are

$$A^+ = A'B'C\,Rb'D'_{711}D'_{2312} + AC' + ARb + AD'_7Eq' \quad (5\text{-}1)$$
$$B^+ = A'B'C\,Rb'(D_{711} + D_{2312}) + BReset' + AC\,Rb'(Eq + D_7)$$
$$C^+ = B'Rb + A'B'C\,D'_{711}D_{2312} + BC\,Reset' + AC\,D_7Eq'$$
$$Win = BC'$$
$$Lose = BC$$
$$Roll = B'CRb$$
$$Sp = A'B'C\,Rb'D'_{711}D'_{2312}$$

These equations can be implemented in any standard technology (using discrete gates, PALs, GALs, CPLDs, or FPGAs).

The dice game controller can also be realized using a ROM. A ROM (LUT) implementation of the game controller will need 512 entries (since there are 9 inputs). Each entry must be 7 bits wide (3 bits for next states and 4 bits for outputs). The ROM is very large because of the large number of inputs involved. The ROM method is hence not very desirable for state machines with a large number of inputs.

We now write a dataflow VHDL model for the dice game controller based on the block diagram of Figure 5-11 and Equations (5-1). The corresponding VHDL architecture is shown in Figure 5-25. The process updates the flip-flop states and the point register when the rising edge of the clock occurs. Generation of the control signals and *D* flip-flop input equations is done using concurrent statements. In particular, D_7, D_{711}, D_{2312}, and *Eq* are implemented using conditional signal assignments. As an alternative, all the signals and *D* input equations could have been implemented in a process with a sensitivity list containing *A, B, C, Sum, Point, Rb,* D_7, D_{711}, D_{2312}, *Eq*, and *Reset*. If the architecture of Figure 5-25 is used with the test bench of Figure 5-19, the results are identical to those obtained with the behavioral architecture in Figure 5-15.

FIGURE 5-25: Dataflow Model for Dice Game (Based on Equations (5-1))

```
architecture Dice_Eq of DiceGame is
signal Sp,Eq,D7,D711,D2312: bit:= '0';
signal DA,DB,DC,A,B,C: bit:='0';
signal Point: integer range 2 to 12;
begin
  process(CLK)
  begin
    if CLK = '1' and CLK'event then
      A <= DA; B <= DB; C <= DC;
      if Sp = '1' then Point <= Sum; end if;
    end if;
  end process;
  Win <= B and not C;
  Lose <= B and C;
  Roll <= not B and C and Rb;
  Sp <= not A and not B and C and not Rb and not D711 and not D2312;
  D7 <= '1' when Sum = 7 else '0';
  D711 <= '1' when (Sum = 11) or (Sum = 7) else '0';
  D2312 <= '1' when (Sum = 2) or (Sum = 3) or (Sum = 12) else '0';
  Eq <= '1' when Point = Sum else '0';
  DA <= (not A and not B and C and not Rb and not D711 and not D2312) or
        (A and not C) or (A and Rb) or (A and not D7 and not Eq);
  DB <= ((not A and not B and C and not Rb) and (D711 or D2312)) or
        (B and not Reset) or ((A and C and not Rb) and (Eq or D7));
  DC <= (not B and Rb) or (not A and not B and C and not D711 and D2312) or
        (B and C and not Reset) or (A and C and D7 and not Eq);
end Dice_Eq;
```

To complete the VHDL implementation of the dice game, we add two modulo-6 counters as shown in Figures 5-26 and 5-27. The counters are initialized to 1, so the sum of the two dice will always be in the range 2 through 12. When *Cnt1* is in state 6, the next clock sets it to state 1, and *Cnt2* is incremented (or *Cnt2* is set to 1 if it is in state 6).

FIGURE 5-26: Counter for Dice Game

```
entity Counter is
  port(Clk, Roll: in bit;
       Sum: out integer range 2 to 12);
end Counter;

architecture Count of Counter is
signal Cnt1, Cnt2: integer range 1 to 6:= 1;
begin
  process(Clk)
  begin
    if Clk = '1' then
      if Roll = '1' then
```

```
        if Cnt1 = 6 then Cnt1 <= 1; else Cnt1 <= Cnt1 + 1; end if;
        if Cnt1 = 6 then
          if Cnt2 = 6 then Cnt2 <= 1; else Cnt2 <= Cnt2 + 1; end if;
        end if;
      end if;
    end if;
  end process;
  Sum <= Cnt1 + Cnt2;
end Count;
```

FIGURE 5-27: Complete Dice Game

```
entity Game is
  port(Rb, Reset, Clk: in bit;
       Win, Lose: out bit);
end Game;

architecture Play1 of Game is
component Counter
  port(Clk, Roll: in bit;
       Sum: out integer range 2 to 12);
end component;

component DiceGame
  port(Rb, Reset, CLK: in bit;
       Sum: in integer range 2 to 12;
       Roll, Win, Lose: out bit);
end component;

signal roll1: bit;
signal sum1: integer range 2 to 12;
begin
  Dice: Dicegame port map (Rb, Reset, Clk, sum1, roll1, Win, Lose);
  Count: Counter port map (Clk, roll1, sum1);
end Play1;
```

This section has illustrated one way of realizing an SM chart. The implementation can use discrete gates, a PLA, a ROM, or a PAL. Alternative procedures are available that make it possible to reduce the size of the PLA or ROM by adding some components to the circuit. These methods are generally based on transformation of the SM chart to different forms and techniques, such as microprogramming.

In this chapter we described a procedure for digital system design based on SM charts. An SM chart is equivalent to a state graph, but it is usually easier to understand the system operation by inspection of the SM chart. After we have drawn a block diagram for a digital system, we can represent the control unit by an SM chart. Next we can write a behavioral VHDL description of the system based on this chart. Using a test bench written in VHDL, we can simulate the VHDL code to verify that the system functions according to specifications. After making any necessary corrections to the VHDL code and SM chart, we can proceed with the detailed logic design of the system. Rewriting the VHDL architecture to describe the system operation, in terms of control signals and logic equations, allows us to verify that our design is correct.

We also presented techniques for implementing control units: hardwiring. We showed how logic equations can easily be derived by tracing link paths on an SM chart. Hardwired control units can easily be implemented from these equations.

5.5 Problems

5.1 **(a)** Construct an SM chart equivalent to the following state table. Test only one variable in each decision box. Try to minimize the number of decision boxes.
(b) Write a VHDL description of the state machine based on the SM chart.

Present State	Next State $X_1X_2 =$ 00	01	10	11	Output (Z_1Z_2) $X_1X_2 =$ 00	01	10	11
S_0	S_3	S_2	S_1	S_0	00	10	11	01
S_1	S_0	S_1	S_2	S_3	10	10	11	11
S_2	S_3	S_0	S_1	S_1	00	10	11	01
S_3	S_2	S_2	S_1	S_0	00	00	01	01

5.2 Construct an SM chart that is equivalent to the following state table. Test only one variable in each decision box. Try to minimize the number of decision boxes. Show Mealy and Moore outputs on the SM chart.

Present State	Next State $X_1X_2 =$ 00	01	10	11	Output $(Z_1Z_2Z_3)$ $X_1X_2 =$ 00	01	10	11
S_0	S_1	S_1	S_1	S_1	000	100	110	010
S_1	S_1	S_1	S_0	S_0	001	001	001	001

5.3 An association has 15 voting members. Executive meetings of this association can be held only if more than half (i.e., at least 8) the members are present (i.e., 8 is the

minimum quorum required to hold meetings). Classified matters can be discussed and voted on only if two-thirds the members are present. The chairman can cast two votes if the quorum is met, but an even number of members (including the chairman) are present. Above the room door there are three lights, GREEN, BLUE, and RED, to indicate the quorum status. Derive an SM chart for a system that will indicate whether minimum quorum is met (GREEN), classified matters can be discussed (BLUE), or quorum met, but even members (RED). GREEN and RED lights may be present at the same time or GREEN, BLUE, and RED lights may be present simultaneously.

Assume that there is a single door to the meeting room and that it is fitted with two photocells. One photocell (PHOTO1) is on the inner side of the door and the other (PHOTO2) is on the outer side. Light beams shine on each photocell, producing a false output from the cell; a true output from a photocell arises when the light beam is interrupted. Assume that once a person starts through a door, the process is completed before another one can enter or leave (i.e., only one person enters or leaves at a time). If PHOTO1 is followed by PHOTO2, a sequencer generates a *LEAVE* signal and if PHOTO2 is followed by PHOTO1, the sequencer generates an *ENTER* signal. At most one *ENTER* or *LEAVE* will be true at any time. Assume that these signals will be true until you read them. Basically you read the signal and provide a signal to the door controller indicating that the door is *READY* to let the next person in or out.

(a) Draw a block diagram for the data section of this circuit. Assume that *ENTER* and *LEAVE* signals are available for you (i.e., you do not need to generate them for this part of the question).
(b) Draw an SM chart for the controller. Write the steps required to accomplish the design. Define all control signals used.
(c) Draw an SM chart for a circuit that generates *ENTER* and *LEAVE*.

5.4 **(a)** Draw the block diagram for a divider that divides an 8-bit dividend by a 5-bit divisor to give a 3-bit quotient. The dividend register should be loaded when $St = 1$.
(b) Draw an SM chart for the control circuit.
(c) Write a VHDL description of the divider based on your SM chart. Your VHDL should explicitly generate the control signals.
(d) Give a sequence of simulator commands that would test the divider for the case 93 divided by 17.

5.5 Draw an SM chart for the BCD to binary converter of Problem 4.13.

5.6 Draw an SM chart for the square root circuit of Problem 4.14.

5.7 Draw an SM chart for the binary multiplier of Problem 4.22.

5.8 Design a binary-to-BCD converter that converts a 10-bit binary number to a 3-digit BCD number. Assume that the binary number is ≤ 999. Initially the binary number is placed in register B. When a *St* signal is received, conversion to BCD takes place, and the resulting BCD number is stored in the A register (12 bits). Initially A contains

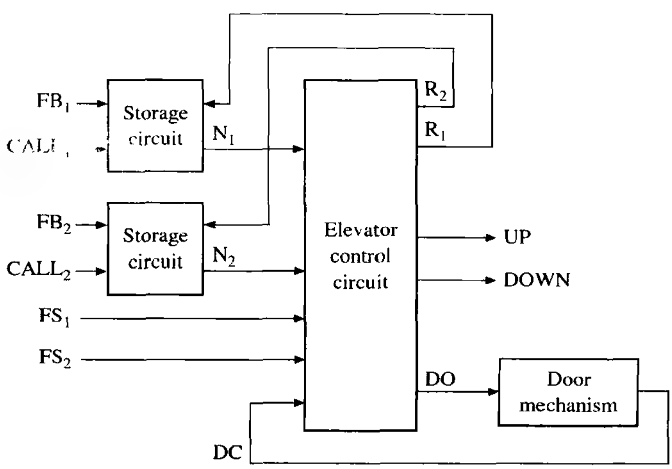
0000 0000 0000. The conversion algorithm is as follows: If the digit in any decade of A is $\geq$ 0101, add 0011 to that decade. Then shift the A register together with the B register one place to the left. Repeat until 10 shifts have occurred. At each step, as the left shift occurs, this effectively multiplies the BCD number by 2 and adds in the next bit of the binary number.

(a) Illustrate the algorithm by converting 100011101 to BCD.
(b) Draw the block diagram of the binary-to-BCD converter. Use a counter to count the number of shifts. The counter should output a signal C_{10} after 10 shifts have occurred.
(c) Draw an SM chart for the converter (three states).
(d) Write a VHDL description of the converter.

5.9 Design a multiplier for 16-bit binary integers. Use a design similar to Figures 4-33 and 4-34.

(a) Draw the block diagram. Add a counter to the control circuit to count the number of shifts.
(b) Draw the SM chart for the controller (three states). Assume that the counter outputs $K = 1$ after 15 shifts have occurred.
(c) Write VHDL code for your design.

5.10 The block diagram for an elevator controller for a building with two floors is shown below. The inputs FB_1 and FB_2 are floor buttons in the elevator. The inputs $CALL_1$ and $CALL_2$ are call buttons in the hall. The inputs FS_1 and FS_2 are floor switches that output a 1 when the elevator is at the first or second floor landing. Outputs UP and $DOWN$ control the motor, and the elevator is stopped when $UP = DOWN = 0$. N_1 and N_2 are flip-flops that indicate when the elevator is needed on the first or second floor. R_1 and R_2 are signals that reset these flip-flops. $DO = 1$ causes the door to open, and $DC = 1$ indicates that the door is closed. Draw an SM chart for the elevator controller (four states).



5.11 Write a test bench for the elevator controller of Problem 5.10. The test bench has two functions: to simulate the operation of the elevator (including the door operation) and to provide a sequence of button pushes to test the operation of the controller.

To simulate the elevator: If the elevator is on the first floor ($FS_1 = 1$) and an UP signal is received, wait 1 second and turn off FS_1; then wait 10 seconds and turn on FS_2; this simulates the elevator moving from the first floor to the second. Similar action should occur if the elevator is on the second floor ($FS_2 = 1$) and a $DOWN$ signal is received. When a door open signal is received ($DO = 1$), set door closed (DC) to 0, wait 5 seconds, and then set $DC = 1$.

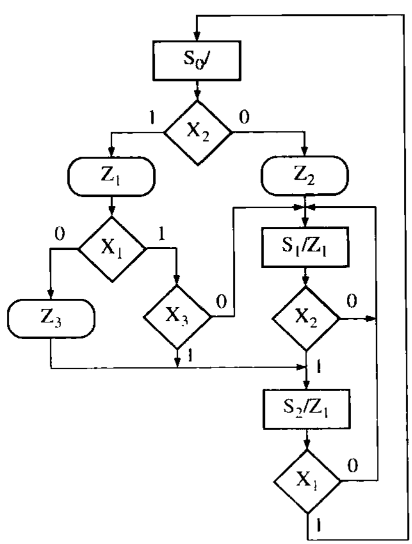
Test sequence: CALL1, 2, FB2, 4, FB1, 1, CALL2, 10, FB2.

Assume each button is held down for 1 s and then released. The numbers between buttons are the delays in seconds between button pushes; this delay is in addition to the 1 s the button is held down.

Complete the following test bench:

```
entity test_el is
end test_el;

architecture eltest of test_el is
  component elev_control
    port(CALL1, CALL2, FB1, FB2, FS1, FS2, DC, CLK: in bit;
         UP, DOWN, DO: out bit);
  end component;
```

5.12 For the following SM chart:



(a) Draw a timing chart that shows the clock, the state (S_0, S_1, or S_2), the inputs (X_1, X_2, and X_3), and the outputs. The input sequence is $X_1\ X_2\ X_3$ = 011, 101, 111,

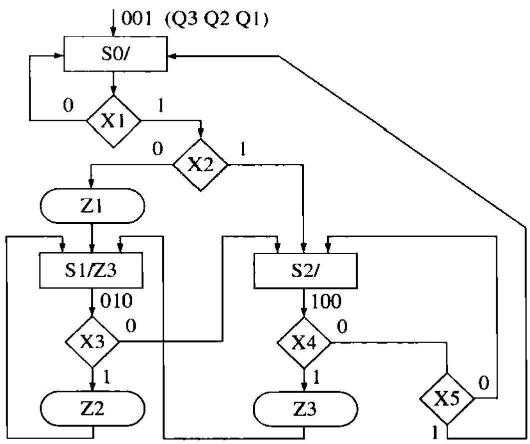
010, 110, 101, 001. Assume that all state changes occur on the rising edge of the clock, and the inputs change between clock pulses.

(b) Use the state assignment S_0: $AB = 00$; S_1: $AB = 01$; S_2: $AB = 10$. Derive the next state and output equations by tracing link paths. Simplify these equations using the don't care state ($AB = 11$).

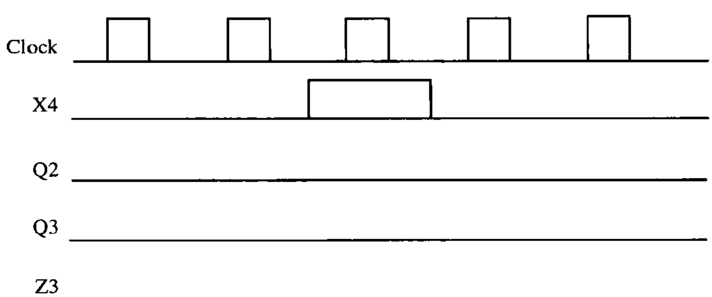
(c) Realize the chart using a PLA and D flip-flops. Give the PLA table (state transition table).

(d) If a ROM is used instead of a PLA, what size ROM is required? Give the first five rows of the ROM table. Assume a naïve ROM method is used (i.e., a full look-up table).

5.13 For the given SM chart:



(a) Complete the following timing diagram (assume that $X_1 = 1$, $X_2 = 0$, $X_3 = 0$, $X_5 = 1$, and X_4 is as shown). Flip-flops change state on falling edge of clock.

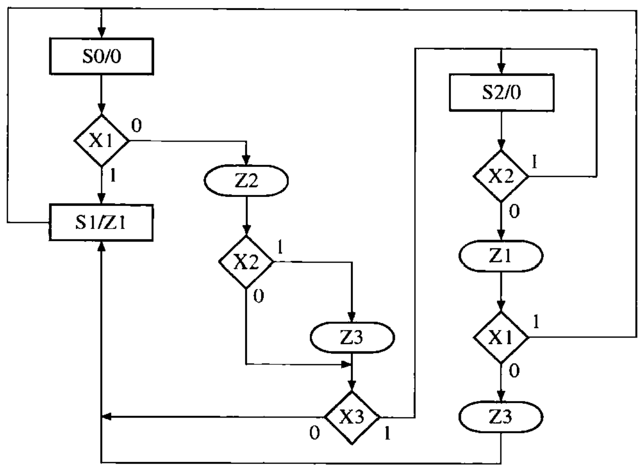


(b) Using the given one-hot state assignment, derive the minimum next state and output equations by inspection of the SM chart.

(c) Write a VHDL description of the digital system.

5.14 **(a)** Draw an SM chart that is equivalent to the state graph of Figure 4-46.
(b) If the SM chart is implemented using a PLA and three flip-flops (A, B, C), give the PLA table (state transition table). Use a straight binary state assignment.
(c) Give the equation for A^+ determined by inspection of the PLA table.
(d) If a one-hot state assignment is used, give the next-state and output equations.

5.15 **(a)** Write VHDL code that describes the following SM chart. Assume that state changes occur on the falling edge of the clock. Use two processes.



(b) The SM chart is to be implemented using a PLA and two flip-flops (A and B). Complete the following state transition table (PLA table) by tracing link paths. Find the equation for A^+ by inspection of the PLA table.

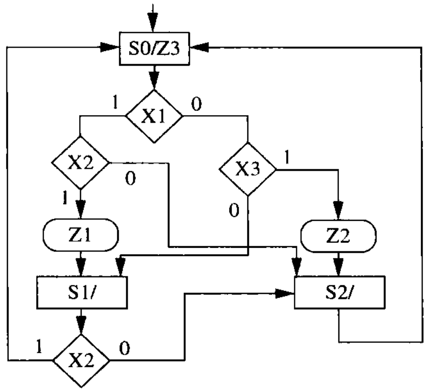
A	B	X_1	X_2	X_3	A^+	B^+	Z_1	Z_2	Z_3

(c) Complete the following timing diagram.

5.16 Realize the following SM chart using a ROM with a minimum number of inputs, a multiplexer, and a loadable counter (like the 74163). The ROM should generate NST. The multiplexer inputs are selected as shown in the table beside the SM chart.

(a) Draw the block diagram.
(b) Convert the SM chart to the proper format. Add a minimum number of extra states.

(c) Make a suitable state assignment and give the first five rows of the ROM table.
(d) Write a VHDL description of the system using a ROM.



T1 T2	
00	1
01	X1
10	X2
11	X3

CHAPTER 6

Floating-Point Arithmetic

Floating-point numbers are frequently used for numerical calculations in computing systems. Arithmetic units for floating-point numbers are considerably more complex than those for fixed-point numbers. Floating-point numbers allow very large or very small numbers to be specified. This chapter first describes a simple representation for floating-point numbers. Then it describes the IEEE floating-point standard. Next, an algorithm for floating-point multiplication is developed and tested using VHDL. Then the design of the floating-point multiplier is completed and implemented using an FPGA. Floating-point addition, subtraction, and division are also briefly described.

6.1 Representation of Floating-Point Numbers

A simple representation of a floating-point (or real) number (N) uses a fraction (F), base (B), and exponent (E), where $N = F \times B^E$. The base can be 2, 10, 16, or any other number. The fraction and the exponent can be represented in many formats. For example, they can be represented by 2's complement formats, sign-magnitude form, or another number representation. There are a variety of floating-point formats depending on how many bits are available for F and E, what the base is, and how negative numbers are represented for F and E. The base can be implied or explicit. Depending on all these choices, a wide variety of floating-point formats have existed in the past.

6.1.1 A Simple Floating-Point Format Using 2's Complement

In this section, we describe a floating-point format where negative exponents and fractions are represented using the 2's complement form. The base for the exponent is 2. Hence, the value of the number is $N = F \times 2^E$. In a typical floating-point number system, F is 16 to 64 bits long and E is 8 to 15 bits long. In order to keep the examples in this section simple and easy to follow, we will use a 4-bit fraction and a 4-bit exponent, but the concepts presented here can easily be extended to more bits.

The fraction and the exponent in this system will use 2's complement. (Refer to Section 4.10 for a discussion of 2's complement fractions.) We will use 4 bits for the fraction and 4 bits for the exponent. The fractional part will have a leading sign bit and three actual fraction bits. The implied binary point is after the first bit. The sign bit is 0 for positive numbers and 1 for negative numbers.

As an example, let us represent decimal 2.5 in this 8-bit 2's complement floating-point format.

$2.5 = 0010.1000$	
$= 1.010 \times 2^1$	(standardized normal representation)
$= 0.101 \times 2^2$	(4-bit 2's complement fraction)

Therefore,

$$F = 0.101 \qquad E = 0010 \qquad N = 5/8 \times 2^2$$

If the number was -2.5, the same exponent can be used, but the fraction must have a negative sign. The 2's complement representation for the fraction is 1.011. Therefore,

$$F = 1.011 \qquad E = 0010 \qquad N = -5/8 \times 2^2$$

Other examples of floating-point numbers using a 4-bit fraction and a 4-bit exponent are

$F = 0.101$	$E = 0101$	$N = 5/8 \times 2^5$
$F = 1.011$	$E = 1011$	$N = -5/8 \times 2^{-5}$
$F = 1.000$	$E = 1000$	$N = -1 \times 2^{-8}$

In order to utilize all the bits in F and have the maximum number of significant figures, F should be normalized so that its magnitude is as large as possible. If F is not normalized, we can normalize F by shifting it left until the sign bit and the next bit are different. Shifting F left is equivalent to multiplying by 2, so every time we shift we must decrement E by 1 to keep N the same. After normalization, the magnitude of F will be as large as possible, since any further shifting would change the sign bit. In the following examples, F is unnormalized to start with and then it is normalized by shifting left.

Unnormalized:	$F = 0.0101$	$E = 0011$	$N = 5/16 \times 2^3 = 5/2$
Normalized:	$F = 0.101$	$E = 0010$	$N = 5/8 \times 2^2 = 5/2$
Unnormalized:	$F = 1.11011$	$E = 1100$	$N = -5/32 \times 2^{-4} = -5 \times 2^{-9}$
(shift F left)	$F = 1.1011$	$E = 1011$	$N = -5/16 \times 2^{-5} = -5 \times 2^{-9}$
Normalized:	$F = 1.011$	$E = 1010$	$N = -5/8 \times 2^{-6} = -5 \times 2^{-9}$

The exponent can be any number between -8 and $+7$. The fraction can be any number between -1 and $+0.875$.

Zero cannot be normalized, so $F = 0.000$ when $N = 0$. Any exponent could then be used; however, it is best to have a uniform representation of 0. In this format, we will associate the negative exponent with the largest magnitude with the fraction 0. In a 4-bit 2's complement integer number system, the most negative number is 1000, which represents -8. Thus when F and E are 4 bits, 0 is represented by

$$F = 0.000 \qquad E = 1000 \qquad N = 0.000 \times 2^{-8}$$

Some floating-point systems use a biased exponent, so $E = 0$ is associated with $F = 0$.

6.1.2 The IEEE 754 Floating-Point Formats

The IEEE 754 is a floating-point standard established by IEEE in 1985. It contains two representations for floating-point numbers, the IEEE single precision format and the IEEE double precision format. The IEEE 754 single precision representation uses 32 bits and the double precision system uses 64 bits.

Although 2's complement representations are very common for negative numbers, the IEEE floating-point representations do not use 2's complement for either the fraction or the exponent. The designers of IEEE 754 desired a format that was easy to sort and hence adopted **a sign-magnitude system** for the **fractional part** and a **biased notation** for the **exponent**.

The IEEE 754 floating-point formats need three subfields: sign, fraction, and exponent. The fractional part of the number is represented using a sign-magnitude representation in the IEEE floating-point formats (i.e., there is an explicit sign bit (S) for the fraction). The sign is 0 for positive numbers and 1 for negative numbers. In a binary normalized scientific notation, the leading bit before the binary point is always 1 and hence the designers of the IEEE format decided to make it implied, representing only the bits after the binary point. In general, the number is of the form

$$N = (-1)^S \times (1 + F) \times 2^E$$

where S is the sign bit, F is the fractional part, and E is the exponent. The base of the exponent is 2. The base is implied (i.e., it is not stored anywhere in the representation). The magnitude of the number is $1 + F$ because of the omitted leading 1. The terms significand means the magnitude of the fraction and is $1 + F$ in the IEEE format. But often the terms *significand* and *fraction* are used interchangeably by many, including in this book.

The exponent in the IEEE floating-point formats uses what is known as a biased notation. A biased representation is one in which every number is represented by the number plus a certain bias. In the IEEE single precision format, the bias is 127. Hence, if the exponent is $+1$, it will be represented by $+1 + 127 = 128$. If the exponent is -2, it will be represented by $-2 + 127 = 125$. Thus, exponents less than 127 indicate actual negative exponents and exponents greater than 127 indicate actual positive exponents. The bias is 1023 in the double precision format.

If a positive exponent becomes too large to fit in the exponent field, the situation is called **overflow**, and if a negative exponent is too large to fit in the exponent field, that situation is called **underflow**.

The IEEE Single Precision Format

The IEEE single precision format uses 32 bits for representing a floating-point number, divided into three subfields, as illustrated in Figure 6-1. The first field is the sign bit for the fractional part. The next field consists of 8 bits which are used for the exponent. The third field consists of the remaining 23 bits and is used for the fractional part.

The sign bit reflects the sign of the fraction. It is 0 for positive numbers and 1 for negative numbers. In order to represent a number in the IEEE single precision

FIGURE 6-1: IEEE Single Precision Floating-Point Format

S	Exponent	Fraction
1 bit	8 bits	23 bits

format, first it should be converted to a normalized scientific notation with exactly one bit before the binary point, simultaneously adjusting the exponent value.

The exponent representation that goes into the second field of the IEEE 754 representation is obtained by adding 127 to the actual exponent of the number when represented in the normalized form. Exponents in the range 1–254 are used for representing normalized floating-point numbers. Exponent values 0 and 255 are reserved for special cases, which will be discussed later.

The representation for the 23-bit fraction is obtained from the normalized scientific notation by dropping the leading 1. Zero cannot be represented in this fashion; hence it is treated as a special case (explained later). Since every number in the normalized scientific notation will have a leading 1, this leading 1 can be dropped so that one more bit can be packed into the significand (fraction). Thus, a 24-bit fraction can be represented using the 23 bits in the representation. The designers of the IEEE formats wanted to make highest use of all the bits in the exponent and fraction fields.

In order to understand the IEEE format, let us represent 13.45 in the IEEE floating-point format. We can see that 0.45 is a recurring binary fraction and hence

13.45 = 1101.01 1100 1100 1100 .... with the bits 1100 continuing to recur

Normalized scientific representation yields

$$13.45 = 1.10101\ 1100\ 1100 \ldots \times 2^3$$

Since the number is positive, the sign bit for the IEEE 754 representation is 0.

The exponent in the biased notation will be 127 + 3 = 130, which in binary format is 10000010.

The fraction is 1.10101 1100 1100 ... ... (with 1100 recurring). Omitting the leading 1, the 23 bits for the fractional part are

10101 1100 1100 1100 1100 11

Thus, the 32 bits are

0 10000010 10101 1100 1100 1100 1100 11

as illustrated in Figure 6-2.

FIGURE 6-2: IEEE Single Precision Floating-Point Representation for 13.45

S	Exponent	Fraction
0	1 0 0 0 0 0 1 0	1 0 1 0 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1

The 32 bits can be expressed more conveniently in a hexadecimal (hex) format as

4157 3333

The number -13.45 can be represented by changing only the sign bit (i.e., the first bit must be 1 instead of 0). Hence, the hex number C157 3333 represents -13.45 in IEEE 754 single precision format.

The IEEE Double Precision Format

The IEEE double precision format uses 64 bits for representing a floating-point number, as illustrated in Figure 6-3. The first bit is the sign bit for the fractional part. The next 11 bits are used for the exponent, and the remaining 52 bits are used for the fractional part.

FIGURE 6-3: IEEE Double Precision Floating-Point Format

S	Exponent	Fraction
1 bit	11 bits	52 bits

As in the single precision format, the sign bit is 0 for positive numbers and 1 for negative numbers.

The exponent representation used in the second field is obtained by adding the bias value of 1023 to the actual exponent of the number in the normalized form. Exponents in the range 1–2046 are used for representing normalized floating-point numbers. Exponent values 0 and 2047 are reserved for special cases.

The representation for the 52-bit fraction is obtained from the normalized scientific notation by dropping the leading 1 and considering only the next 52 bits.

As an example, let us represent 13.45 in IEEE double precision floating-point format. Converting 13.45 to a binary representation,

13.45 = 1101.01 1100 1100 1100 . . . . with the bits 1100 continuing to recur

In normalized scientific representation,

$$13.45 = 1.10101\ 1100\ 1100 \ldots \times 2^3$$

The exponent in biased notation will be $1023 + 3 = 1026$, which in binary representation is

10000000010

The fraction is 1.10101 1100 1100 . . . . (with 1100 recurring). Omitting the leading 1, the 52 bits of the fractional part are

10101 1100 1100 1100 1100 1100 1100 1100 1100 1100 1100 1100 110

Thus, the 64 bits are

0 10000000010 10101 1100 1100 1100 1100 1100 1100 1100 1100 1100 1100 1100 110

as illustrated in Figure 6-4. The 64 bits can be expressed more conveniently in a hexadecimal format as

402A E666 6666 6666

FIGURE 6-4: IEEE Double Precision Floating-Point Representation for 13.45

S	Exponent	Fraction
0	1 0 0 0 0 0 0 0 0 1 0	1 0 1 0 1 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0

Fraction (cont'd)
0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0

The number −13.45 can be represented by changing only the sign bit (i.e., the first bit must be 1 instead of 0). Hence, the hex number C02A E666 6666 6666 represents −13.45 in IEEE 754 double precision format.

Special Cases in the IEEE 754 Standard

The IEEE 754 standard has several special cases, which are illustrated in Figure 6-5. These include 0, infinity, denormalized numbers, and NaN (Not a Number) representations. The smallest and the highest exponents are used to denote these special cases.

FIGURE 6-5: Special Cases in the IEEE 754 Floating-Point Formats

Single Precision		Double Precision		Object Represented
Exponent	Fraction	Exponent	Fraction	
0	0	0	0	0
0	nonzero	0	nonzero	± denormalized number
255	0	2047	0	± infinity
255	nonzero	2047	nonzero	NaN (not a number)

Zero The IEEE format specifies 0 to be the representation with 0's in all bits (i.e., all exponent and fraction bits are 0). Zero is specified as a special case in the format due to the difficulty in representing 0 in a normalized format. When using the usual convention for IEEE format normalized numbers, we would add a leading 1 to the fractional part, but that would make it impossible to represent 0.

Denormalized Numbers The smallest normalized number that the single precision format can represent is

$$1.0 \times 2^{-126}$$

Numbers between this number and 0 cannot be expressed in the normalized format. If normalization is not made a requirement of the format, we could represent numbers smaller than 1.0×2^{-126}. Hence, the IEEE floating-point format allows denormalized numbers as a special case. If the exponent is 0 and the fraction is nonzero, the number is considered denormalized. Now, the smallest number that can be represented is

$$0.00000000000000000000001 \times 2^{-126}\text{, which is } 1.0 \times 2^{\ 149}.$$

Thus, denormalization allows numbers between 1.0×2^{-126} and 1.0×2^{-149} to be represented.

For double precision, the denormalized range allows numbers between 1.0×2^{-1022} and 1.0×2^{-1074}.

Infinity Infinity is represented by the highest exponent value together with a fraction of 0. In the case of single precision representation, the exponent is 255, and for double precision, it is 2047.

Not a Number (NaN) The IEEE 754 standard has a special representation to represent the result of invalid operations, such as 0/0. This special representation is called NaN or Not a Number. If the exponent is 255 and the fraction is any nonzero number, it is considered to be NaN or Not a Number.

Rounding When the number of bits available is fewer than the number of bits required to represent a number, rounding is employed. It is desirable to round to the nearest value. We can round up if the number is higher than halfway between and round down if the number is less than halfway between. Another option is to truncate, ignoring the bits beyond the allowable number of bits. We must keep more bits in intermediate representations to achieve higher accuracy. The IEEE standard requires two extra bits in intermediate representations in order to facilitate better rounding. The two bits are called **guard and round**. Sometimes, a third intermediate bit is used in rounding in addition to the guard and round bits. It is called **sticky bit**. The sticky bit is set whenever there are nonzero bits to the right of the round bit.

The biggest challenge is when the number falls halfway in between. The IEEE standard has four different rounding modes:

- **Round up** Round toward positive infinity. Round up to the next higher number.
- **Round down** Round toward negative infinity. Round down to the nearest smaller number.
- **Truncate** Round toward zero. Ignore bits beyond the allowable number of bits. Same as truncation in sign magnitude.
- **Unbiased** If the number falls halfway between, round up half the time and round down half the time. In order to achieve rounding up half the time, add 1 if the lowest bit retained is 1, and truncate if it is 0. This is based on the assumption that a 0 or 1 appears in the lowest retained bit with an equal probability. One consequence of this rounding scheme is that the rounded number always has a 0 in the lowest place.

6.2 Floating-Point Multiplication

Given two floating-point numbers, $(F_1 \times 2^{E_1})$ and $(F_2 \times 2^{E_2})$, the product is

$$(F_1 \times 2^{E_1}) \times (F_2 \times 2^{E_2}) = (F_1 \times F_2) \times 2^{(E_1 + E_2)} = F \times 2^E$$

The fraction part of the product is the product of the fractions, and the exponent part of the product is the sum of the exponents. Hence, a floating-point multiplier consists of two major components: a fraction multiplier, and an exponent adder. The details of floating-point multiplication will depend on the precise formats in which the fraction multiplication and exponent addition are performed.

Fraction multiplication can be done in many ways. If the IEEE format is used, multiplication of the magnitude can be done and then the signs can be adjusted. If 2's complement fractions are used, we can use a fraction multiplier that handles signed 2's complement numbers directly. We discussed such a fraction multiplier in Chapter 4.

Addition of the exponents can be done with a binary adder. If the IEEE formats are directly used, the representations must be carefully adjusted in order to obtain the correct result. For instance, if exponents of two floating-point numbers in the biased format are added, the sum contains twice the bias value. To get the correct exponent, the bias value must be subtracted from the sum.

The 2's complement system has several interesting properties for performing arithmetic. Hence, many floating-point arithmetic units convert the IEEE notation to 2's complement and then use the 2's complement internally for carrying out the floating-point operations. Then the final result is converted back to IEEE standard notation.

The general procedure for performing floating-point multiplication is the following:

1. Add the two exponents.
2. Multiply the two fractions (significands).
3. If the product is 0, adjust the representation to the proper representation for 0.
4. a. If the product fraction is too big, normalize by shifting it right and incrementing the exponent.
   b. If the product fraction is too small, normalize by shifting left and decrementing the exponent.
5. If an exponent underflow or overflow occurs, generate an exception or error indicator.
6. Round to the appropriate number of bits. If rounding resulted in loss of normalization, go to step 4 again.

Note that, in addition to adding the exponents and multiplying the fractions, several steps—such as normalizing the product, handling overflow and underflow, and rounding to the appropriate number of bits—also need to be done. We assume that the two numbers are properly normalized to start with, and we want the final result to be normalized.

Now, we discuss the design of a floating-point multiplier. We use 4-bit fractions and 4-bit exponents, with negative numbers represented in 2's complement.

The fundamental steps are to add the exponents (step 1) and multiply the fractions (step 2). However, several special cases must be considered. If F is 0, we must set the exponent E to the largest negative value (1000) (step 3). A special situation occurs if we multiply -1 by -1 (1.000×1.000). The result should be $+1$. Since we cannot represent $+1$ as a 2's complement fraction with a 4-bit fraction, this special case necessitates right shifting as in step 4. To correct this situation, we right shift the significand (fraction) and increment the exponent. Essentially, we set $F = 1/2$ (0.100) and add 1 to E. This results in the correct answer, since $1 \times 2^E = 1/2 \times 2^{E+1}$.

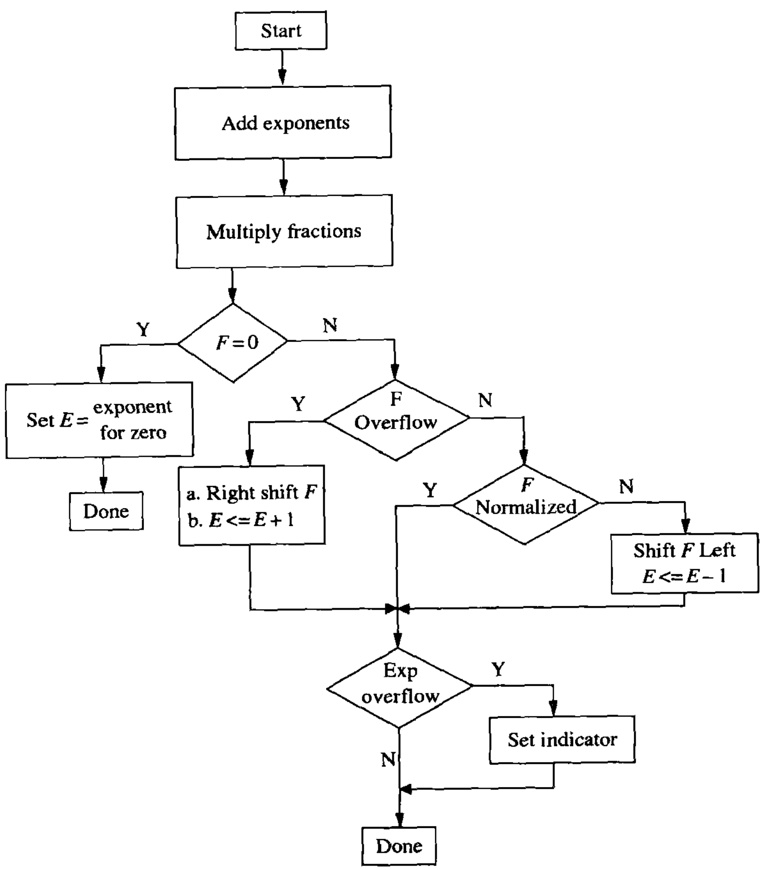
When we multiply the fractions, the result could be unnormalized. For example,

$$(0.1 \times 2^{E_1}) \times (0.1 \times 2^{E_2}) = 0.01 \times 2^{E_1 + E_2} = 0.1 \times 2^{E_1 + E_2 - 1}$$

This is situation 4.b in the preceding list. In this case, we normalize the result by shifting the fraction left one place and subtracting 1 from the exponent to compensate. Finally, if the resulting exponent is too large in magnitude to represent in our number system, we have an exponent overflow. (An overflow in the negative direction is referred to as an *underflow*.) Since we are using 4-bit exponents, if the exponent is not in the range 1000 to 0111 (-8 to $+7$), an overflow has occurred. Since an exponent overflow cannot be corrected, an overflow indicator should be turned on (step 5).

A flow chart for this floating-point multiplier is shown in Figure 6-6. After multiplying the fraction, all the special cases are tested for. Since F_1 and F_2 are normalized, the smallest possible magnitude for the product is 0.01, as indicated in the preceding example. Therefore, only one left shift is required to normalize F.

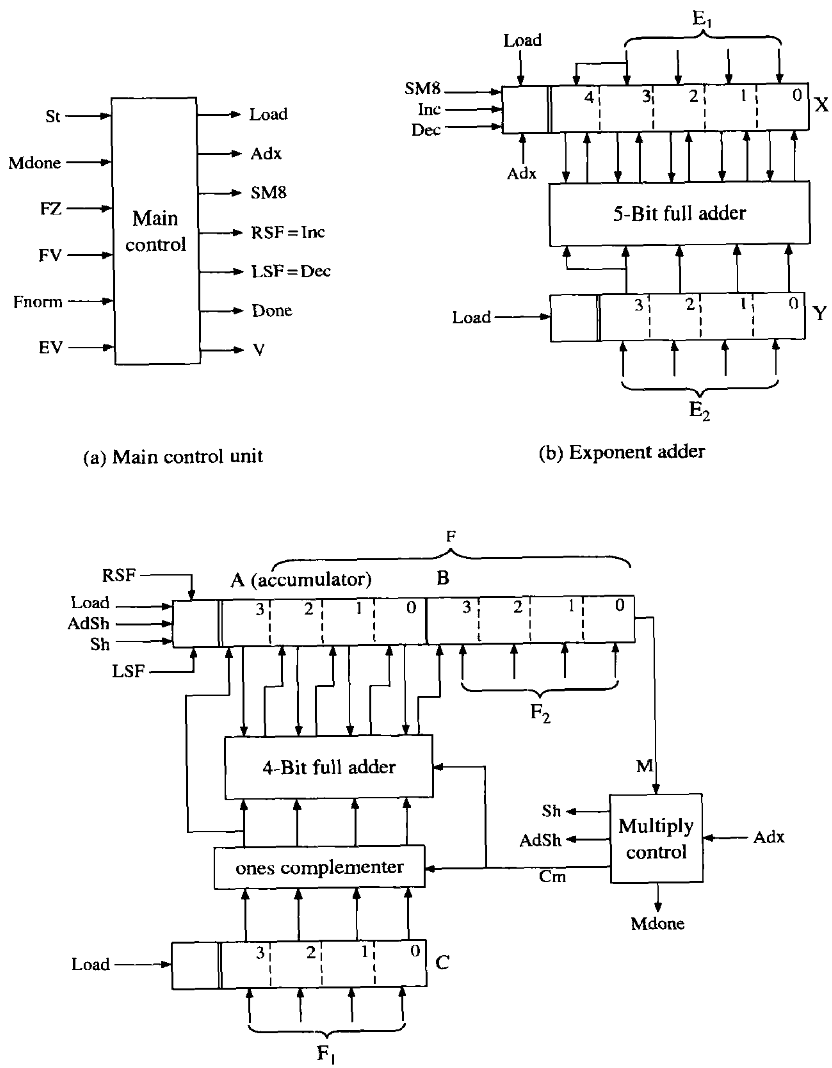
FIGURE 6-6: Flow Chart for Floating-Point Multiplication with 2's Complement Fractions/ Exponents



The hardware required to implement the multiplier (Figure 6-7) consists of an exponent adder, a fraction multiplier, and a control unit that provides the signals to perform the appropriate operations of right shifting, left shifting, exponent incrementing/decrementing, and so on.

Exponent Adder: Since 2's complement addition results with the sum in the proper format, the design of the exponent adder is straightforward. A 5-bit full adder is used as the exponent adder as demonstrated in Figure 6-7. When the fraction is normalized, the exponent will have to be correspondingly incremented or decremented. Also, in

FIGURE 6-7: Major Components of a Floating-Point Multiplier



(a) Main control unit

(b) Exponent adder

(c) Fraction multiplier

the special case when product is 0, the register should be set to the value 1000. The register has control signals for incrementing, decrementing, and setting to the most negative value (*SM8*).

The register which holds the sum is made into a 5-bit register to handle special situations. When the exponents are added, an overflow can occur. If E_1 and E_2 are positive and the sum (E) is negative, or if E_1 and E_2 are negative and the sum is positive, the result is a 2's complement overflow. However, this overflow might be corrected when 1 is added to or subtracted from E during normalization or correction of fraction overflow. To allow for this case, we have made the X register 5 bits long. When E_1 is loaded into X, the sign bit must be extended so that we have a correct 2's complement representation. Since there are two sign bits, if the addition of E_1 and E_2 produces an overflow, the lower sign bit will get changed, but the high-order sign bit will be unchanged. Each of the following examples has an overflow, since the lower sign bit has the wrong value:

$$7 + 6 = 00111 + 00110 = 01101 = 13 \quad \text{(maximum allowable value is 7)}$$

$$-7 + (-6) = 11001 + 11010 = 10011 = -13 \quad \text{(maximum allowable negative value is } -8\text{)}$$

The following example illustrates the special case where an initial fraction overflow and exponent overflow occurs, but the exponent overflow is corrected when the fraction overflow is corrected:

$$(1.000 \times 2^{-3}) \times (1.000 \times 2^{-6}) = 01.000000 \times 2^{-9} = 00.100000 \times 2^{-8}$$

Fraction Multiplier: The fraction multiplier that we designed in Section 4.10 handles 2's complement fractions in a straightforward manner. Hence, we adapt that design for the floating-point multiplier. It implements a shift and add multiplier algorithm. Since we are multiplying 3 bits plus sign by 3 bits plus sign, the result will be 6 bits plus sign. After the fraction multiplication, the 7-bit result (F) will be the lower 3 bits of A concatenated with B. The multiplier has its own control unit that generates appropriate shift and add signals depending on the multiplier bits.

Main Control Unit: The SM chart for the main controller (Figure 6-8) of the floating-point multiplier is based on the flow chart. This controller is called main controller to distinguish it from the controller for the multiplier, which is a separate state machine that is linked into the main controller.

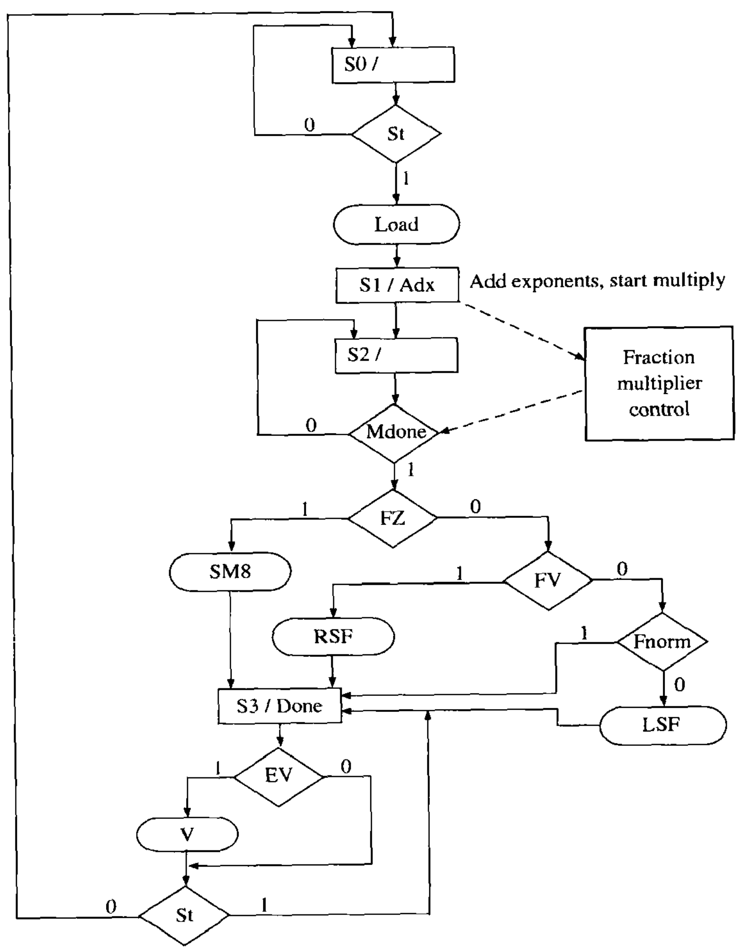
The SM chart uses the following inputs and control signals:

St	Start the floating-point multiplication.
Mdone	Fraction multiply is done.
FZ	Fraction is zero.
FV	Fraction overflow (fraction is too big).
Fnorm	F is normalized.
EV	Exponent overflow.
Load	Load F_1, E_1, F_2, E_2 into the appropriate registers (also clear A in preparation for multiplication).
Adx	Add exponents; this signal also starts the fraction multiplier.

SM8	Set exponent to minus 8 (to handle special case of 0).
RSF	Shift fraction right; also increment E.
LSF	Shift fraction left; also decrement E.
V	Overflow indicator.
Done	Floating-point multiplication is complete.

The SM chart for the main controller has four states. In S_0, the registers are loaded when the start signal is 1. In S_1, the exponents are added, and the fraction multiply is started. In S_2, we wait until the fraction multiply is done and then test for special cases and take appropriate action. It may seem surprising that the tests on *FZ, FV*, and *Fnorm* can all be done in the same state since they are done in sequence on the flow chart. However, *FZ, FV*, and *Fnorm* are generated by combinational circuits that operate in parallel and hence can be tested in the same state. However, we must wait until the exponent has been incremented or decremented at the next clock

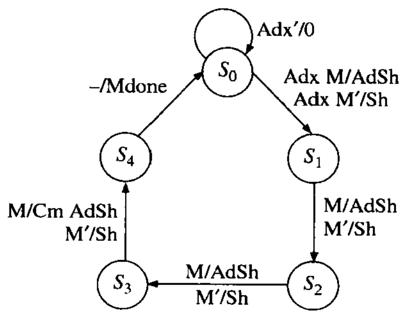
FIGURE 6-8: SM Chart for Floating-Point Multiplication



before we can check for exponent overflow in S_3. In S_3, the *Done* signal is turned on and the controller waits for $St = 0$ before returning to S_0.

The state graph for the multiplier control (Figure 6-9) is similar to Figure 4-34, except that the load state is not needed because the registers are loaded by the main controller. Add and shift operations are performed in one state because as seen in Figure 6-7(c), the sum wires from the adder are shifted by 1 before loading into the accumulator register. When $Adx = 1$, the multiplier is started, and *Mdone* is turned on when the multiplication is completed.

FIGURE 6-9: State Graph for Multiplier Control



The VHDL behavioral description (Figure 6-10) uses three processes. The main process generates control signals based on the SM chart. A second process generates the control signals for the fraction multiplier. The third process tests the control signals and updates the appropriate registers on the rising edge of the clock. In state S_2 of the main process, A = "0000" implies that $F = 0$ ($FZ = 1$ on the SM chart). If we multiply 1.000×1.000, the result is A & B = "01000000", and a fraction overflow has occurred ($FV = 1$). If $A(2) = A(1)$, the sign bit of F and the following bit are the same and F is unnormalized ($Fnorm = 0$). In state S_3, if the two high-order bits of X are different, an exponent overflow has occurred ($EV = 1$).

The registers are updated in the third process. The variable *addout* represents the output of the 4-bit full adder, which is part of the fraction multiplier. This adder adds the 2's complement of C to A when $Cm = 1$. When $Load = 1$, the sign-extended exponents are loaded into X and Y. When $Adx = 1$, vectors X and Y are

FIGURE 6-10: VHDL Code for Floating-Point Multiplier

```
library IEEE;
use IEEE.numeric_bit.all;

entity FMUL is
  port(CLK, St: in bit;
       F1, E1, F2, E2: in unsigned(3 downto 0);
       F: out unsigned(6 downto 0);
       V, done: out bit);
end FMUL;
```

```
architecture FMULB of FMUL is
signal A, B, C: unsigned(3 downto 0);          -- fraction registers
signal X, Y: unsigned(4 downto 0);             -- exponent registers
signal Load, Adx, SM8, RSF, LSF: bit;
signal AdSh, Sh, Cm, Mdone: bit;
signal PS1, NS1: integer range 0 to 3;         -- present and next state
signal State, Nextstate: integer range 0 to 4; -- multiplier control state
begin
  main_control: process(PS1, St, Mdone, X, A, B)
  begin
    Load <= '0'; Adx <= '0'; NS1 <= 0;         -- clear control signals
    SM8 <= '0'; RSF <= '0'; LSF <= '0'; V <= '0'; F <= "0000000";
    done <= '0';
    case PS1 is
      when 0 => F <= "0000000";                -- clear outputs
        done <= '0'; V <= '0';
        if St = '1' then Load <= '1'; NS1 <= 1; end if;
      when 1 => Adx <= '1'; NS1 <= 2;
      when 2 =>
        if Mdone = '1' then                    -- wait for multiply
          if A = 0 then                        -- zero fraction
            SM8 <= '1';
          elsif A = 4 and B = 0 then
            RSF <= '1';                        -- shift AB right
          elsif A(2) = A(1) then               -- test for unnormalized
            LSF <= '1';                        -- shift AB left
          end if;
          NS1 <= 3;
        else
          NS1 <= 2;
        end if;
      when 3 =>                                -- test for exp overflow
        if X(4) /= X(3) then V <= '1'; else V <= '0'; end if;
        done <= '1';
        F <= A(2 downto 0) & B;                -- output fraction
        if ST = '0' then NS1 <= 0; end if;
    end case;
  end process main_control;

  mul2c: process(State, Adx, B)                -- 2's complement multiply
  begin
    AdSh <= '0'; Sh <= '0'; Cm <= '0'; Mdone <= '0';  -- clear control signals
    Nextstate <= 0;
    case State is
      when 0 =>                                   -- start multiply
        if Adx = '1' then
          if B(0) = '1' then AdSh <= '1'; else Sh <= '1'; end if;
          Nextstate <= 1;
        end if;
```

```
      when 1 | 2 =>                                 -- add/shift state
        if B(0) = '1' then AdSh <= '1'; else Sh <= '1'; end if;
        Nextstate <= State + 1;
      when 3 =>
        if B(0) = '1' then Cm <= '1'; AdSh <= '1'; else Sh <= '1'; end if;
        Nextstate <= 4;
      when 4 =>
        Mdone <= '1'; Nextstate <= 0;
    end case;
  end process mul2c;

  update: process                                   -- update registers
  variable addout: unsigned(3 downto 0);
  begin
    wait until CLK = '1' and CLK'event;
    PS1 <= NS1;
    State <= Nextstate;
    if Cm = '0' then addout := A + C;
    else addout := A - C;
    end if;                                         -- add 2's comp. of C
    if Load = '1' then
      X <= E1(3) & E1; Y <= E2(3) & E2;
      A <= "0000"; B <= F1; C <= F2;
    end if;
    if ADX = '1' then X <= X + Y; end if;
    if SM8 = '1' then X <= "11000"; end if;
    if RSF = '1' then A <= '0' & A(3 downto 1);
      B <= A(0) & B(3 downto 1);
      X <= X + 1;
    end if;                                         -- increment X
    if LSF = '1' then
      A <= A(2 downto 0) & B(3); B <= B(2 downto 0) & '0';
      X <= X + 31;
    end if;                                         -- decrement X
    if AdSh = '1' then
      A <= (C(3) xor Cm) & addout(3 downto 1);      -- load shifted adder
      B <= addout(0) & B(3 downto 1);               -- output into A & B
    end if;
    if Sh = '1' then
      A <= A(3) & A(3 downto 1);                    -- right shift A & B
      B <= A(0) & B(3 downto 1);                    -- with sign extend
    end if;
  end process update;
end FMULB;
```

added. When $SM8 = 1$, -8 is loaded into X. When $AdSh = 1$, A is loaded with the sign bit of C (or the complement of the sign bit if $Cm = 1$), concatenated with bits 3 **downto** 1 of the adder output, and the remaining bit of *addout* is shifted into the B register.

Testing the VHDL code for the floating-point multiplier must be done carefully to account for all the special cases in combination with positive and negative fractions, as well as positive and negative exponents. Figure 6-11 shows a command file and some test results. This is not a complete test.

When the VHDL code was synthesized for the Xilinx Spartan-3/Virtex-4 architectures using the Xilinx ISE tools, the result was 38 slices, 29 flip-flops, 72 four-input LUTs, 27 I/O blocks, and one global clock circuitry. The output signals *V*, *Done*, and

FIGURE 6-11: Test Data and Simulation Results for Floating-Point Multiplier

```
add list f x f1 e1 f2 e2 v done
force f1 0111 0, 1001 200, 1000 400, 0000 600, 0111 800
force e1 0001 0, 1001 200, 0111 400, 1000 600, 0111 800
force f2 0111 0, 1001 200, 1000 400, 0000 600, 1001 800
force e2 1000 0, 0001 200, 1001 400, 1000 600, 0001 800
force st 1 0, 0 20, 1 200, 0 220, 1 400, 0 420, 1 600, 0 620, 1 800, 0 820
force clk 0 0, 1 10 -repeat 20
run 1000

ns  delta   f       x     f1   e1   f2   e2   v done
 0   +0  0000000 00000 0000 0000 0000 0000 0 0
 0   +1  0000000 00000 0111 0001 0111 1000 0 0   (0.111 × 2^1) × (0.111 × 2^-8)
 10  +1  0000000 00001 0111 0001 0111 1000 0 0
 30  +1  0000000 11001 0111 0001 0111 1000 0 0
150  +2  0110001 11001 0111 0001 0111 1000 0 1   = 0.110001 × 2^7
170  +2  0000000 11001 0111 0001 0111 1000 0 0
200  +0  0000000 11001 1001 1001 1001 0001 0 0   (1.001 × 2^-7) × (1.001 × 2^1)
250  +1  0000000 11010 1001 1001 1001 0001 0 0
370  +2  0110001 11010 1001 1001 1001 0001 0 1   = 0.110001 × 2^6
390  +2  0000000 11010 1001 1001 1001 0001 0 0
400  +0  0000000 11010 1000 0111 1000 1001 0 0   (1.000 × 2^7) × (1.000 × 2^-7)
430  +1  0000000 00111 1000 0111 1000 1001 0 0
450  +1  0000000 00000 1000 0111 1000 1001 0 0
570  +1  0000000 00001 1000 0111 1000 1001 0 0
570  +2  0100000 00001 1000 0111 1000 1001 0 1   = 0.100000 × 2^1
590  +2  0000000 00001 1000 0111 1000 1001 0 0
600  +0  0000000 00001 0000 1000 0000 1000 0 0   (0.000 × 2^-8) × (0.000 × 2^-8)
630  +1  0000000 11000 0000 1000 0000 1000 0 0
650  +1  0000000 10000 0000 1000 0000 1000 0 0
770  +1  0000000 11000 0000 1000 0000 1000 0 0
770  +2  0000000 11000 0000 1000 0000 1000 0 1   = 0.0000000 × 2^-8
790  +2  0000000 11000 0000 1000 0000 1000 0 0
800  +0  0000000 11000 0111 0111 1001 0001 0 0   (0.111 × 2^7) × (1.001 × 2^1)
830  +1  0000000 00111 0111 0111 1001 0001 0 0
850  +1  0000000 01000 0111 0111 1001 0001 0 0
970  +2  1001111 01000 0111 0111 1001 0001 1 1   = 1.001111 × 2^8 (overflow)
990  +2  0000000 01000 0111 0111 1001 0001 0 0
```

F were set to zero at the beginning of the process to eliminate unwanted latches. An RTL-level design was also attempted, but the RTL design was not superior to the synthesized behavioral design.

Now that the basic design has been completed, we need to determine how fast the floating-point multiplier will operate and determine the maximum clock frequency. Most CAD tools provide a way of simulating the final circuit taking into account both the delays within the logic blocks and the interconnection delays. If this timing analysis indicates that the design does not operate fast enough to meet specifications, several options are possible. Most FPGAs come in several different speed grades, so one option is to select a faster part. Another approach is to determine the longest delay path in the circuit and attempt to reroute the connections or redesign that part of the circuit to reduce the delays.

6.3 Floating-Point Addition

Next, we consider the design of an adder for floating-point numbers. Two floating-point numbers will be added to form a floating-point sum:

$$(F_1 \times 2^{E_1}) + (F_2 \times 2^{E_2}) = F \times 2^E$$

Again, we will assume that the numbers to be added are properly normalized and that the answer should be put in normalized form. In order to add two fractions, the associated exponents must be equal. Thus, if the exponents E_1 and E_2 are different, we must unnormalize one of the fractions and adjust the exponent accordingly. The smaller number is the one that should be adjusted so that if significant digits are lost, the effect is not significant. To illustrate the process, we add

$$F_1 \times 2^{E_1} = 0.111 \times 2^5 \text{ and } F_2 \times 2^{E_2} = 0.101 \times 2^3$$

Since $E_2 \neq E_1$, we unnormalize the smaller number F_2 by shifting right two times and adding 2 to the exponent:

$$0.101 \times 2^3 = 0.0101 \times 2^4 = 0.00101 \times 2^5$$

Note that shifting right one place is equivalent to dividing by 2, so each time we shift we must add 1 to the exponent to compensate. When the exponents are equal, we add the fractions:

$$(0.111 \times 2^5) + (0.00101 \times 2^5) = 01.00001 \times 2^5$$

This addition caused an overflow into the sign bit position, so we shift right and add 1 to the exponent to correct the fraction overflow. The final result is

$$F \times 2^E = 0.100001 \times 2^6$$

When one of the fractions is negative, the result of adding fractions may be unnormalized, as illustrated in the following example:

$(1.100 \times 2^{2}) + (0.100 \times 2^{-1})$
$= (1.110 \times 2^{1}) + (0.100 \times 2^{-1})$ (after shifting F_1)
$= 0.010 \times 2^{-1}$ (result of adding fractions is unnormalized)
$= 0.100 \times 2^{2}$ (normalized by shifting left and subtracting 1 from exponent)

In summary, the steps required to carry out floating-point addition are as follows:

1. Compare exponents. If the exponents are not equal, shift the fraction with the smaller exponent right and add 1 to its exponent; repeat until the exponents are equal.
2. Add the fractions (significands).
3. If the result is 0, set the exponent to the appropriate representation for 0 and exit.
4. If fraction overflow occurs, shift right and add 1 to the exponent to correct the overflow.
5. If the fraction is unnormalized, shift left and subtract 1 from the exponent until the fraction is normalized.
6. Check for exponent overflow. Set overflow indicator, if necessary.
7. Round to the appropriate number of bits.
   Still normalized? If not, go back to step 4.

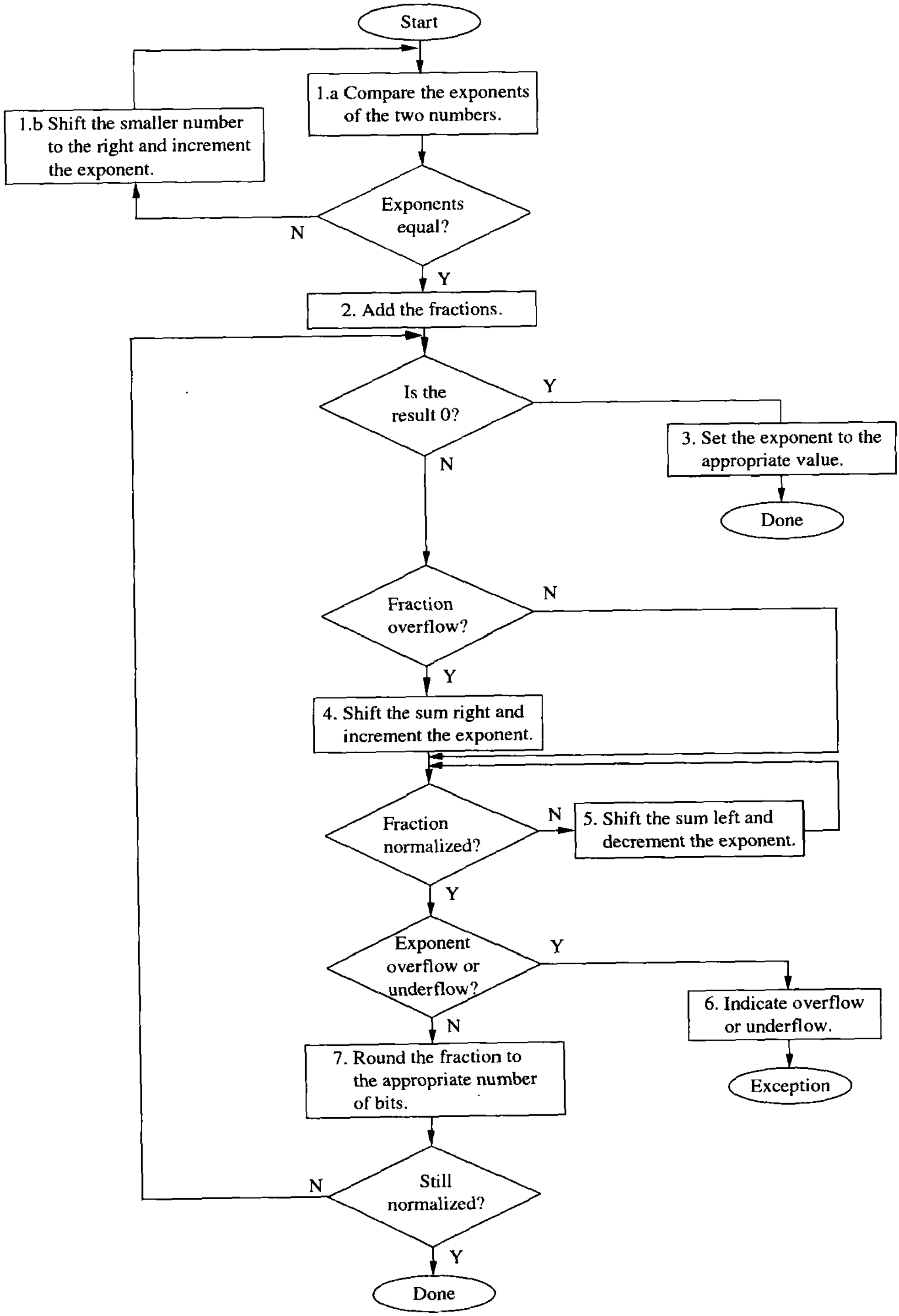
Figure 6-12 illustrates this procedure graphically. An optimization can be added to step 1. We can identify cases where the two numbers are vastly different. If $E_1 >> E_2$ and F_2 is positive, F_2 will become all 0's as we right shift F_2 to equalize the exponents. In this case, the result is $F = F_1$ and $E = E_1$, so it is a waste of time to do the shifting. If $E_1 >> E_2$ and F_2 is negative, F_2 will become all 1's (instead of all 0's) as we right shift F_2 to equalize the exponents. When we add the fractions, we will get the wrong answer. To avoid this problem, we can skip the shifting when $E_1 >> E_2$ and set $F = F_1$ and $E = E_1$. Similarly, if $E_2 >> E_1$, we can skip the shifting and set $F = F_2$ and $E = E_2$.

For the 4-bit fractions in our example, if $|E_1 - E_2| > 3$, we can skip the shifting. For IEEE single precision numbers, there are 23 bits after the binary point; hence if the exponent difference is greater than 23, the smaller number will become 0 before the exponents are equal. In general, if the exponent difference is greater than the number of available fractional bits, the sum should be set to the larger number. If $E_1 >> E_2$, set $F = F_1$ and $E = E_1$. If $E_2 >> E_1$, set $F = F_2$ and $E = E_2$.

Inspection of this procedure illustrates that the following hardware units are required to implement a floating-point adder:

- Adder (subtractor) to compare exponents (step 1a)
- Shift register to shift the smaller number to the right (step 1b)
- ALU (adder) to add fractions (step 2)
- Bidirectional shifter, incrementer/decrementer (steps 4, 5)
- Overflow detector (step 6)
- Rounding hardware (step 7)

FIGURE 6-12: Flow Chart for Floating-Point Addition



Many of these components can be combined. For instance, the register that stores the fractions can be made a shift register in order to perform the shifts. The register that stores the exponent can be a counter with increment/decrement capability. Figure 6-13 shows a hardware arrangement for the floating-point adder. The major components are the exponent comparator and the fraction adder. Fraction addition can be done using 2's complement addition. It is assumed that the operands are delivered on an I/O bus. If the numbers are in a sign-magnitude form as in the IEEE format, they can be converted to 2's complement numbers and then added. Special cases should be handled according to the requirements of the format. The sum is written back into the Addend register in Figure 6-13.

FIGURE 6-13: Overview of a Floating-Point Adder

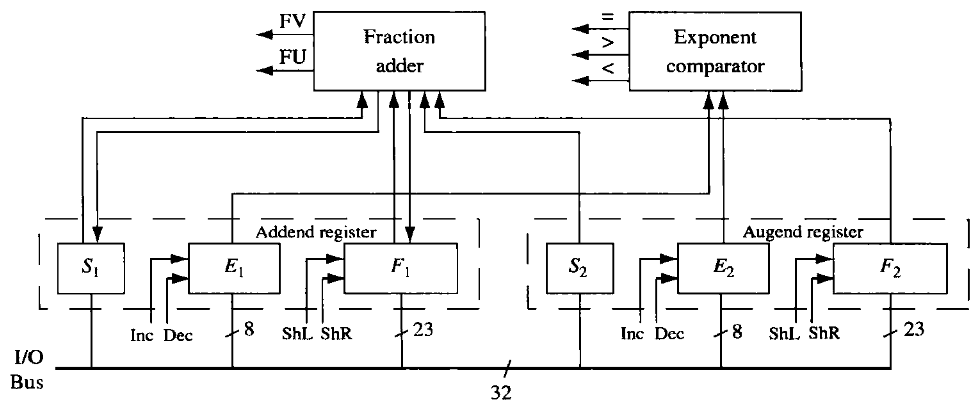


Figure 6-14 shows VHDL code for a floating-point adder based on the IEEE single precision floating-point format. This code is not a complete implementation of the standard. It handles the special case of 0, but it does not deal with infinity, unnormalized, and not-a-number formats. The final result is truncated instead of rounded. Sign and magnitude format and biased exponents are used throughout, except 2's complement is used for the fraction addition.

`FPinput` is an input bus, and we assume that the input numbers represent normalized floating-point numbers in IEEE standard format. In state 0, the first number is split and loaded into S_1, F_1, and E_1. These represent the sign of the fraction, the magnitude of the fraction, and the biased exponent. When F_1 is loaded, the 23-bit fraction is prefixed by a 1 except in the special case of 0, in which case the leading bit is a 0. Two 0's are appended at the end of the fraction to conform to the IEEE standard requirements (guard and round bits). In state 1, the second number to be added is loaded into S_2, F_2, and E_2. In state 2, the fraction with the smallest exponent is unnormalized by shifting right and incrementing the exponent. When this operation is complete, the exponents are equal, except in the special case when F_1 or F_2 equals 0.

FIGURE 6-14: VHDL Code for a Floating-Point Adder

```
library IEEE;
use IEEE.numeric_bit.all;

entity FPADD is
  port(CLK, St: in bit; done, ovf, unf: out bit;
       FPinput: in unsigned(31 downto 0);  -- IEEE single precision FP format
       FPsum: out unsigned(31 downto 0));  -- IEEE single precision FP format
end FPADD;

architecture FPADDER of FPADD is
 -- F1 and F2 store significand with leading 1 and trailing 0's added
signal F1, F2: unsigned(25 downto 0);
signal E1, E2: unsigned(7 downto 0);  -- exponents
signal S1, S2, FV, FU: bit;
 -- intermediate results for 2's complement addition
signal F1comp, F2comp, Addout, Fsum: unsigned(27 downto 0);
signal State: integer range 0 to 6;
begin  -- convert fractions to 2's comp and add
  F1comp <= not ("00" & F1) + 1 when S1 = '1' else "00" & F1;
  F2comp <= not ("00" & F2) + 1 when S2 = '1' else "00" & F2;
  Addout <= F1comp + F2comp;
   -- find magnitude of sum
  Fsum <= Addout when Addout(27) = '0' else not Addout + 1;
  FV <= Fsum(27) xor Fsum(26);          -- fraction overflow
  FU <= not F1(25);                     -- fraction underflow
  FPsum <= S1 & E1 & F1(24 downto 2); -- pack output word
  process(CLK)
  begin
    if CLK'event and CLK = '1' then
      case State is
        when 0 =>
          if St = '1' then -- load E1 and F1
            E1 <= FPinput(30 downto 23); S1 <= FPinput(31);
            F1(24 downto 0) <= FPinput(22 downto 0) & "00";
            -- insert 1 in significand (or 0 if the input number is 0)
            if FPinput = 0 then F1(25) <= '0'; else F1(25) <= '1'; end if;
            done <= '0'; ovf <= '0'; unf <= '0'; State <= 1;
          end if;
        when 1 =>  -- load E2 and F2
          E2 <= FPinput(30 downto 23); S2 <= FPinput(31);
          F2(24 downto 0) <= FPinput(22 downto 0) & "00";
          if FPinput = 0 then F2(25) <= '0'; else F2(25) <= '1'; end if;
          State <= 2;
        when 2 =>  -- unnormalize fraction with smallest exponent
         if F1 = 0 or F2 = 0 then State <= 3;
         else
           if E1 = E2 then State <= 3;
```

```
          elsif E1 < E2 then
            F1 <= '0' & F1(25 downto 1); E1 <= E1 + 1;
          else
            F2 <= '0' & F2(25 downto 1); E2 <= E2 + 1;
          end if;
        end if;
      when 3 => -- add fractions and check for fraction overflow
        S1 <= Addout(27);
        if FV = '0' then F1 <= Fsum(25 downto 0);
        else F1 <= Fsum(26 downto 1); E1 <= E1 + 1; end if;
        State <= 4;
      when 4 => -- check for sum of fractions = 0
        if F1 = 0 then E1 <= "00000000"; State <= 6;
        else State <= 5; end if;
      when 5 => -- normalize
        if E1 = 0 then unf <= '1'; State <= 6;
        elsif FU = '0' then State <= 6;
        else F1 <= F1(24 downto 0) & '0'; E1 <= E1 - 1;
        end if;
      when 6 => -- check for exponent overflow
        if E1 = 255 then ovf <= '1'; end if;
        done <= '1'; State <= 0;
    end case;
  end if;
 end process;
end FPADDER;
```

The fractions are added using 2's complement arithmetic, which is performed by concurrent statements. The input numbers are first converted to 2's complement representation. Two sign bits (00) are prefixed to F_1, and the 2's complement is formed if S_1 is 1 (negative). Two sign bits are used so that the sign is not lost if the fraction addition overflows into the first sign bit. F_2 is processed in a similar way. The resulting numbers, *F1comp* and *F2comp*, are added and the sum is assigned to *Addout*. The adder output is read in state 3. *Fsum* represents the magnitude of the fraction, so Addout must be complemented if it is negative. Normally the two sign bits of `Fsum` are "00", so they are discarded and the result is stored back in F_1, which serves as a floating-point accumulator. The sign bit is extracted from the MSB of *Addout*. Fraction overflow and underflow are indicated by *FV* and *FU*, respectively. Fraction overflow can be detected by exclusive-OR of the highest two bits of *Addout*. This is done as a concurrent statement. In case of fraction overflow, the sign bits of *Fsum* are "01", so *FV* = '1', Fsum is right shifted before it is stored in F_1, and E_1 is incremented. If the result of addition F1 = 0, E1 is set to 0 in state 4, and the floating-point addition is complete. If F_1 is unnormalized, it is normalized in state 5 by shifting F_1 left and decrementing E_1. Exponent overflow and underflow are represented by `ovf` and `unf`, respectively. Since the normal range of biased exponents is 1 to 254, an underflow occurs if E_1 is decremented to 0, and unf is set to '1' before exiting state 5. In state 6, if $E_1 = 255$, this indicates an exponent overflow, and `ovf` is set to '1'. The done signal is turned on before exiting state 6. S_1, E_1, and F_1 are merged by a concurrent statement to give the final sum, *FPsum*, in IEEE format.

The floating-point adder was tested for the following cases.

Addend		Augend		Expected Result	
Number (Binary)	IEEE Single Precision	Number (Binary)	IEEE Single Precision	Number (Binary)	IEEE Single Precision
0	x00000000	0	x00000000	0	x00000000
1×2^0	x3F800000	1×2^0	x3F800000	1×2^1	x40000000
-1×2^0	xBF800000	-1×2^0	xBF800000	-1×2^1	xC0000000
1×2^0	x3F800000	-1×2^0	xBF800000	0	x00000000
$1.111 \ldots \times 2^{127}$	x7F7FFFFF	1×2^0	x3F800000	$1.111 \ldots \times 2^{127}$	x7F7FFFFF
$-1.111 \ldots \times 2^{127}$	xFF7FFFFF	-1×2^0	xBF800000	$-1.111 \ldots \times 2^{127}$	xFF7FFFFF
$1.111 \ldots \times 2^{127}$	x7F7FFFFF	$1.111 \ldots \times 2^{127}$	x7F7FFFFF	overflow	
$-1.111 \ldots \times 2^{127}$	xFF7FFFFF	$-1.111 \ldots \times 2^{127}$	xFF7FFFFF	overflow	
1.11×2^8	x43E00000	-1.11×2^6	xC2E00000	1.0101×2^8	x43A80000
-1.11×2^8	xC3E00000	1.11×2^6	x42E00000	-1.0101×2^8	xC3A80000
$1.111 \ldots \times 2^{127}$	x7F7FFFFF	$0.0 \ldots 01 \times 2^{127}$	x73800000	overflow	
$-1.111 \ldots \times 2^{127}$	xFF7FFFFF	$-0.0 \ldots 01 \times 2^{127}$	xF3800000	overflow	
$1.1 \ldots 10 \times 2^{127}$	x7F7FFFFE	$0.0 \ldots 01 \times 2^{127}$	x73800000	$1.111 \ldots \times 2^{127}$	x7F7FFFFF
$-1.1 \ldots 10 \times 2^{127}$	xFF7FFFFE	$-0.0 \ldots 01 \times 2^{127}$	xF3800000	$-1.111 \ldots \times 2^{127}$	xFF7FFFFF
1.1×2^{-126}	X00C00000	-1.0×2^{-126}	x80800000	underflow	

6.4 Other Floating-Point Operations

6.4.1 Subtraction

Floating-point subtraction is the same as floating-point addition, except that we must subtract the fractions instead of adding them. The rest of the steps remain the same.

6.4.2 Division

The quotient of two floating-point numbers is

$$(F_1 \times 2^{E_1}) \div (F_2 \times 2^{E_2}) = (F_1/F_2) \times 2^{(E_1 - E_2)} = F \times 2^E$$

Thus, the basic rule for floating-point division is to divide the fractions and subtract the exponents. In addition to considering the same special cases as for multiplication, we must test for divide by 0 before dividing. If F_1 and F_2 are normalized, then the largest positive quotient (F) will be

$$0.1111 \ldots / 0.1000 \ldots = 01.111 \ldots$$

which is less than 10_2, so the fraction overflow is easily corrected. For example,

$$(0.110101 \times 2^2) \div (0.101 \times 2^{-3}) = 01.010 \times 2^5 = 0.101 \times 2^6$$

Alternatively, if $F_1 \geq F_2$, we can shift F_1 right before dividing and avoid fraction overflow in the first place. In the IEEE format, when divide by 0 is involved, the result can be set to NaN (Not a Number).

In this chapter, we presented different representations of floating-point numbers. IEEE floating-point single precision and double precision formats were discussed. A floating-point format with 2's complement numbers was also presented. Then we presented a floating-point multiplier. We also presented a procedure to perform addition of floating-point numbers. In the process of designing the multiplier, we used the following steps:

1. Develop an algorithm for floating-point multiplication, taking all of the special cases into account.
2. Draw a block diagram of the system and define the necessary control signals.
3. Construct an SM chart (or state graph) for the control state machine using a separate linked state machine for controlling the fraction multiplier.
4. Write behavioral VHDL code.
5. Test the VHDL code to verify that the high-level design of the multiplier is correct.
6. Use the CAD software to synthesize the multiplier. Then implement the multiplier in the desired target technology (e.g., ASIC, FPGA, etc.).

6.5 Problems

6.1 **(a)** What is the biggest number that can be represented in the 8-bit 2's complement floating-point format with 4 bits for exponent and 4 for fraction?
(b) What is the smallest number that can be represented in the 8-bit 2's complement format with 4 bits for exponent and 4 for fraction?
(c) What is the biggest normalized number that can be represented in the IEEE single precision floating-point format?
(d) What is the smallest normalized number that can be represented in the IEEE single precision floating-point format?
(e) What is the biggest normalized number that can be represented in the IEEE double precision floating-point format?
(f) What is the smallest normalized number that can be represented in the IEEE double precision floating-point format?

6.2 Convert the following decimal numbers in the IEEE single precision format.

(i) 25.25, **(ii)** 2000.25, **(iii)** 1, **(iv)** 0, **(v)** 1000, **(vi)** 8000, **(vii)** 10^6, **(viii)** -5.4, **(ix)** 1.0×2^{-140}, **(x)** 1.5×10^9

6.3 Convert the following decimal numbers to IEEE double precision format.

(i) 25.25, **(ii)** 2000.25, **(iii)** 1, **(iv)** 0, **(v)** 1000, **(vi)** 8000, **(vii)** 10^6, **(viii)** -5.4, **(ix)** 1.0×2^{-140}, **(x)** 1.5×10^9

6.4 What do the following hex representations mean if they are in IEEE single precision format?

(i) ABABABAB, **(ii)** 45454545, **(iii)** FFFFFFFF, **(iv)** 00000000, **(v)** 11111111, **(vi)** 01010101

6.5 What do the following hex representations mean if they are in IEEE double precision format?

(i) ABABABAB 00000000, **(ii)** 45454545 00000001, **(iii)** FFFFFFFF 10001000, **(iv)** 00000000 00000000, **(v)** 11111111 10001000, **(vi)** 01010101 01010101

6.6 **(a)** Represent −35.25 in IEEE single precision floating-point format.
(b) What does the hex number ABCD0000 represent if it is in IEEE single precision floating-point format?

6.7 **(a)** Represent 25.625 in IEEE single precision floating-point format.
(b) Represent −15.6 in IEEE single precision floating-point format.

6.8 This problem concerns the design of a digital system that converts an 8-bit signed integer (negative numbers are represented in 2's complement) to a floating-point number. Use a floating-point format similar to the ones used in Section 6.1.1 except the fraction should be 8 bits and the exponent 4 bits. The fraction should be properly normalized.

(a) Draw a block diagram of the system and develop an algorithm for doing the conversion. Assume that the integer is already loaded into an 8-bit register, and when the conversion is complete the fraction should be in the same register. Illustrate your algorithm by converting –27 to floating point.
(b) Draw a state diagram for the controller. Assume that the start signal is present for only one clock time. (Two states are sufficient.)
(c) Write a VHDL description of the system.

6.9 **(a)** Multiply the following two floating-point numbers to give a properly normalized result. Assume 4-bit 2's complement format.

$$F_1 = 1.011, E_1 = 0101, F_2 = 1.001, E_2 = 0011$$

(b) Repeat (a) for

$$F_1 = 1.011, E_1 = 1011, F_2 = 0.110, E_2 = 1101$$

6.10 A floating-point number system uses a 4-bit fraction and a 4-bit exponent with negative numbers expressed in 2's complement. Design an *efficient* system that will multiply the number by −4 (minus four). Take all special cases into account, and give a properly normalized result. Assume that the initial fraction is properly normalized or zero. *Note:* This system multiplies *only* by −4.

(a) Give examples of the normal and special cases that can occur (for multiplication by −4).
(b) Draw a block diagram of the system.
(c) Draw an SM chart for the control unit. Define all signals used.

6.11 Redesign the floating-point multiplier in Figure 6-7 using a common 5-bit full adder connected to a bus instead of two separate adders for the exponents and fractions.

(a) Redraw the block diagram and be sure to include the connections to the bus and include all control signals.
(b) Draw a new SM chart for the new control.
(c) Write the VHDL description for the multiplier or specify what changes need to be made to an existing description.

6.12 This problem concerns the design of a circuit to find the square of a floating-point number, $F \times 2^E$. F is a normalized 5-bit fraction, and E is a 5-bit integer; negative numbers are represented in 2's complement. The result should be properly normalized. Take advantage of the fact that $(-F)^2 = F^2$.

(a) Draw a block diagram of the circuit. (Use only one adder and one complementer.)
(b) State your procedure, taking all special cases into account. Illustrate your procedure for

$$F = 1.0110 \qquad E = 00100$$

(c) Draw an SM chart for the main controller. You may assume that multiplication is carried out using a separate control circuit, which outputs $Mdone = 1$ when multiplication is complete.
(d) Write a VHDL description of the system.

6.13 Write a behavioral VHDL code for a floating-point multiplier using the IEEE single precision floating-point format. Use an overloaded multiplication operator instead of using an add-shift multiplier. Ignore special cases like infinity, denormalized, and not-a-number formats. Truncate the final result instead of rounding.

6.14 Write a test bench for the floating-point adder of Figure 6-14.

6.15 Add the following floating-point numbers (show each step). Assume that each fraction is 5 bits (including sign) and each exponent is 5 bits (including sign) with negative numbers in 2's complement.

$$F_1 = 0.1011 \qquad E_1 = 11111$$
$$F_2 = 1.0100 \qquad E_2 = 11101$$

6.16 Two floating-point numbers are added to form a floating-point sum:

$$(F_1 \times 2^{E_1}) + (F_2 \times 2^{E_2}) = F \times 2^E$$

Assume that F_1 and F_2 are normalized, and the result should be normalized.

(a) List the steps required to carry out floating-point addition, including all special cases.
(b) Illustrate these steps for $F_1 = 1.0101$, $E_1 = 1001$, $F_2 = 0.1010$, $E_2 = 1000$. Note that the fractions are 5 bits, including sign, and the exponents are 4 bits, including sign.
(c) Write a VHDL description of the system.

6.17 For the floating-point adder of Figure 6-14, modify the VHDL code so that

(a) It handles IEEE standard single precision denormalized numbers both as input and output.
(b) In state 2, it speeds up the processing when the exponents differ by more than 23.
(c) It rounds up instead of truncating the resulting fraction.

6.18 **(a)** Add the floating-point numbers $0.111 \times 2^5 + 0.101 \times 2^3$ and normalize the result.
(b) Draw an SM chart for a floating-point adder that adds $(F_1 \times 2^{E_1})$ and $(F_2 \times 2^{E_2})$. Assume that the fractions are initially normalized (or zero) and the final result should be normalized (or zero). A zero fraction should have an exponent of -8. Set an exponent overflow flag (EV) if the final answer has an exponent overflow. Each number to be added consists of a 4-bit fraction and a 4-bit exponent, with negative numbers represented in 2's complement. Assume that all registers (F_1, E_1, F_2, and E_2) can be loaded in one clock time when a start signal (St) is received. If $E_1 > E_2$, the control signal $GT = 1$, and if $E_1 < E_2$, the control signal $LT = 1$. Define all other control signals used. Include the special case where $|E_1 - E_2| > 3$.

6.19 **(a)** Draw a block diagram for a floating-point subtracter. Assume that the inputs to the subtracter are properly normalized, and the answer should be properly normalized. The fractions are 8 bits including sign, and the exponents are 5 bits including sign. Negative numbers are represented in 2's complement.
(b) Draw an SM chart for the control circuit for the floating-point subtracter. Define the control signals used, and give an equation for each control signal used as an input to the control circuit.
(c) Write the VHDL description of the floating-point subtracter.

6.20 **(a)** State the steps necessary to carry out floating-point subtraction, including special cases. Assume that the numbers are initially in normalized form, and the final result should be in normalized form.
(b) Subtract the following (fractions are in 2's complement):

$$(1.0111 \times 2^{-3}) - (1.0101 \times 2^{\ 5})$$

(c) Write a VHDL description of the system. Fractions are 5 bits including sign, and exponents are 4 bits including sign.

6.21 This problem concerns the design of a divider for floating point numbers:

$$(F_1 \times 2^{E_1}) / (F_2 \times 2^{E_2}) = F \times 2^E$$

Assume that F_1 and F_2 are properly normalized fractions (or 0), with negative fractions expressed in 2's complement. The exponents are integers with negative numbers expressed in 2's complement. The result should be properly normalized if it is not zero. Fractions are 8 bits including sign, and exponents are 5 bits including sign.

(a) Draw a flow chart for the floating-point divider. Assume that a divider is available that will divide two binary fractions to give a fraction as a result. Do not show the individual steps in the division of the fractions on your flowchart, just say "divide." The divider requires that $|F_2| > |F_1|$ before division is carried out.

(b) Illustrate your procedure by computing

$$0.111 \times 2^3 / 1.011 \times 2^{-2}$$

When you divide F_1 by F_2, you don't need to show the individual steps, just the result of the division.

(c) Write a VHDL description for the system.

6.22 Assume that A, B, and C are floating-point numbers expressed in IEEE single precision floating-point format and that floating-point addition is performed.

If $A = 2^{40}$, $B = -2^{40}$, $C = 1$, then

What is $A + (B + C)$? (i.e., $B + C$ done first and then A added to it)
What is $(A + B) + C$? (i.e., $A + B$ done first and then C added to it)

6.23 Assume that A, B, and C are floating-point numbers expressed in IEEE double precision floating-point format and that floating-point addition is performed.

If $A = 2^{40}$, $B = -2^{40}$, $C = 1$, then

What is $A + (B + C)$? (i.e., $B + C$ done first and then A added to it)
What is $(A + B) + C$? (i.e., $A + B$ done first and then C added to it)

6.24 Assume that A, B, and C are floating-point numbers expressed in IEEE single precision floating-point format and that floating-point addition is performed.

If $A = 2^{65}$, $B = -2^{65}$, $C = 1$, then

What is $A + (B + C)$? (i.e., $B + C$ done first and then A added to it)
What is $(A + B) + C$? (i.e., $A + B$ done first and then C added to it)

6.25 Assume that A, B, and C are floating-point numbers expressed in IEEE double precision floating-point format and that floating-point addition is performed.

If $A = 2^{65}$, $B = -2^{65}$, $C = 1$, then

What is $A + (B + C)$? (i.e., $B + C$ done first and then A added to it)
What is $(A + B) + C$? (i.e., $A + B$ done first and then C added to it)

CHAPTER 7

Hardware Testing and Design for Testability

This chapter introduces digital system testing and design methods that make the systems easier to test. We have already discussed the use of testing during the design process. We have written VHDL test benches to verify that the overall design and algorithms used are correct. We have used simulation at the logic level to verify that a design is logically correct and that it meets specifications. After the logic level design of an IC is completed, additional testing can be done by simulating it at the circuit level to verify that the design has been correctly implemented and that the timing is correct.

When a digital system is manufactured, further testing is required to verify that it functions correctly. When multiple copies of an IC are manufactured, each copy must be tested to verify that it is free from manufacturing defects. This testing process can become very expensive and time consuming. With today's complex ICs, the cost of testing is a major component of the manufacturing cost. Therefore, it is very important to develop efficient methods of testing digital systems and to design the systems so that they are easy to test. **Design for testability (DFT)** is thus an important issue in modern IC design.

In this chapter, we first discuss methods of testing combinational logic for the basic types of faults that can occur. Then we describe methods for determining test sequences for sequential logic. **Automatic test pattern generators (ATPGs)** are employed in order to generate test sequences required for testing circuits and systems. One of the problems encountered is that normally we have access only to the inputs and outputs of the circuit being tested and not to the internal state. To remedy this problem, internal test points may be brought out to additional pins on the IC. To reduce the number of test pins required, we introduce the concept of **scan design**, in which the state of the system can be stored in a shift register and shifted out serially. Finally, we discuss the concept of **built-in self-test (BIST)**. By adding more components to the IC, we can generate test sequences and verify the response to these sequences internally without the need for expensive external testing.

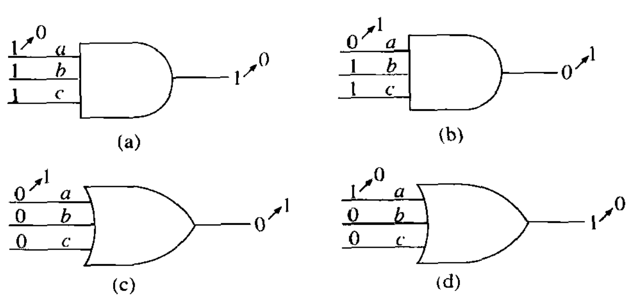
7.1 Testing Combinational Logic

Two common types of faults are short circuits and open circuits. If the input to a gate is shorted to ground, the input acts as if it is stuck at a logic 0. If the input to a gate is shorted to a positive power supply voltage, the gate input acts as if it is stuck at a logic

1. If the input to a gate is an open circuit, the input may act as if it is stuck at 0 or stuck at 1, depending on the type of logic being used. Thus, it is common practice to model faults in logic circuits as stuck-at-1 (s-a-1) or stuck-at-0 (s-a-0) faults. To test a gate input for s-a-0, the gate input must be 1 so a change to 0 can be detected. Similarly, to test a gate input for s-a-1, the normal gate input must be 0 so a change to 1 can be detected.

We can test an AND gate for s-a-0 faults by applying 1's to all inputs, as shown in Figure 7-1(a). The normal gate output is then 1, but if any input is s-a-0, the output becomes 0. The notation 1→0 on the gate input a means that the normal value of a is 1, but the value has changed to 0 because of the s-a-0 fault. The notation 1→0 at the gate output indicates that this change has propagated to the gate output. We can test an AND gate input for s-a-1 by applying 0 to the input being tested and 1's to the other inputs, as shown in Figure 7-1(b). The normal gate output then is 0, but if the input being tested is s-a-1, the output becomes 1. To test OR gate inputs for s-a-1, we apply 0's to all inputs, and if any input is s-a-1, the output will change to 1 (Figure 7-1(c)). To test an OR gate input for s-a-0, we apply a 1 to the input under test and 0's to the other inputs. If the input under test is s-a-0, the output will change to 0 (Figure 7-1(d)). In the process of testing the inputs to a gate for s-a-0 and s-a-1, we also can detect s-a-0 and s-a-1 faults at the gate output.

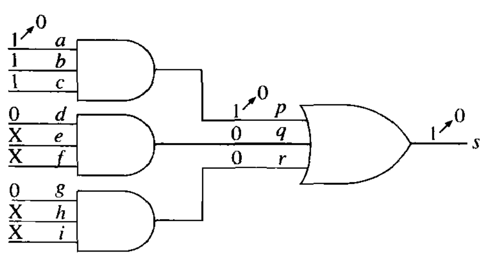
FIGURE 7-1: Testing AND and OR Gates for Stuck-At Faults



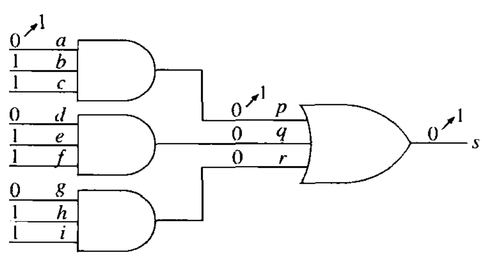
The two-level AND-OR circuit of Figure 7-2 has nine inputs and one output. We assume that the OR gate inputs (p, q, and r) are not accessible, so the gates cannot be tested individually. One approach to testing the circuit would be to apply all $2^9 = 512$ different input combinations and observe the output. A more efficient approach is based on testing for all s-a-0 and s-a-1 faults, as shown in Table 7-1. To test the abc AND gate inputs for s-a-0, we must apply 1's to a, b, and c, as shown in Figure 7-2(a). Then, if any gate input is s-a-0, the gate output (p) will become 0. In order to transmit the change to the OR gate output, the other OR gate inputs must be 0. To achieve this, we can set $d = 0$ and $g = 0$ (e, f, h, and i are then don't cares). This test vector will detect $p0$ (p stuck-at-0) as well as $a0$, $b0$, and $c0$. In a similar manner, we can test for $d0$, $e0$, $f0$, and $q0$ by setting $d = e = f = 1$ and $a = g = 0$. A third test with $g = h = i = 1$ and $a = d = 0$ will test the remaining s-a-0 faults. To

test a for s-a-1 ($a1$), we must set $a = 0$ and $b = c = 1$, as shown in Figure 7-2(b). Then, if a is s-a-1, p will become 1. In order to transmit this change to the output, we must have $q = r = 0$, as before. However, if we set $d = g = 0$ and $e = f = h = i = 1$, we can test for $d1$ and $g1$ at the same time as $a1$. This same test vector also tests for $p1$, $q1$, and $r1$. As shown in the table, we can test for $b1$, $e1$, and $h1$ with a single test vector and test similarly for $c1$, $f1$, and $i1$. Thus, we can test all s-a-0 and s-a-1 faults with only six tests, whereas the brute-force approach would require 512 tests. When we apply the six tests, we can determine whether or not a fault is present, but we cannot determine the exact location of the fault. In the preceding analysis, we have assumed that only one fault occurs at a time. In many cases the presence of multiple faults will also be detected.

FIGURE 7-2: Testing an AND-OR Circuit



(a) stuck-at-0 test



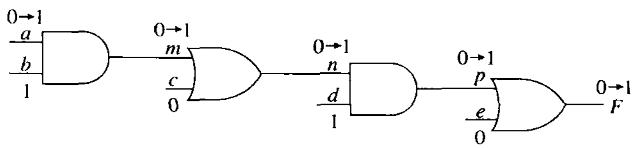
(b) stuck-at-1 test

TABLE 7-1: Test Vectors for Figure 7-2

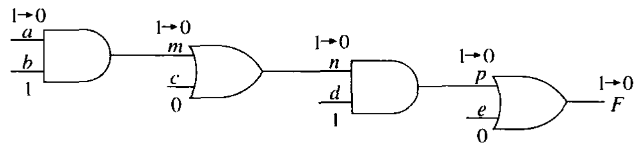
a	b	c	d	e	f	g	h	i	Faults Tested
1	1	1	0	X	X	0	X	X	$a0$, $b0$, $c0$, $p0$
0	X	X	1	1	1	0	X	X	$d0$, $e0$, $f0$, $q0$
0	X	X	0	X	X	1	1	1	$g0$, $h0$, $i0$, $r0$
0	1	1	0	1	1	0	1	1	$a1$, $d1$, $g1$, $p1$, $q1$, $r1$
1	0	1	1	0	1	1	0	1	$b1$, $e1$, $h1$, $p1$, $q1$, $r1$
1	1	0	1	1	0	1	1	0	$c1$, $f1$, $i1$, $p1$, $q1$, $r1$

Testing multilevel circuits is considerably more complex than testing two-level circuits. In order to test for an internal fault in a circuit, we must choose a set of inputs that will excite that fault and then propagate the effect of that fault to the circuit output. In Figure 7-3, a, b, c, d, and e are circuit inputs. If we want

FIGURE 7-3: Fault Detection Using Path Sensitization



(a) s-a-1 tests



(b) s-a-0 tests

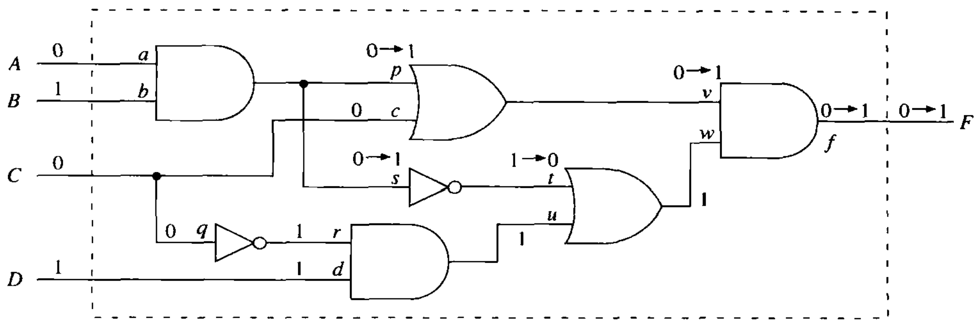
to test for gate input n s-a-1, n must be 0. This can be achieved if we make $c = 0$, $a = 0$, and $b = 1$, as shown. In order to propagate the fault n s-a-1 to the output F, we must make $d = 1$ and $e = 0$. With this set of inputs, if a, m, n, or p is s-a-1, the output F will have the incorrect value and the fault can be detected. Furthermore, if we change a to 1 and gate input a, m, n, or p is s-a-0, the output F will change from 1 to 0. We say that the path through a, m, n, and p has been sensitized, since any fault along that path can be detected. The method of **path sensitization** allows us to test for a number of different stuck-at faults using one set of circuit inputs.

Next, we try to determine a minimum set of test vectors to test the circuit of Figure 7-4 for all single stuck-at-1 and stuck-at-0 faults. We assume that we can apply inputs to A, B, C, and D and observe the output F and that the internal gate inputs and outputs cannot be accessed. The general procedure to determine the test vectors is the following:

1. Select an untested fault.
2. Determine the required $ABCD$ inputs.
3. Determine the additional faults that are tested.
4. Repeat this procedure until tests are found for all of the faults.

Let us start by testing input p for s-a-1. In order to do this, we must choose inputs A, B, C, and D such that $p = 0$, and if p is s-a-1, we must propagate this fault to the output F so it can be observed. In order to propagate the fault, we must make $c = 0$ and $w = 1$. We can make $w = 1$ by making $t = 1$ or $u = 1$. To make $u = 1$, we must have both D and $r = 1$. Fortunately, our choice of $C = 0$ makes $r = 1$. To make $p = 0$, we choose $A = 0$. By choosing $B = 1$, we can sensitize the path A-a-p-v-f-F so that the set of inputs $ABCD = 0101$ will test for faults $a1$, $p1$, $v1$, and $f1$. This set of inputs also tests for c s-a-1. We assume that c s-a-1 is a fault internal to the gate, so it is still possible to have $q = 0$ and $r = 1$ if c s-a-1 occurs.

FIGURE 7-4: Example Circuit for Stuck-At Fault Testing (*p* stuck at 1)



To test for s-a-0 inputs along the path *A-a-p-v-f-F*, we can use the inputs *ABCD* = 1101. In addition to testing for faults *a*0, *p*0, *v*0, and *f*0, this input vector also tests the following faults: *b*0, *w*0, *u*0, *r*0, *q*1, and *d*0. To determine tests for the remaining stuck-at faults, we select an untested fault, determine the required *ABCD* inputs, and then determine the additional faults that are tested. Then we can repeat this procedure until tests are found for all of the faults. Table 7-2 lists a set five test vectors that will test for all single stuck-at faults in Figure 7-4.

TABLE 7-2: Tests for Stuck-At Faults in Figure 7-4

Test Vectors *A B C D*	Normal Gate Inputs *a b p c q r d s t u v w*	*F*	Faults Tested
0 1 0 1	0 1 0 0 0 1 1 0 1 1 0 1	0	*a*1 *p*1 *c*1 *v*1 *f*1
1 1 0 1	1 1 1 0 0 1 1 1 0 1 1 1	1	*a*0 *b*0 *p*0 *q*1 *r*0 *d*0 *u*0 *v*0 *w*0 *f*0
1 0 1 1	1 0 0 1 1 0 1 0 1 0 1 1	1	*b*1 *c*0 *s*1 *t*0 *v*0 *w*0 *f*0
1 1 0 0	1 1 1 0 0 1 0 1 0 0 1 0	0	*a*0 *b*0 *d*1 *s*0 *t*1 *u*1 *w*1 *f*1
1 1 1 1	1 1 1 1 1 0 1 1 0 0 1 0	0	*a*0 *b*0 *q*0 *r*1 *s*0 *t*1 *u*1 *w*1 *f*1

In addition to stuck-at faults, other types of faults, such as bridging faults, may occur. A bridging fault occurs when two unconnected signal lines are shorted together. For a large combinational circuit, finding a minimum set of test vectors that will test for all possible faults is very difficult and time consuming. For circuits that contain redundant gates, testing for some of the faults may be impossible. Even if a comprehensive set of test vectors can be found, applying all of the vectors may take too much time and cost too much. For these reasons, it is common practice to use a relatively small set of test vectors that will test most of the faults. In general, determining such a set of vectors is a difficult and computationally intensive problem. Many algorithms and corresponding computer programs have been developed to generate such sets of test vectors. Computer programs have also been developed to simulate faulty circuits. Such programs allow the user to determine what percentage of possible faults are tested by a given set of input vectors. The percentage of possible faults that can be tested by a set of input vectors is called the **coverage** of the test vectors.

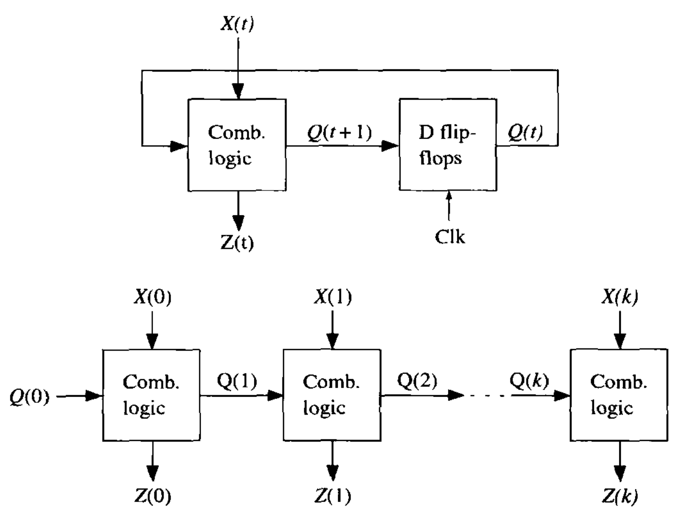
7.2 Testing Sequential Logic

Testing sequential logic is generally much more difficult than testing combinational logic, because we must use sequences of inputs for testing. If we can observe only the input and output sequences and not the state of the flip-flops in a sequential circuit, a very large number of test sequences may be required. Basically, the problem is to determine if the circuit under test is equivalent to a correctly functioning circuit. We will assume that the sequential circuit being tested has a reset input so we can reset it to a known initial state. If we attempted to test the circuit using the brute-force approach, we would reset the circuit to the initial state, apply a test sequence, and observe the output sequence. If the output sequence was correct, then we would repeat the test for another sequence. This process has to be repeated for all possible input sequences. A large number of tests are required to test exhaustively all states and all state transitions in the machine. Since the brute-force approach is totally impractical, the question arises: Can we derive a relatively small set of test sequences that will adequately test the circuit?

One way to derive test sequences for a sequential circuit is to convert it to an iterative circuit. The iterative circuit means that the combinational part of the sequential circuit is repeated several times to indicate the condition of the combinational part of the circuit at each time. Since the iterative circuit is a combinational circuit, we could derive test vectors for the iterative circuit using one of the standard methods for combinational circuits.

As an example, Figure 7-5 shows a standard Mealy sequential circuit and the corresponding iterative circuit. In these figures, X, Z, and Q can either be single variables or vectors. The iterative circuit has $k + 1$ identical copies of the combinational network used in the sequential circuit, where $k + 1$ is the length of the sequence used to test the sequential circuit. For the sequential circuit, $X(t)$ represents a sequence of

FIGURE 7-5: Sequential and Iterative Circuits



inputs in time. In the iterative circuit, $X(0)$ $X(1)$ ... $X(\mathrm{k})$ represents the same sequence in space. Each cell of the iterative circuit computes $Z(t)$ and $Q(t+1)$ in terms of $Q(t)$ and $X(t)$. The leftmost cell computes the values for $t = 0$, the next cell for $t = 1$, and so on. After the test vectors have been derived for the iterative circuit, these vectors become the input sequences used to test the original sequential circuit. The number of cells in the iterative circuit depends on the length of the sequences required to test the sequential circuit.

Derivation of a small set of test sequences that will adequately test a sequential circuit is generally difficult to do. Consider the state graph shown in Figure 7-6 and the corresponding state table (Table 7-3). We assume that we can reset the circuit to state S_0. It is necessary that the test sequence cause the circuit to go through all possible state transitions, but this is not an adequate test. For example, the input sequence

$$X = 0\,1\,0\,1\,1\,0\,0\,1\,1$$

traverses all the arcs connecting the states and produces the output sequence

$$Z = 0\,0\,1\,0\,1\,1\,1\,1\,0$$

If we replace the arc from S_3 to S_0 with a self-loop, as shown by the dashed line, the output sequence will be the same, but the new sequential machine is not equivalent to the old one.

FIGURE 7-6: State Graph for Test Example

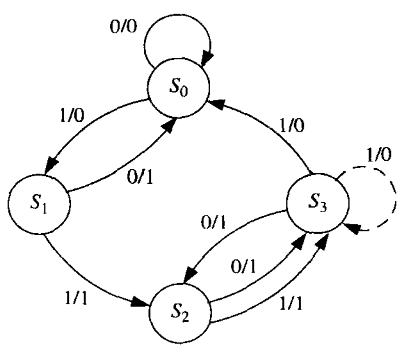


TABLE 7-3: State Table for Figure 7-6

Q1Q2	State	Next State $X = 0$	1	Output $X = 0$	1
00	S_0	S_0	S_1	0	0
10	S_1	S_0	S_2	1	1
01	S_2	S_3	S_3	1	1
11	S_3	S_2	S_0	1	0

A state graph in which every state can be reached from every other state is referred to as **strongly connected**. A general test strategy for a sequential circuit with a strongly connected state graph and no equivalent states is first to find an input

sequence that will distinguish each state from the other states. Such an input sequence is referred to as a **distinguishing sequence**. Two states of a state machine M are distinguishable if and only if there exists at least one finite input sequence, which, when applied to M, causes different output sequences. If the output sequence is identical for every possible input sequence, then obviously the states are equivalent. It has been proved that if two states of machine M are distinguishable, they can be distinguished by a sequence of length $n - 1$ or less, where n is the number of states in M [28]. Given a distinguishing sequence, each entry in the state table can be verified.

For the example of Figure 7-6, one distinguishing sequence is 11. This distinguishing sequence can be obtained as follows. Divide the states S_0, S_1, S_2, and S_3 into two groups, where the states in each group are equivalent if the test sequence is only one-bit long. For instance, Table 7-3 shows that by applying a one bit test sequence, we can distinguish between groups $\{S_0, S_3\}$ and $\{S_1, S_2\}$. If the input is 1, output is 0 for $\{S_0, S_3\}$ and 1 for $\{S_1, S_2\}$. States inside each partition are equivalent if the test sequence is only a 1. Now, from Table 7-3, we can see that if we applied a test input of 1 again, states in group $\{S_0, S_3\}$ can be distinguished. The states in group $\{S_1, S_2\}$ can also be distinguished by the test input 1. Hence, the sequence 11 is sufficient to distinguish among the four states. In the worst case, a sequence of three bits would have been sufficient since there are only four states in the machine. If we start in S_0, the input sequence 11 gives the output sequence 01; for S_1 the output is 11; for S_2, 10; and for S_3, 00. Thus, we can distinguish the four states by using the input sequence 11. We can then verify every entry in the state table using the following sequences, where R means reset to state S_0:

Input	Output	Transition Verified
R 0 1 1	0 0 1	(S_0 to S_0)
R 1 1 1	0 1 1	(S_0 to S_1)
R 1 0 1 1	0 1 0 1	(S_1 to S_0)
R 1 1 1 1	0 1 1 0	(S_1 to S_2)
R 1 1 0 1 1	0 1 1 0 0	(S_2 to S_3)
R 1 1 1 1 1	0 1 1 0 0	(S_2 to S_3)
R 1 1 0 0 1 1	0 1 1 1 1 0	(S_3 to S_2)
R 1 1 0 1 1 1	0 1 1 0 0 1	(S_3 to S_0)

Another approach to deriving test sequences is based on testing for stuck-at faults. Figure 7-7 shows the realization of Figure 7-6 using the following state assignment: S_0, 00; S_1, 10; S_2, 01; S_3, 11. If we want to test for a s-a-1, we must first excite the fault by going to state S_1, in which $Q1Q2 = 10$ and then setting $X = 0$. In normal operation, the next state will be S_0. However, if a is s-a-1, then next state is $Q1Q2 = 01$, which is S_2. This test sequence can then be constructed as follows:

- To go to S_1: reset followed by $X = 1$.
- To test a s-a-1: $X = 0$.
- To distinguish the state that is reached: $X = 11$.

The final sequence is R1011. The normal output is 0101, and the faulty output is 0110.

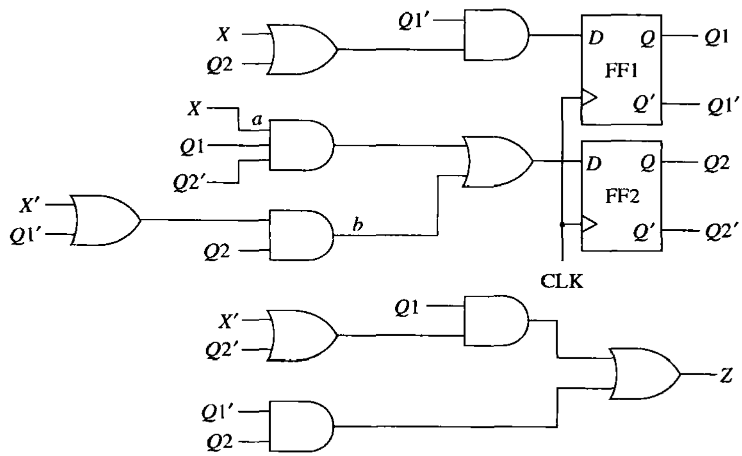


FIGURE 7-7: Realization of Figure 7-6

We have shown some simple examples that illustrate some of the methods used to derive test sequences for sequential circuits. As the number of inputs and states in the circuit increases, the number and length of the required test sequence increases rapidly, and the derivation of these test sequences becomes much more difficult. This, in turn, means that the time and expense required to test the circuits increases rapidly with the number of inputs and states.

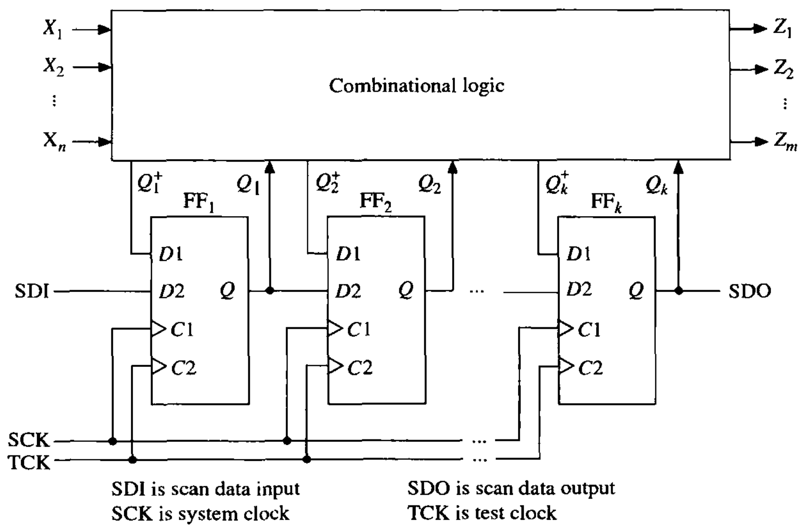
7.3 Scan Testing

The problem of testing a sequential circuit is greatly simplified if we can observe the state of all the flip-flops instead of just observing the circuit outputs. For each state of the flip-flops and for each input combination, we need to verify that the circuit outputs are correct and that the circuit goes to the correct next state. One approach would be to connect the output of each flip-flop within the IC being tested to one of the IC pins. Since the number of pins on the IC is very limited, this approach is not very practical. So the question arises: How can we observe the state of all the flip-flops without using up a large number of pins on the IC? If the flip-flops were arranged to form a shift register, then we could shift out the state of the flip-flops bit by bit using a single serial output pin on the IC. This leads to the concept of **scan path testing**.

Figure 7-8 shows a method of scan path testing based on two-port flip-flops. In the usual way, the sequential circuit is separated into a combinational logic part and a state register composed of flip-flops. Each of the flip-flops has two D inputs and two clock inputs. When $C1$ is pulsed, the $D1$ input is stored in the flip-flop. When $C2$ is pulsed, $D2$ is stored in the flip-flop. The Q output of each flip-flop is connected to the $D2$ input of the next flip-flop to form a shift register. The next state ($Q_1^+Q_2^+ \ldots Q_k^+$) generated by the combinational logic is loaded into the flip-flops when $C1$ is pulsed, and the new state (Q_1 Q_2 $\ldots$ Q_k) feeds back into

the combinational logic. When the circuit is not being tested, the system clock ($SCK = C1$) is used. A set of inputs ($X_1X_2 \ldots X_n$) is applied, the outputs ($Z_1Z_2 \ldots Z_m$) are generated, SCK is pulsed, and the circuit goes to the next state.

FIGURE 7-8: Scan Path Test Circuit Using Two-Port Flip-Flops



When the circuit is being tested, the flip-flops are set to a specified state by shifting the state code into the register using the scan data input (SDI) and the test clock (TCK). A test input vector ($X_1X_2 \ldots X_n$) is applied, the outputs ($Z_1Z_2 \ldots Z_m$) are verified, and SCK is pulsed to take the circuit to the next state. The next state is then verified by pulsing TCK to shift the state code out of the scan data register via the scan data output (SDO). This method reduces the problem of testing a sequential circuit to that of testing a combinational circuit. Any of the standard methods can be used to generate a set of test vectors for the combinational logic. Each test vector contains ($n + k$) bits, since there are n X inputs and k state inputs to the combinational logic. The X part of the test vector is applied directly, and the Q part is shifted in via the SDI. In summary, the test procedure is as follows:

1. Scan in the test vector Q_i values via SDI using the test clock TCK.
2. Apply the corresponding test values to the X_i inputs.
3. After sufficient time for the signals to propagate through the combinational circuit, verify the output Z_i values.
4. Apply one clock pulse to the system clock SCK to store the new values of Q_i^+ into the corresponding flip-flops.
5. Scan out and verify the Q_i values by pulsing the test clock TCK.
6. Repeat steps 1 through 5 for each test vector.

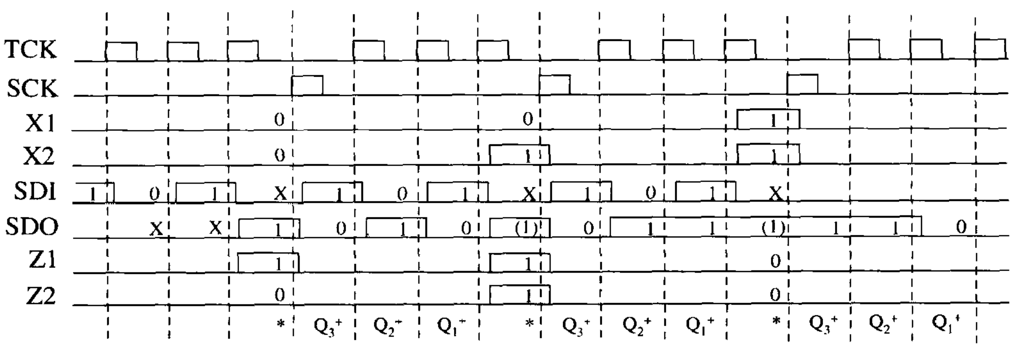
Steps 5 and 1 can overlap, since it is possible to scan in one test vector while scanning out the previous test result.

We will apply this method to test a sequential circuit with two inputs, three flip-flops, and two outputs. The circuit is configured as in Figure 7-8 with inputs X_1X_2, flip-flops $Q_1Q_2Q_3$, and outputs Z_1Z_2. One row of the state transition table is as follows:

	$Q_1^+ Q_2^+ Q_3^+$				Z_1Z_2			
$Q_1Q_2Q_3$ | X_1X_2 =	00	01	11	10	00	01	11	10
101	010	110	011	111	10	11	00	01

Figure 7-9 shows the timing diagram for testing this row of the transition table. First, 101 is shifted in using *TCK*, least significant bit (Q_3) first. The input X_1X_2 = 00 is applied, and Z_1Z_2 = 10 is then read. *SCK* is pulsed and the circuit goes to state 010. As 010 is shifted out using *TCK*, 101 is shifted in for the next test. This process continues until the test is completed.

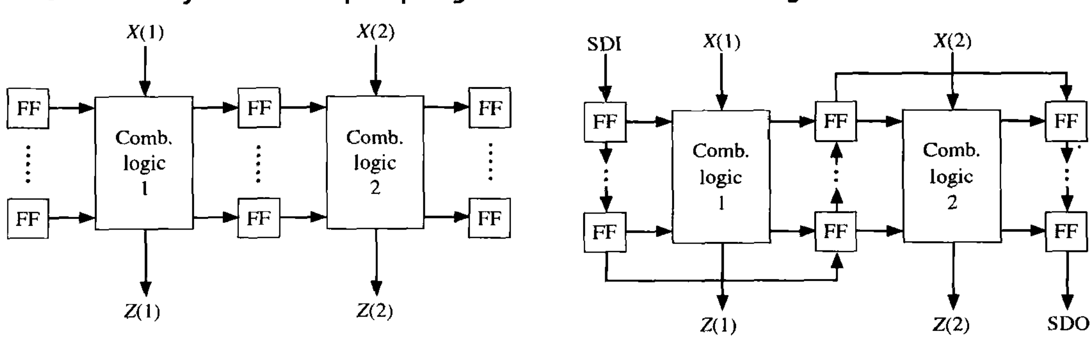
FIGURE 7-9: Timing Chart for Scan Test



*Read output (output at other times not shown)

In general, a digital system implemented by an IC consists of flip-flop registers separated by blocks of combinational logic, as shown in Figure 7-10(a). In order to apply scan test to the IC, we need to replace the flip-flops with two-port flip-flops

FIGURE 7-10: System with Flip-Flop Registers and Combinational Logic Blocks



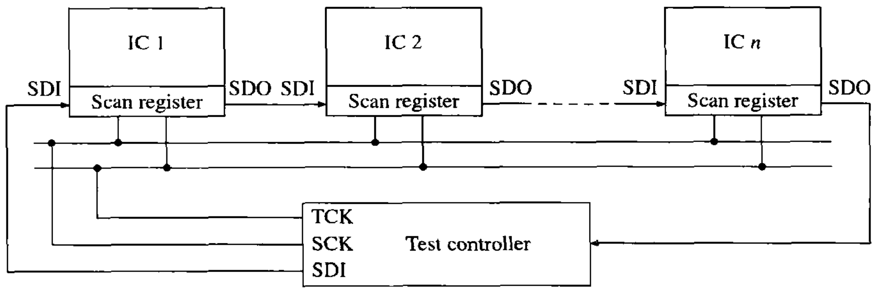
(a) Without scan chain

(b) With scan chain added

(or other types of scannable flip-flops) and link all the flip-flops into a scan chain, as shown in Figure 7-10(b). Then we can scan test data into all the registers, apply the test clock, and scan out the results.

When multiple ICs are mounted on a PC board, it is possible to chain together the scan registers in each IC so that the entire board can be tested using a single serial access port (Figure 7-11).

FIGURE 7-11: Scan Test Configuration with Multiple ICs



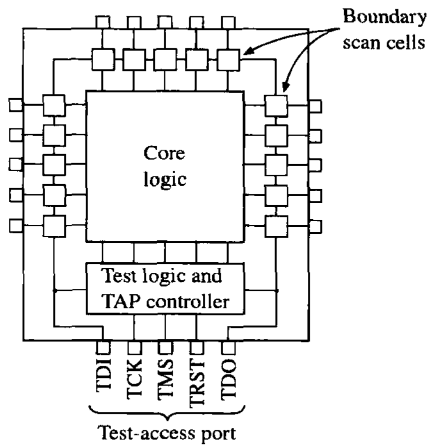
7.4 Boundary Scan

As ICs have become more complex, with more and more pins, printed circuit boards have become denser, with multiple layers and very fine traces. Testing these PC boards after they have been loaded with complex ICs has become very difficult. Testing a board by means of its edge connector does not provide adequate testing and may require very long test sequences. When PC boards were less dense with wider traces, testing was often done using a **bed-of-nails test fixture**. This method used sharp probes to contact the traces on the board so test data could be applied to and read from various ICs on the board. Bed-of-nails testing is not practical for high-density PC boards with fine traces and complex ICs.

Boundary scan test methodology was introduced to facilitate the testing of complex PC boards. It is an integrated method for testing circuit boards with many ICs. A standard for boundary scan testing was developed by the Joint Test Action Group (**JTAG**), and this standard has been adopted as **ANSI/IEEE Standard 1149.1**, "Standard Test Access Port and Boundary-Scan Architecture." Many IC manufacturers make ICs that conform to this standard. Such ICs can be linked together on a PC board so that they can be tested using only a few pins on the PC board edge connector.

Figure 7-12 shows an IC with added boundary scan logic according to the IEEE standard. One cell of the boundary scan register (BSR) is placed between each input or output pin and the internal core logic. Four or five pins of the IC are devoted to the **test-access port**, or **TAP**. The TAP controller and additional test logic are also

FIGURE 7-12: IC with Boundary Scan Register and Test-Access Port



added to the core logic on the IC. The functions of the TAP pins (according to the standard) are as follows:

TDI	Test data input (this data is shifted serially into the BSR)
TCK	Test clock
TMS	Test mode select
TDO	Test data output (serial output from the BSR)
TRST	Test reset (resets the TAP controller and test logic; optional pin)

A PC board with several boundary scan ICs is shown in Figure 7-13. The boundary scan registers in the ICs are linked serially in a single chain with input *TDI* and output *TDO*. *TCK*, *TMS*, and *TRST* (if used) are connected in parallel to all of the ICs. Using these signals, test instructions and test data can be clocked into every IC on the board.

FIGURE 7-13: PC Board with Boundary Scan ICs

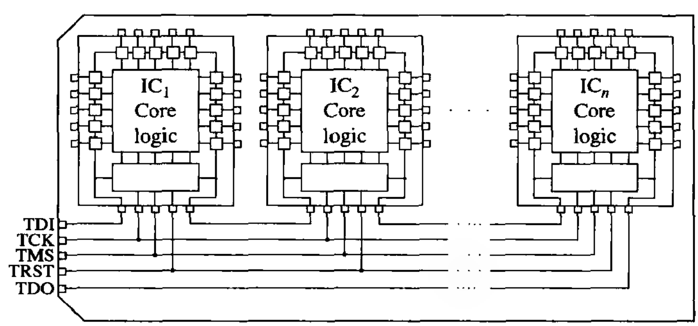
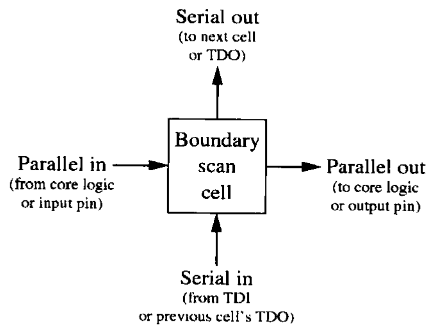
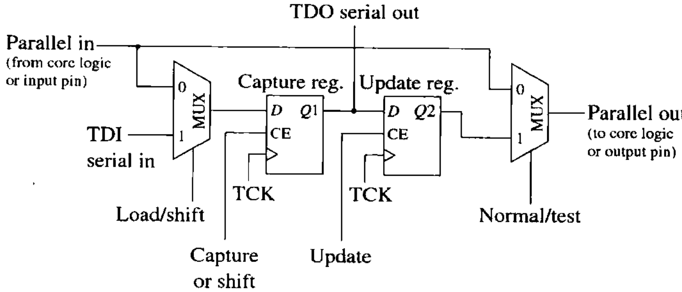


Figure 7-12 illustrated the boundary scan cells on the periphery of each IC that conforms to the boundary scan standard. The structure of a typical boundary scan cell is shown in Figure 7-14. A boundary scan cell has two inputs, TDI serial input and the parallel input pin. Similarly, it has two outputs, the serial out and the parallel data out. When in the normal mode, data from the parallel input pin is routed to the internal

FIGURE 7-14: Typical Boundary Scan Cell



(a)



(b)

core logic in the IC, or data from the core logic is routed to the output pin. When in the shift mode, serial data from the previous cell is clocked into flip-flop $Q1$ at the same time as the data stored in $Q1$ is clocked into the next boundary scan cell. After $Q2$ is updated, test data can be supplied to the internal logic or to the output pin.

Figure 7-15 shows the basic boundary scan architecture that is implemented on each boundary scan IC. The boundary scan register is divided into two parts. BSR1 represents the shift register, which consists of the $Q1$ flip-flops in the boundary scan cells. BSR2 represents the $Q2$ flip-flops, which can be parallel-loaded from BSR1 when an update signal is received. The serial input data (*TDI*) can be shifted into the boundary scan register (BSR1), through a bypass register, or into the instruction register. The TAP controller on each IC contains a state machine (Figure 7-16). The input to the state machine is *TMS*, and the sequence of 0's and 1's applied to *TMS* determines whether the *TDI* data is shifted into the instruction register or through the boundary scan cells. The TAP controller and the instruction register control the operation of the boundary scan cells.

The TAP controller state machine has 16 states. States 9 through 15 are used for loading and updating the instruction register, and states 2 through 8 are used for loading and updating the data register (BSR1). The TRST signal, if used, resets the state to Test-Logic-Reset. The state graph has the interesting property that, regardless of the initial state, a sequence of five 1's on the TMS input will always reset the machine to state 0.

FIGURE 7-15: Basic Boundary Scan Architecture

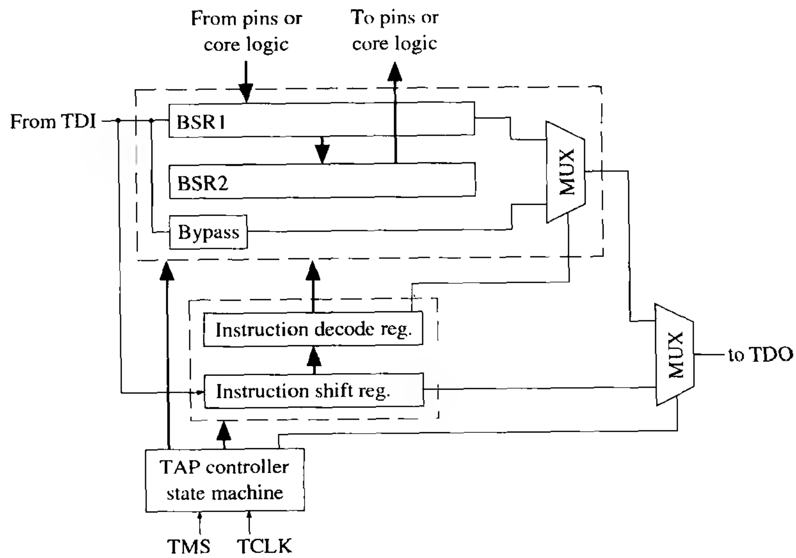
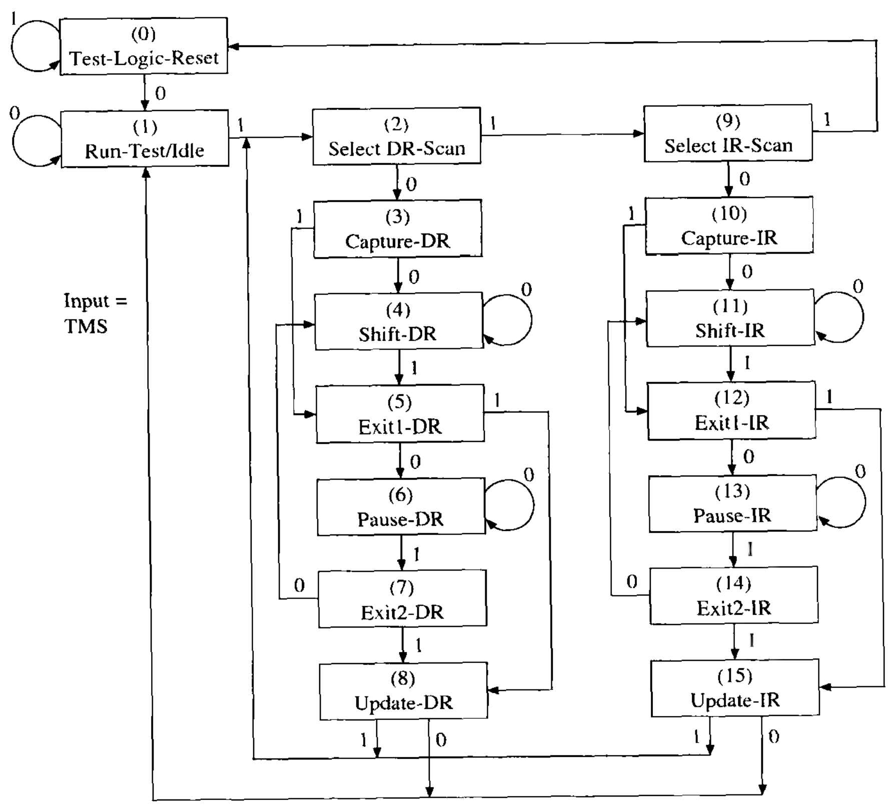


FIGURE 7-16: State Machine for TAP Controller



The following instructions are defined in the IEEE standard:

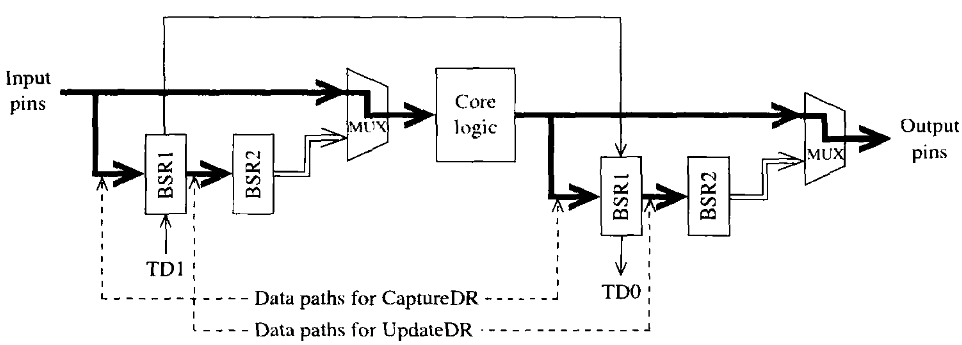
- BYPASS: This instruction allows the *TDI* serial data to go through a 1-bit bypass register on the IC instead of through the boundary scan register. In this way, one or more ICs on the PC board may be bypassed while other ICs are being tested.
- SAMPLE/PRELOAD: This instruction is used to scan the boundary scan register without interfering with the normal operation of the core logic. Data is transferred to or from the core logic from or to the IC pins without interference. Samples of this data can be taken and scanned out through the boundary scan register. Test data can be shifted into the BSR.
- EXTEST: This instruction allows board-level interconnect testing, and it also allows testing of clusters of components that do not incorporate the boundary scan test features. Test data is shifted into the BSR and then it goes to the output pins. Data from the input pins is captured by the BSR.
- INTEST (optional): This instruction allows testing of the core logic by shifting test data into the boundary scan register. Data shifted into the BSR takes the place of data from the input pins, and output data from the core logic is loaded into the BSR.
- RUNBIST (optional): This instruction causes special built-in self-test (BIST) logic within the IC to execute. (Section 7.5 explains how BIST logic can be used to generate test sequences and check the test results.)

Several other optional and user-defined instructions may also be included.

The data paths between the IC pins, the boundary scan registers, and the core logic depend on the instruction being executed as well as the state of the TAP controller. Figures 7-17, 7-18, and 7-19 highlight the data paths for the Sample/Preload, Extest, and Intest instructions. In each case, the boundary scan registers BSR1 and BSR2 have been split into two sections—one associated with the input pins and one associated with the output pins. Test data can be shifted into BSR1 from TDI and shifted out to TDO.

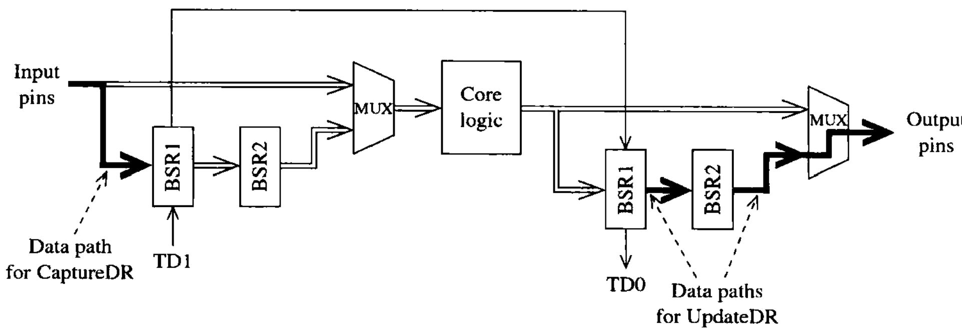
For the Sample/Preload instruction (Figure 7-17) the core logic operates in the normal mode with inputs from the input pins of the IC and outputs going to the output pins. When the controller is in the CaptureDR state, BSR1 is parallel-loaded from the input pins and from the outputs of the core logic. In the UpdateDR state, BSR2 is loaded from BSR1.

FIGURE 7-17: Signal Paths for Sample/Preload Instruction (highlighted)



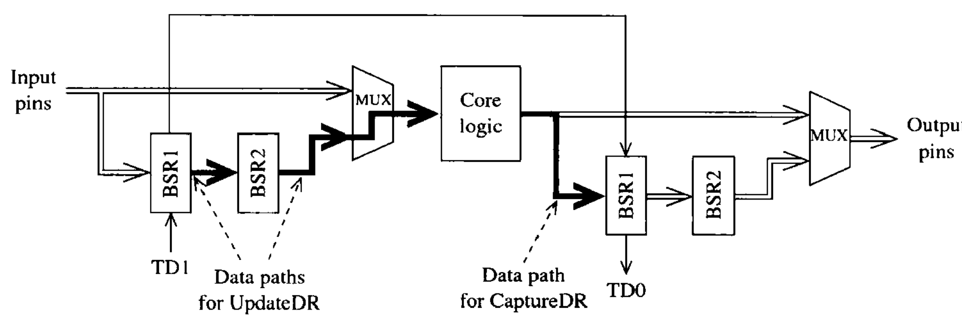
For the Extest instruction (Figure 7-18) the core logic is not used. In the UpdateDR state, BSR1 is loaded into BSR2 and the data is routed to the output pins of the IC. In the CaptureDR state, data from the input pins is loaded into BSR1.

FIGURE 7-18: Signal Paths for Extest Instruction (highlighted)



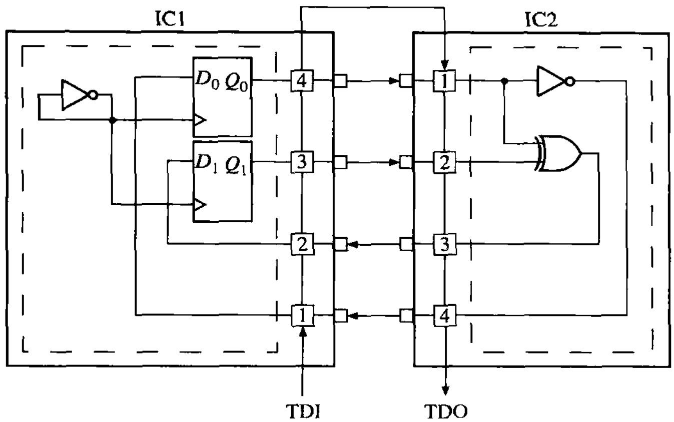
For the Intest instruction (Figure 7-19) the IC pins are not used. In the UpdateDR state, test data that has previously been shifted into BSR1 is loaded into BSR2 and routed to the core logic inputs. In the CaptureDR state, data from the core logic is loaded into BSR1.

FIGURE 7-19: Signal Paths for Intest Instruction (highlighted)



The following simplified example illustrates how the connections between two ICs can be tested using the SAMPLE/PRELOAD and EXTEST instructions. The test is intended to check for shorts and opens in the PC board traces. Both ICs have two input pins and two output pins, as shown in Figure 7-20. Test data is shifted into the BSRs via *TDI*. Then data from the input pins is parallel-loaded into the BSRs and shifted out via *TDO*. We assume that the instruction register on each IC is three bits long with EXTEST coded as 000 and SAMPLE/PRELOAD as 001. The core logic in IC1 is an inverter connected as a clock oscillator and two flip-flops. The core logic in IC2 is an inverter and XOR gate. The two ICs are interconnected to form a 2-bit counter.

FIGURE 7-20: Interconnection Testing Using Boundary Scan



The steps required to test the connections between the ICs are as follows:

1. Reset the TAP state machine to the Test-Logic-Reset state by inputting a sequence of five 1's on *TMS*.
2. Scan in the SAMPLE/PRELOAD instruction to both ICs using the sequences for *TMS* and *TDI* given here. The state numbers refer to Figure 7-16.

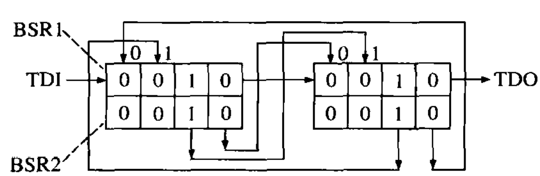
State:	0	1	2	9	10	11	11	11	11	11	11	12	15	2
TMS:	0	1	1	0	0	0	0	0	0	0	1	1	1	
TDI:	–	–	–	–	–	1	0	0	1	0	0	–	–	

The *TMS* sequence 01100 takes the TAP controller to the Shift-IR state. In this state, copies of the SAMPLE/PRELOAD instruction (code 001) are shifted into the instruction registers on both ICs. In the Update-IR state, the instructions are loaded into the instruction decode registers. Then the TAP controller goes back to the Select DR-scan state.

3. Preload the first set of test data into the ICs using the following sequences for *TMS* and *TDI*:

State:	2	3	4	4	4	4	4	4	4	4	5	8	2
TMS:	0	0	0	0	0	0	0	0	0	1	1	1	
TDI:	–	–	0	1	0	0	0	1	0	0	–	–	

Data is shifted into BSR1 in the Shift-DR state, and it is transferred to BSR2 in the Update-DR state. The result is as follows:



4. Scan in the EXTEST instruction to both ICs using the following sequences:

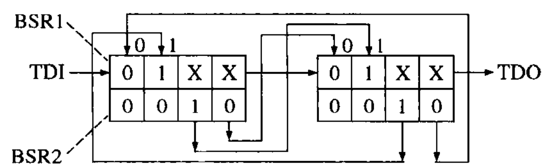
State:	2	9	10	11	11	11	11	11	11	12	15	2
TMS:	1	0	0	0	0	0	0	0	1	1	1	
TDI:	–	–	–	0	0	0	0	0	0	–	–	

The EXTEST instruction (000) is scanned into the instruction register in state Shift-IR and loaded into the instruction decode register in state Update-IR. At this point, the preloaded test data goes to the output pins, and it is transmitted to the adjacent IC input pins via the printed circuit board traces.

5. Capture the test results from the IC inputs. Scan this data out to *TDO* and scan the second set of test data in using the following sequences:

State:	2	3	4	4	4	4	4	4	4	4	5	8	2
TMS:	0	0	0	0	0	0	0	0	0	1	1	1	
TDI:	–	–	1	0	0	0	1	0	0	0	–	–	
TDO:	–	–	x	x	1	0	x	x	1	0	–	–	

The data from the input pins is loaded into BSR1 in state Capture-DR. At this time, if no faults have been detected, the BSRs should be configured as shown below, where the X's indicate captured data that is not relevant to the test.



The test results are then shifted out of BSR1 in state Shift-DR as the new test data is shifted in. The new data is loaded into BSR2 in the Update-IR state.

6. Capture the test results from the IC inputs. Scan this data out to *TDO* and scan all 0's in using the following sequences:

State:	2	3	4	4	4	4	4	4	4	4	5	8	2	9	0
TMS:	0	0	0	0	0	0	0	0	0	1	1	1	1	1	
TDI:	–	–	0	0	0	0	0	0	0	0	–	–	–	–	
TDO:	–	–	x	x	0	1	x	x	0	1	–	–	–	–	

The data from the input pins is loaded into BSR1 in state Capture-DR. Then it is shifted out in state Shift-DR as all 0's are shifted in. The 0's are loaded into BSR2 in the Update-DR state. The controller then returns to the Test-Logic-Reset state, and normal operation of the ICs can then occur. The interconnection test passes if the observed *TDO* sequences match the ones given above.

VHDL code for the basic boundary scan architecture of Figure 7-15 is given in Figure 7-21. Only the three mandatory instructions (EXTEST, SAMPLE/PRELOAD, and BYPASS) are implemented using a 3-bit instruction register. These instructions are

coded as 000, 001, and 111, respectively. The number of cells in the *BSR* is a generic parameter. A second generic parameter, *CellType*, is a bit_vector that specifies whether each cell is an input cell or output cell. The case statement implements the TAP controller state machine. The instruction code is scanned in and loaded into *IDR* in states Capture-IR, Shift-IR, and Update-IR. The instructions are executed in states Capture-DR, Shift-DR, and Update-DR. The actions taken in these states depend on the instruction being executed. The register updates and state changes all occur on the rising edge of *TCK*. The VHDL code implements most of the functions required by the IEEE boundary scan standard, but it does not fully comply with the standard.

FIGURE 7-21: VHDL Code for Basic Boundary Scan Architecture

```
-- VHDL for Boundary Scan Architecture of Figure 7-15

entity BS_arch is
  generic(NCELLS: natural range 2 to 120 := 2);
        -- number of boundary scan cells
  port(TCK, TMS, TDI: in bit;
       TDO: out bit;
       BSRin: in bit_vector(1 to NCELLS);
       BSRout: inout bit_vector(1 to NCELLS);
       CellType: bit_vector(1 to NCELLS));
       -- '0' for input cell, '1' for output cell
end BS_arch;

architecture behavior of BS_arch is
  signal IR, IDR: bit_vector(1 to 3);          -- instruction registers
  signal BSR1, BSR2: bit_vector(1 to NCELLS); -- boundary scan cells
  signal BYPASS: bit;                          -- bypass bit
  type TAPstate is (TestLogicReset, RunTest_Idle,
    SelectDRScan, CaptureDR, ShiftDR, Exit1DR, PauseDR, Exit2DR, UpdateDR,
    SelectIRScan, CaptureIR, ShiftIR, Exit1IR, PauseIR, Exit2IR, UpdateIR);
  signal St: TAPstate;                         -- TAP Controller State
begin
  process (TCK)
  begin
    if TCK'event and TCK='1' then
     -- TAP Controller State Machine
      case St is
        when TestLogicReset =>
          if TMS='0' then St <= RunTest_Idle; else St<=TestLogicReset; end if;
        when RunTest_Idle =>
          if TMS='0' then St <= RunTest_Idle; else St <= SelectDRScan; end if;
        when SelectDRScan =>
          if TMS='0' then St <= CaptureDR; else St <= SelectIRScan; end if;
        when CaptureDR =>
          if IDR = "111" then BYPASS <= '0';
          elsif IDR = "000" then  -- EXTEST (input cells capture pin data)
```

```
          BSR1 <= (not CellType and BSRin) or (CellType and BSR1);
        elsif IDR = "001" then - SAMPLE/PRELOAD
          BSR1 <= BSRin;
        end if;  -- all cells capture cell input data
        if TMS='0' then St <= ShiftDR; else St <= Exit1DR; end if;
      when ShiftDR =>
        if IDR = "111" then BYPASS <= TDI; -- shift data through bypass reg.
        else BSR1 <= TDI & BSR1(1 to NCELLS-1); end if;
         -- shift data into BSR
        if TMS='0' then St <= ShiftDR; else St <= Exit1DR; end if;
      when Exit1DR =>
        if TMS='0' then St <= PauseDR; else St <= UpdateDR; end if;
      when PauseDR =>
        if TMS='0' then St <= PauseDR; else St <= Exit2DR; end if;
      when Exit2DR =>
        if TMS='0' then St <= ShiftDR; else St <= UpdateDR; end if;
      when UpdateDR =>
        if IDR = "000" then -- EXTEST (update output reg. for output cells)
          BSR2 <= (CellType and BSR1) or (not CellType and BSR2);
        elsif IDR = "001" then    -- SAMPLE/PRELOAD
          BSR2 <= BSR1;           -- update output reg. in all cells
        end if;
        if TMS='0' then St <= RunTest_Idle; else St <= SelectDRScan; end if;
      when SelectIRScan =>
        if TMS='0' then St <= CaptureIR; else St <= TestLogicReset; end if;
      when CaptureIR =>
        IR <= "001"; -- load 2 LSBs of IR with 01 as required by the standard
        if TMS='0' then St <= ShiftIR; else St <= Exit1IR; end if;
      when ShiftIR =>
        IR <= TDI & IR(1 to 2);  -- shift in instruction code
        if TMS='0' then St <= ShiftIR; else St <= Exit1IR; end if;
      when Exit1IR =>
        if TMS='0' then St <= PauseIR; else St <= UpdateIR; end if;
      when PauseIR =>
        if TMS='0' then St <= PauseIR; else St <= Exit2IR; end if;
      when Exit2IR =>
        if TMS='0' then St <= ShiftIR; else St <= UpdateIR; end if;
      when UpdateIR =>
        IDR <= IR;   -- update instruction decode register
        if TMS='0' then St <= RunTest_Idle; else St <= SelectDRScan; end if;
    end case;
  end if;
end process;

TDO <= BYPASS when St = ShiftDR and IDR = "111"  -- BYPASS
  else BSR1(NCELLS) when St=ShiftDR   -- EXTEST or SAMPLE/PRELOAD
  else IR(3) when St=ShiftIR;

BSRout <= BSRin when (St = TestLogicReset or not (IDR = "000"))
  else BSR2;                          -- define cell outputs
end behavior;
```

VHDL code that implements the interconnection test example of Figure 7-20 is given in Figure 7-22. The *TMS* and *TDI* test patterns are the concatenation of the test patterns used in steps 2 through 6. A copy of the basic boundary scan architecture is instantiated for IC1 and for IC2. The external connections and internal logic for each IC are then specified. The internal clock frequency was arbitrarily chosen to be different than the test clock frequency. The test process runs the internal logic, then runs the scan test, and then runs the internal logic again. The test results verify that the IC logic runs correctly and that the scan test produces the expected results.

FIGURE 7-22: VHDL Code for Interconnection Test Example

```
 -- Boundary Scan Tester

entity system is
end system;

architecture IC_test of system is
  component BS_arch is
    generic(NCELLS:natural range 2 to 120 := 4);
    port(TCK, TMS, TDI: in bit;
         TDO: out bit;
         BSRin: in bit_vector(1 to NCELLS);
         BSRout: inout bit_vector(1 to NCELLS);
         CellType: in bit_vector(1 to NCELLS));
         -- '0' for input cell, '1' for output cell
  end component;

  signal TCK, TMS, TDI, TDO, TDO1: bit;
  signal Q0, Q1, CLK1: bit;
  signal BSR1in, BSR1out, BSR2in, BSR2out: bit_vector(1 to 4);
  signal count: integer := 0;

  constant TMSpattern: bit_vector(0 to 62) :=
    "011000000011100000000111110000000111000000000111000000000111111";
  constant TDIpattern: bit_vector(0 to 62) :=
    "000001001000000010001000000000000000000100010000000000000000000";
begin
  BS1: BS_arch port map(TCK, TMS, TDI, TDO1, BSR1in, BSR1out, "0011");
  BS2: BS_arch port map(TCK, TMS, TDO1, TDO, BSR2in, BSR2out, "0011");
  -- each BSR has two input cells and two output cells
  BSR1in(1) <= BSR2out(4);                -- IC1 external connections
  SR1in(2) <= BSR2out(3);
  BSR1in(3) <= Q1;                        -- IC1 internal logic
  BSR1in(4) <= Q0;
  CLK1 <= not CLK1 after 7 ns;             -- internal clock
```

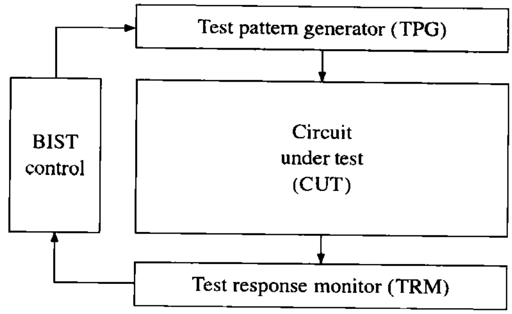
```
  process(CLK1)
  begin
    if CLK1 = '1' then                              -- D flip-flops
      Q0 <= BSR1out(1);
      Q1 <= BSR1out(2);
    end if;
  end process;

  BSR2in(1) <= BSR1out(4);                        -- IC2 external connections
  BSR2in(2) <= BSR1out(3);
  BSR2in(3) <= BSR2out(1) xor BSR2out(2);         -- IC2 internal logic
  BSR2in(4) <= not BSR2out(1);

  TCK <= not TCK after 5 ns;                                 -- test clock

  process
  begin
    TMS <= '1';
    wait for 70 ns;                               -- run internal logic
    wait until TCK = '1';
    for i in TMSpattern'range loop                -- run scan test
      TMS <= TMSpattern(i);
      TDI <= TDIpattern(i);
      wait for 0 ns;
      count <= i + 1;                               -- count triggers listing output
      wait until TCK = '1';
    end loop;
    wait for 70 ns;                               -- run internal logic
    wait;                                         -- stop
  end process;
end IC_test;
```

7.5 Built-In Self-Test

As digital systems become more and more complex, they become much harder and more expensive to test. One solution to this problem is to add logic to the IC so that it can test itself. This is referred to as built-in self-test, or BIST. Figure 7-23 illustrates the general method for using BIST. An on-chip test generator applies test patterns to the circuit under test. The resulting output is observed by the response monitor, which produces an error signal if an incorrect output pattern is detected.

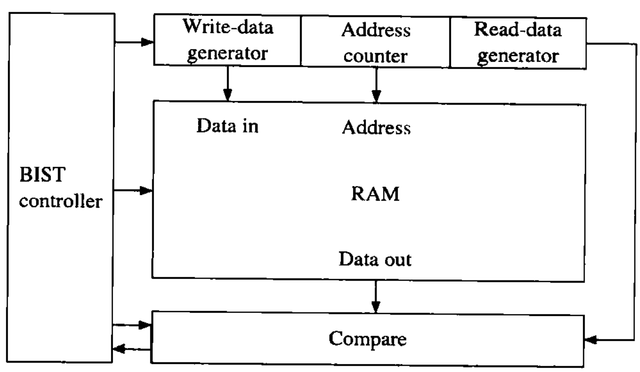
BIST is often used for testing memory. The regular structure of a memory chip makes it easy to generate test patterns. Figure 7-24 shows a block diagram of a self-test circuit for a RAM. The BIST controller enables the write-data generator and address counter so that data is written to each location in the RAM. Then the address counter and read-data generator are enabled, and the data read from each

FIGURE 7-23: Generic BIST Scheme



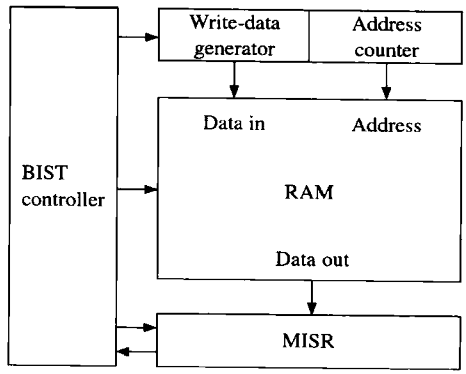
RAM location is compared with the output of the read-data generator to verify that it is correct. Memory is often tested by writing **checkerboard patterns** (alternating 0's and 1's) in all memory locations and reading them back. For instance, we could first write alternating 0's and 1's in all even addresses and alternating 1's and 0's in all odd addresses. After reading these back, the odd and even address patterns can be swapped to complete the test. In another test, the **March test**, each cell is read and then the complemented value is written. This process is continued until the entire memory array has been traversed. Then the process is repeated in the reverse order of addresses.

FIGURE 7-24: Self-Test Circuit for RAM



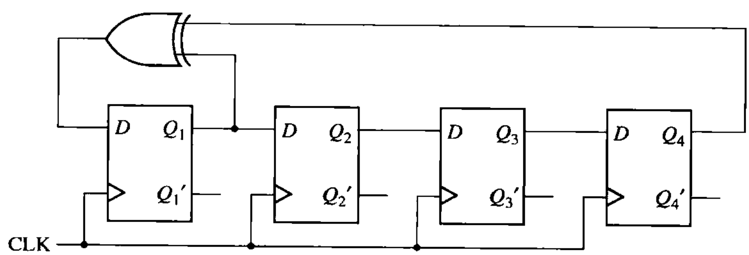
The test circuit can be simplified by using a signature register. The signature register compresses the output data into a short string of bits called a **signature**, and this signature is compared with the signature for a correctly functioning component. A **multiple-input signature register (MISR)** combines and compresses several output streams into a single signature. Figure 7-25 shows a simplified version of the RAM self-test circuit. The read-data generator and comparator have been eliminated and replaced with a MISR. One type of MISR simply forms a check sum by adding up all the data bytes stored in the RAM. When testing a ROM, Figure 7-25 can be simplified further, since no write-data generator is needed.

FIGURE 7-25: Self-Test Circuit for RAM with Signature Register



Linear feedback shift registers (LFSRs) are often used to generate test patterns and to compress test outputs into signatures. An LFSR is a shift register whose serial input bit is a linear function of some bits of the current shift register content. The bit positions that affect the serial input are called **taps**. The general form of a LFSR is a shift register with two or more flip-flop outputs XOR'ed together and fed back into the first flip-flop. The name **linear** comes from the fact that exclusive OR is equivalent to modulo-2 addition, and addition is a linear operation. Figure 7-26 shows an example of a LFSR. The outputs from the first and fourth flip-flops are XOR'ed together and fed back into the D input of the first flip-flop; the taps are positions 1 and 4.

FIGURE 7-26: Four-Bit Linear Feedback Shift Register (LFSR)



By proper choice of the outputs that are fed back through the exclusive OR gate, it is possible to generate $2^n - 1$ different bit patterns using an n-bit shift register. All possible patterns can be generated except for all 0's. The patterns generated by the LFSR of Figure 7-26 are

$$1000, 1100, 1110, 1111, 0111, 1011, 0101, 1010, 1101, 0110, 0011,$$
$$1001, 0100, 0010, 0001, 1000, \ldots$$

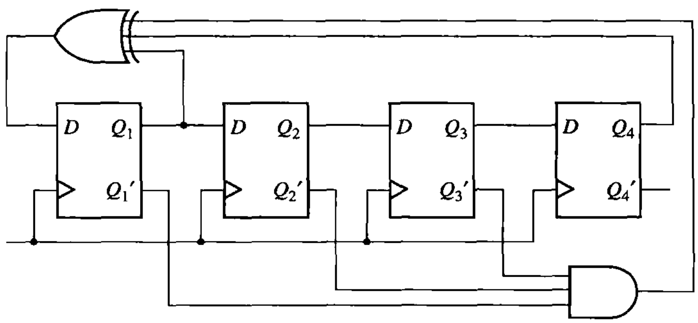
These patterns have no obvious order, and they have certain randomness properties. Such an LFSR is often referred to as a **pseudo-random pattern generator**, or **PRPG**. PRPGs are obviously very useful for BIST, since they can generate a large number of test patterns with a small amount of logic circuitry. Table 7-4 gives a feedback combination that will generate all $2^n - 1$ bit patterns for some LFSRs with lengths in the range $n = 4$ to 32.

TABLE 7-4: Feedback for Maximum-Length LFSR Sequence

n	Feedback
4, 6, 7	$Q_1 \oplus Q_n$
5	$Q_2 \oplus Q_5$
8	$Q_2 \oplus Q_3 \oplus Q_4 \oplus Q_8$
12	$Q_1 \oplus Q_4 \oplus Q_6 \oplus Q_{12}$
14, 16	$Q_3 \oplus Q_4 \oplus Q_5 \oplus Q_n$
24	$Q_1 \oplus Q_2 \oplus Q_7 \oplus Q_{24}$
32	$Q_1 \oplus Q_2 \oplus Q_{22} \oplus Q_{32}$

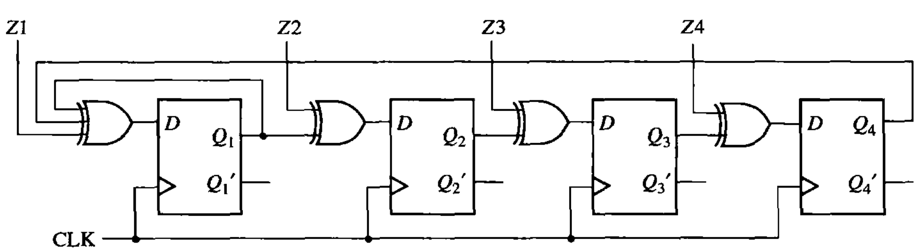
If the all-0s test pattern is required, an n-bit LFSR can be modified by adding an AND gate with $n - 1$ inputs, as shown in Figure 7-27 for $n = 4$. When in state 0001, the next state is 0000; when in state 0000, the next state is 1000; otherwise, the sequence is the same as for Figure 7-26.

FIGURE 7-27: Modified LFSR with 0000 State



An MISR can be constructed by modifying a LFSR by adding XOR gates, as shown in Figure 7-28. The test data ($Z_1Z_2Z_3Z_4$) is XOR'ed into the register with each clock, and the final result represents a signature that can be compared with the signature for a known correctly functioning component. This type of signature analysis will catch many, but not all, possible errors. An n-bit signature register maps all possible input streams into one of the 2^n possible signatures. One of these is the correct signature, and the others indicate that errors have occurred. The probability that an incorrect input sequence will map to the correct signature is of the order of $1/2^n$.

FIGURE 7-28: Multiple-Input Signature Register (MISR)



For the MISR of Figure 7-28, assume that the correct input sequence is 1010, 0001, 1110, 1111, 0100, 1011, 1001, 1000, 0101, 0110, 0011, 1101, 0111, 0010, 1100. This

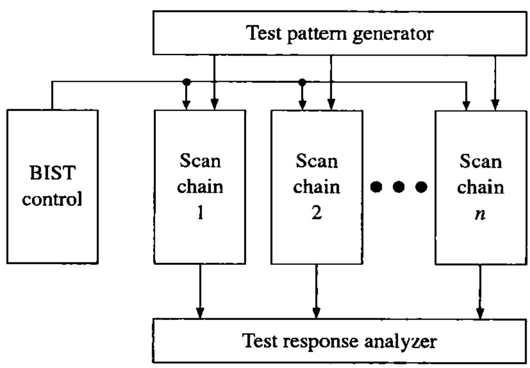
sequence maps to the signature 0010, assuming the initial contents of the MISR to be 0000. Any input sequence that differs in one bit will map to a different signature. For example, if 0001 in the sequence is changed to 1001, the resulting sequence maps to 0000. Most sequences with two errors will be detected, but if we change 0001 to 1001 and 0010 to 0110 in the original sequence, the result maps to 0010, which is the correct signature, so the errors would not be detected.

Several types of architectures have been proposed for BIST. Two popular examples are the STUMPS architecture and the BILBO architecture.

STUMPS stands for **S**elf-**T**esting **U**sing an **MISR** and **P**arallel **SRSG**. SRSG, in turn, stands for Shift Register Sequence Generator. STUMPS is a BIST architecture that uses scan chains. An overview of the STUMPS architecture is shown in Figure 7-29. A pseudo-random pattern generator feeds test stimulus to the scan chains, and after a capture cycle, the test response analyzer receives the test responses. The test procedure in STUMPS is the following:

1. Scan in patterns from the test pattern generator (LFSR) into all scan chains.
2. Switch to normal function mode and clock once with system clock.
3. Shift out scan chain into test response analyzer (MISR) where test signature is generated.

FIGURE 7-29: The STUMPS Architecture



If the scan chain contains 100 scan cells, steps 1 and 3 will take 100 clocks. All scan chains should first be filled by the pseudo-random generator; hence, long scan chains necessitate long testing times. Since one test is done per scan, the STUMPS architecture is called a **test-per-scan** scheme. In order to reduce the testing time, a large number of parallel scan chains can be used, which reduces the time for filling the scan chains with the test since all scan chains can be loaded in parallel.

The STUMPS architecture was originally developed for self-testing of multichip modules [7]. The scan chain on each logic chip (module) is loaded in parallel from the pseudo-random pattern source. The number of clock cycles required is equal to the number of flip-flops in the longest scan chain. If there are m scan cells in the longest scan chain, it will take $2m + 1$ cycles to perform one test (m cycles for scan-in, one for capture, and m cycles for scan out). The shorter scan chains will overflow into the MISR, but that will not affect the final correct signature.

In order to reduce test-times, steps 1 and 3 can be overlapped. When the scan chain is unloaded into the MISR after one test, simultaneously the next pseudo-random pattern set from the SRSG can be loaded into the scan chain (i.e., when test response from test I is being shifted out, test pattern for test $I + 1$ can be shifted in). Assuming overlap between scan-out of a test and scan-in of the following test, each test vector will take $m + 1$ cycles, and it will take $n(m + 1) + m$ cycles to apply n test vectors, including the m cycles taken for the last scan-out.

As opposed to the test-per-scan scheme just discussed, a **test-per-clock** scheme can be used for faster testing. One such scheme is called the **BILBO (Built-In Logic Block Observer)** technique. In BILBO schemes, the scan register is modified so that parts of the scan register can serve as a state register, pattern generator, signature register, or shift register. When used as a shift register, the test data can be scanned in and out in the usual way. During testing, part of the scan register can be used as a pattern generator (PRPG) and part as a signature register (MISR) to test one of the combinational blocks. The roles can then be changed to test another combinational block. When the testing is finished, the scan register is placed in the state register mode for normal operation. After the BILBO registers are initialized, since there is no loading of test patterns as in the case of scan chains, a test can be applied in each clock cycle. Hence, this is categorized as a test-per-clock BIST scheme. BILBO involves shorter test lengths, but more test hardware.

Figure 7-30 shows the placement of BILBO registers for testing a circuit with two combinational blocks. Combinational circuit 1 is tested when the first BILBO is used as a PRPG and the second as an MISR. The roles of the registers are reversed to test combinational circuit 2. In the normal operating mode, both BILBOs serve as registers for the associated combinational logic. To scan data in and out, both BILBOs operate in the shift register mode.

FIGURE 7-30: BIST Using BILBO Registers

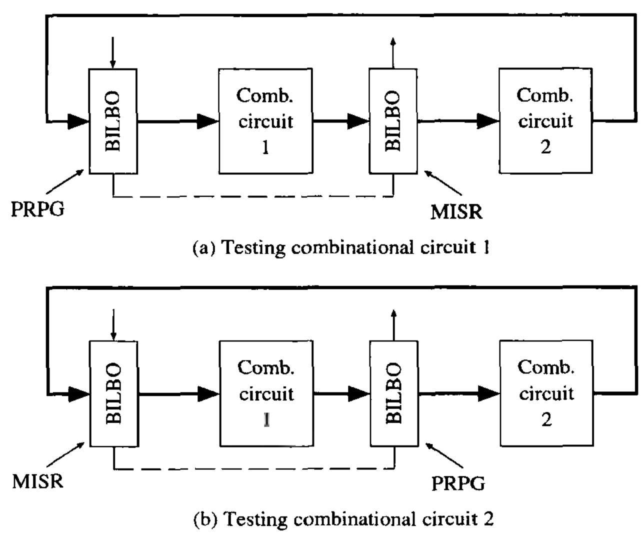


Figure 7-31 shows the structure of one version of a 4-bit BILBO register. The control inputs B_1 and B_2 determine the operating mode. Si and So are the serial

input and output for the shift register mode. The Z's are inputs from the combinational logic. The equations for this BILBO register are

$$D_1 = Z_1 B_1 \oplus (Si\, B_2' + FB\, B_2)(B_1' + B_2)$$
$$D_i = Z_i B_1 \oplus Q_{i-1}(B_1' + B_2)\ (i > 1)$$

When $B_1 = B_2 = 0$, these equations reduce to

$$D_1 = Si \text{ and } D_i = Q_{i-1}\ (i > 1)$$

which corresponds to the shift register mode. When $B_1 = 0$ and $B_2 = 1$, the equations reduce to

$$D_1 = FB, \qquad D_i = Q_{i-1}$$

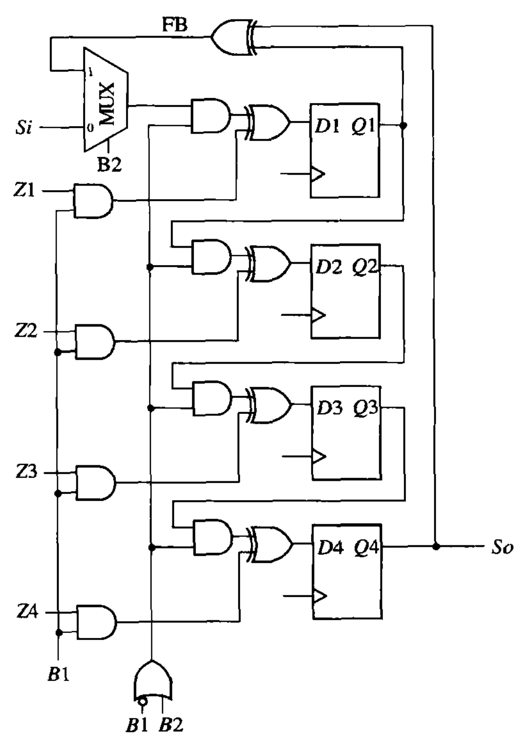
which corresponds to the PRPG mode, and the BILBO register is equivalent to Figure 7-26. When $B_1 = 1$ and $B_2 = 0$, the equations reduce to

$$D_1 = Z_1, \qquad D_i = Z_i$$

which corresponds to the normal operating mode. When $B_1 = B_2 = 1$, the equations reduce to

$$D_1 = Z_1 \oplus FB, \qquad D_i = Z_i \oplus Q_{i-1}$$

FIGURE 7-31: Four-Bit BILBO Register



which corresponds to the MISR mode, and the BILBO register is equivalent to Figure 7-28. In summary, the BILBO operating modes are as follows:

B1B2	Operating Mode
00	Shift register
01	PRPG
10	Normal
11	MISR

Figure 7-32 shows the VHDL description of an *n*-bit BILBO register. NBITS, which equals the number of bits, is a generic parameter in the range 4 through 8. The

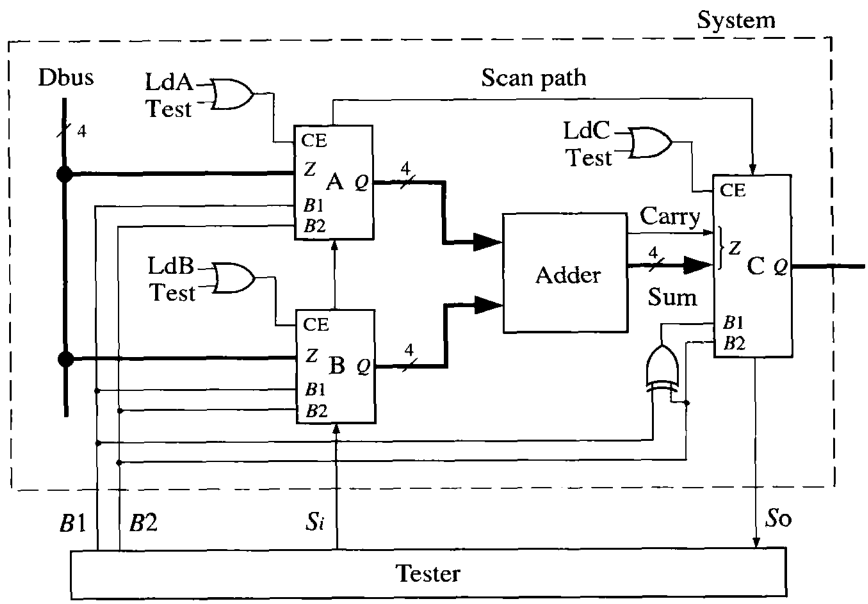
FIGURE 7-32: VHDL Code for BILBO Register of Figure 7-31

```
entity BILBO is                    -- BILBO Register
  generic (NBITS: natural range 4 to 8 := 4);
  port (Clk, CE, B1, B2, Si: in bit;
        So: out bit;
        Z: in bit_vector(1 to NBITS);
        Q: inout bit_vector(1 to NBITS));
end BILBO;

architecture behavior of BILBO is
  signal FB: bit;
begin
  Gen8: if NBITS = 8 generate
    FB <= Q(2) xor Q(3) xor Q(NBITS); end generate;
  Gen5: if NBITS = 5 generate
    FB <= Q(2) xor Q(NBITS); end generate;
  GenX: if not(NBITS = 5 or NBITS = 8) generate
    FB <= Q(1) xor Q(NBITS); end generate;
  process(Clk)
    variable mode: bit_vector(1 downto 0);
  begin
    if (Clk = '1' and CE = '1') then
      mode := B1 & B2;
      case mode is
        when "00" =>      -- Shift register mode
          Q <= Si & Q(1 to NBITS-1);
        when "01" =>      -- Pseudo Random Pattern Generator mode
          Q <= FB & Q(1 to NBITS-1);
        when "10" =>      -- Normal Operating mode
          Q <= Z;
        when "11" =>      -- Multiple Input Signature Register mode
          Q <= Z(1 to NBITS) xor (FB & Q(1 to NBITS-1));
      end case;
    end if;
  end process;
  So <= Q(NBITS);
end behavior;
```

register is functionally equivalent to Figure 7-31, except that we have added a clock enable (*CE*). The feedback (*FB*) for the LFSR depends on the number of bits.

The system shown in Figure 7-33 illustrates the use of BILBO registers. In this system, registers *A* and *B* can be loaded from the Dbus using the *LDA* and *LDB* signals. Then the registers are added and the sum and carry are stored in register *C*. When *B*1 & *B*2 = 10, the registers are in the normal mode (*Test* = 0), and loading of the registers is controlled by *LDA*, *LDB*, and *LDC*. To test the adder, we first set *B*1 & *B*2 = 00 to place the registers in the shift register mode and scan in initial values for *A*, *B*, and *C*. Then we set *B*1 & *B*2 = 01, which places registers *A* and *B* in PRPG mode and register *C* in MISR mode. After 15 clocks, the test is complete. Then we can set *B*1 & *B*2 = 00 and scan out the signature.

FIGURE 7-33: System with BILBO Registers and Tester



The VHDL code for the system is given in Figure 7-34, and a test bench is given in Figure 7-35. The system uses three BILBO registers and the 4-bit adder of Figure 3-20. The test bench scans in a test vector to initialize the BILBO registers; then it runs the test with registers *A* and *B* used as PRPGs and register *C* as a MISR. The resulting signature is shifted out and compared with the correct signature.

FIGURE 7-34: VHDL Code for System with BILBO Registers and Tester

```
entity BILBO_System is
  port(Clk, LdA, LdB, LdC, B1, B2, Si: in bit;
       So: out bit;
       DBus: in bit_vector(3 downto 0);
       Output: inout bit_vector(4 downto 0));
end BILBO_System;
```

```
architecture BSys1 of BILBO_System is
  component Adder4 is
    port(A, B: in bit_vector(3 downto 0); Ci: in bit;
         S: out bit_vector(3 downto 0); Co: out bit);
  end component;
  component BILBO is
    generic(NBITS: natural range 4 to 8 := 4);
    port(Clk, CE, B1, B2, Si : in bit;
         So: out bit;
         Z: in bit_vector(1 to NBITS);
         Q: inout bit_vector(1 to NBITS));
  end component;

  signal Aout, Bout: bit_vector(3 downto 0);
  signal Cin: bit_vector(4 downto 0);
  alias Carry: bit is Cin(4);
  alias Sum: bit_vector(3 downto 0) is Cin(3 downto 0);
  signal ACE, BCE, CCE, CB1, Test, S1, S2: bit;
begin
  Test <= not B1 or B2;
  ACE <= Test or LdA;
  BCE <= Test or LdB;
  CCE <= Test or LdC;
  CB1 <= B1 xor B2;
  RegA: BILBO generic map (4) port map(Clk, ACE, B1, B2, S1, S2, DBus, Aout);
  RegB: BILBO generic map (4) port map(Clk, BCE, B1, B2, Si, S1, DBus, Bout);
  RegC: BILBO generic map (5) port map(Clk, CCE, CB1, B2, S2, So, Cin, Output);
  Adder: Adder4 port map(Aout, Bout, '0', Sum, Carry);
end BSys1;
```

FIGURE 7-35: Test Bench for BILBO System

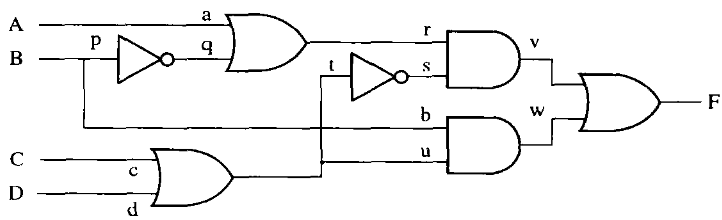
```
-- System with BILBO test bench

entity BILBO_test is
end BILBO_test;

architecture Btest of BILBO_test is
  component BILBO_System is
    port(Clk, LdA, LdB, LdC, B1, B2, Si: in bit;
         So: out bit;
         DBus: in bit_vector(3 downto 0);
         Output: inout bit_vector(4 downto 0));
  end component;
  signal Clk: bit := '0';
  signal LdA, LdB, LdC, B1, B2, Si, So: bit := '0';
```

```
  signal DBus: bit_vector(3 downto 0);
  signal Output: bit_vector(4 downto 0);
  signal Sig: bit_vector(4 downto 0);

  constant test_vector: bit_vector(12 downto 0) := "1000110000000";
  constant test_result: bit_vector(4 downto 0) := "01011";
begin
  clk <= not clk after 25 ns;
  Sys: BILBO_System port map(Clk,Lda,LdB,LdC,B1,B2,Si,So,DBus,Output);
  process
  begin
    B1 <= '0'; B2 <= '0';            -- Shift in test vector
    for i in test_vector'right to test_vector'left loop
      Si <= test_vector(i);
      wait until clk = '1';
    end loop;

    B1 <= '0'; B2 <= '1';            -- Use PRPG and MISR
    for i in 1 to 15 loop
      wait until clk = '1';
    end loop;

    B1 <= '0'; B2 <= '0';            -- Shift signature out
    for i in 0 to 5 loop
      Sig <= So & Sig(4 downto 1);
      wait until clk = '1';
    end loop;

    if (Sig = test_result) then      -- Compare signature
      report "System passed test.";
    else
      report "System did not pass test!";
    end if;

    wait;
  end process;
end Btest;
```

In this chapter, we introduced the subject of testing hardware, including combinational circuits, sequential circuits, complex ICs, and PC boards. Use of scan techniques for testing and built-in self-test has become a necessity as digital systems have become more complex. It is very important that design for testability be considered early in the design process so that the final hardware can be tested efficiently and economically.

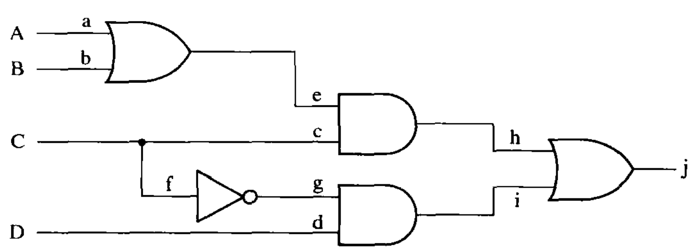
7.6 Problems

7.1 **(a)** Determine the necessary inputs to the following circuit to test for *u* stuck-at-0.
(b) For this set of inputs, determine which other stuck-at faults can be tested.
(c) Repeat (a) and (b) for *r* stuck-at-1.

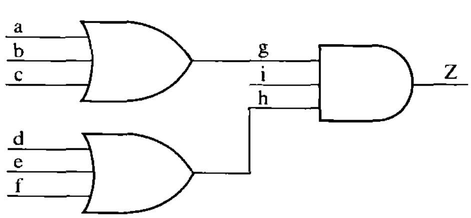


7.2 For the following circuit,

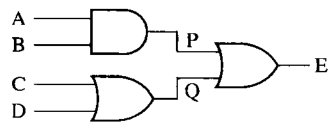
(a) Determine the values of *A, B, C*, and *D* necessary to test for *e* s-a-1. Specify the other faults tested by this input vector.
(b) Repeat (a) for *g* s-a-0.



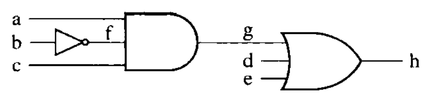
7.3 Find a minimum set of tests that will test all single stuck-at-0 and stuck-at-1 faults in the following circuit. For each test, specify which faults are tested for s-a-0 and for s-a-1.



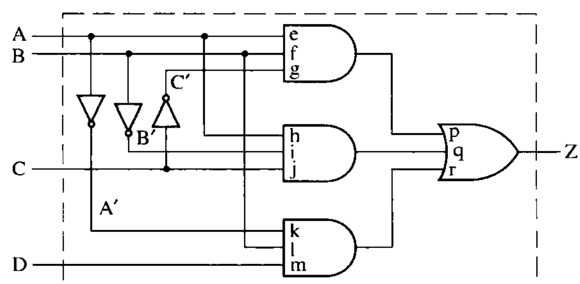
7.4 Give a minimum set of test vectors that will test for all stuck-at faults in the following circuit. List the faults tested by each test vector.



7.5 For the following circuit, specify a minimum set of test vectors for *a*, *b*, *c*, *d*, and *e* that will test for all stuck-at faults. Specify the faults tested by each vector.

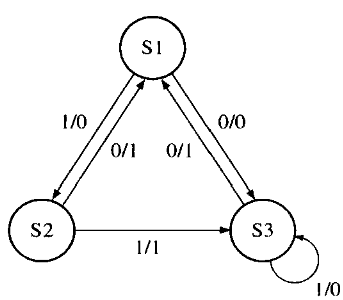


7.6 For the following circuit, find a minimum number of test vectors that will test all s-a-0 and s-a-1 faults at the AND and OR gate inputs. For each test vector, specify the values of *A*, *B*, *C* and *D*, and the stuck-at faults that are tested.



7.7 Find a test sequence to test for *b* s-a-0 in the sequential circuit of Figure 7-7.

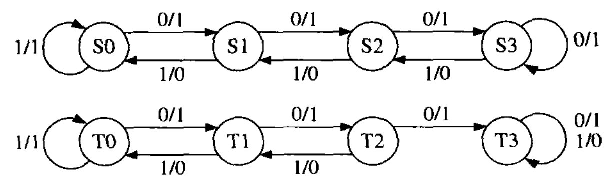
7.8 A sequential circuit has the following state graph:



The three states can be distinguished using the input sequence 11 and observing the output. The circuit has a reset input, *R*, that resets the circuit to state S_1. Give a

set of test sequences that will test every state transition and give the transition tested by each sequence. (When you test a state transition, you must verify that the output and the next state are correct by observing the output sequence.)

7.9 State graphs for two sequential machines are given below. The first graph represents a correctly functioning machine, and the second represents the same machine with a malfunction. Assuming that the two machines can be reset to their starting states (S_0 and T_0), determine the shortest input sequence that will distinguish the two machines.



7.10 When testing a sequential circuit, what are the major advantages of using scan-path testing compared to applying input sequences and observing output sequences?

7.11 A scan path test circuit of the type shown in Figure 7-8 has three flip-flops, two inputs, and two outputs. One row of the state table of the sequential circuit to be tested is as follows:

	$Q_1^+Q_2^+Q_3^+$				Z_1Z_2			
$Q_1Q_2Q_3$	$X_1X_2 =$ 00	01	11	10	00	01	11	10
011	010	110	011	111	10	11	00	01

For this row of the table, complete a timing chart similar to Figure 7-9 to show how the circuit can be tested to verify the next states and outputs for inputs 00, 01, and 10. Show the expected Z_1 and Z_2 outputs only at the time when they should be read.

7.12 **(a)** Redraw the code converter circuit of Figure 1-26 in the form of Figure 7-8 using dual-port flip-flops.
(b) Determine a test sequence that will verify the first two rows of the transition table of Figure 1-24(b). Draw a timing diagram similar to Figure 7-9 for your test sequence.

7.13 **(a)** Write VHDL code for a dual-port flip-flop.
(b) Write VHDL code for your solution to Problem 7.12(a).
(c) Write a test bench that applies the test sequence from Problem 7.12(b), and compare the resulting waveforms with your solution to Problem 7.12(b).

7.14 Instead of using dual-port flip-flops of the type shown in Figure 7-8, scan testing can be accomplished using standard D flip-flops with a mux on each D input to select D_1 or D_2. Redraw the circuit of Figure 1-22 to establish a scan chain using D flip-flops and muxes. A test signal (T) should control the muxes.

7.15 Referring to Figure 7-16, determine the sequence of *TMS* and *TDI* inputs required to load the instruction register with 011 and the boundary scan register BSR2 with 1101. Start in state 0 and end in state 1. Give the sequence of states along with the *TMS* and *TDI* inputs.

7.16 The INTEST instruction (code 010) allows testing of the core logic by shifting test data into the boundary scan register (BSR1) and then updating BSR2 with this test data. For input cells this data takes the place of data from the input pins. Output data from the core logic is captured in BSR1 and then shifted out. For this problem, assume that the BSR has three cells.

(a) Referring to Figure 7-16, give the sequence for *TMS* and *TDI* that will load the instruction register with 010 and BSR2 with 011. Also give the state sequence, starting in state 0.

(b) In the code of Figure 7-21, what changes or additions must be made in the last BSRout assignment statement, in the CaptureDR state, and in the UpdateDR state to implement the INTEST instruction?

7.17 Based on the VHDL code of Figure 7-21, design a two-cell boundary scan register. The first cell should be an input cell, and the second cell an output cell. Do not design the TAP controller; just assume that the necessary control signals like *shift-DR*, *capture-DR*, and *update-DR* are available. Do not design the instruction register or instruction decoding logic; just assume that the following signals are available: *EXT* (EXTEST instruction is being executed), *SPR* (Sample/Preload instruction is being executed), and *BYP* (Bypass instruction is being executed). Use two flip-flops for BSR1, two flip-flops for BSR2, and one BYPASS flip-flop. In addition to the control signals mentioned above, the inputs are *Pin*1 (from a pin), *Core*2 (from the core logic), *TDI*, and *TCK*; outputs are *Core*1 (to core logic), *Pin*2 (to a pin), and *TDO*. Use *TCK* as the clock input for all of the flip-flops. Draw a block diagram showing the flip-flops, muxes, and so on. Then give the logic equations or connections for each flip-flop D input, each CE (clock enable), and each MUX control input.

7.18 Simulate the boundary scan tester of Figure 7-22 and verify that the results are as expected. Change the code to represent the case where the lower input to IC1 is shorted to ground, simulate again, and interpret the results.

7.19 Write VHDL code for the boundary scan cell of Figure 7-14(b). Rewrite the VHDL code of Figure 7-21 to use this boundary scan cell as a component in place of some of the behavioral code for the BSR. Use a generate statement to instantiate NCELLS copies of this component. Test your new code using the boundary scan tester example of Figure 7-22.

7.20 **(a)** Draw a circuit diagram for an LFSR with $n = 5$ that generates a maximum length sequence.
(b) Add logic so that 00000 is included in the state sequence.
(c) Determine the actual state sequence.

7.21 **(a)** Draw a circuit diagram for an LFSR with $n = 6$ that generates a maximum length sequence.
(b) Add logic so that 000000 is included in the sequence.
(c) Determine the 10 elements of the sequence starting in 101010.

7.22 **(a)** Write VHDL for an 8-bit MISR that is similar to Figure 7-28.
(b) Design a self-test circuit, similar to Figure 7-25, for a 6116 static RAM (see Figure 3-15). The write-data generator should store data in the following sequence: 00000000, 10000000, 11000000, . . . , 11111111, 01111111, 00111111, . . . , 00000000.
(c) Write VHDL code to test your design. Simulate the system for at least one example with no errors, one error, two errors, and three errors.

7.23 In the system of Figure 7-33, A, B, and C are BILBO registers. The B_1 and B_2 inputs to each of the registers determine its BILBO operating mode as follows:

$B_1B_2 = 00$, shift register; $B_1B_2 = 01$, PRPG (pattern generator);
$B_1B_2 = 10$, normal system mode; $B_1B_2 = 11$, MISR (signature register).

The shifting into A, B, and C is always LSB first. When in the test mode, the Dbus is not used. Specify the sequence of the Tester outputs (B_1, B_2, and S_i) needed to perform the following operations:

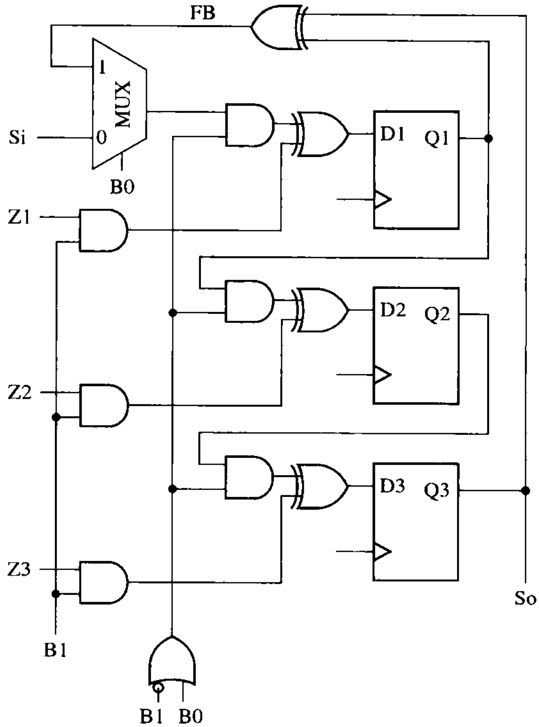
(1) Load A with 1011 and B with 1110, clear C.
(2) Test the system by using A and B as pattern generators and C as a signature register for four clock times.
(3) Shift the C register output into the tester.
(4) Return to the normal system mode.

$B_1\ B_2\ S_i = 0\ 0\ 0, \ldots$

7.24 Given the BILBO register shown below, specify B_1 and B_0 for each of the following modes:

normal mode
shift register mode
PRPG (LSFR) mode
MISR mode

When in the PRPG mode, what sequence of states would be generated for Q_1, Q_2, and Q_3, assuming that the initial state is 001?


FB
1
MUX
0
Si
B0
Z1
D1
Q1
D2
Q2
Z2
D3
Q3
Z3
So
B1
B1
B0

CHAPTER 8 Additional Design Examples

In this chapter, we present additional examples that show how VHDL, together with synthesis tools, can be used to simulate and design complex digital systems. We first design a wristwatch with alarm and stopwatch functions. Simulation models for memory chips with specific timing specifications are presented next. Finally, a receiver-transmitter for a serial data port is presented.

8.1 Design of a Wristwatch

In this section, we will design a multifunction wristwatch that has time-keeping, alarm, and stopwatch functions. The wristwatch has three buttons (B1, B2, and B3) that are used to change the mode, set the time, set the alarm, start and stop the stopwatch, and so on. Pushing button B1 changes the mode from **Time** to **Alarm** to **Stopwatch** and back to **Time**. The functions of buttons B2 and B3 vary depending on the mode and are explained in the following paragraphs.

8.1.1 Specifications

Operation in time mode: Display indicates the time and whether it is A.M. or P.M. using the format hh:mm:ss (A or P). When in time mode, the alarm can be shut off manually by pressing B3. Pushing B2 changes the state to Set Hours or Set Minutes and back to Time mode. When in the Set Hours or Set Minutes state, each press of B3 advances the hours or minutes by 1.

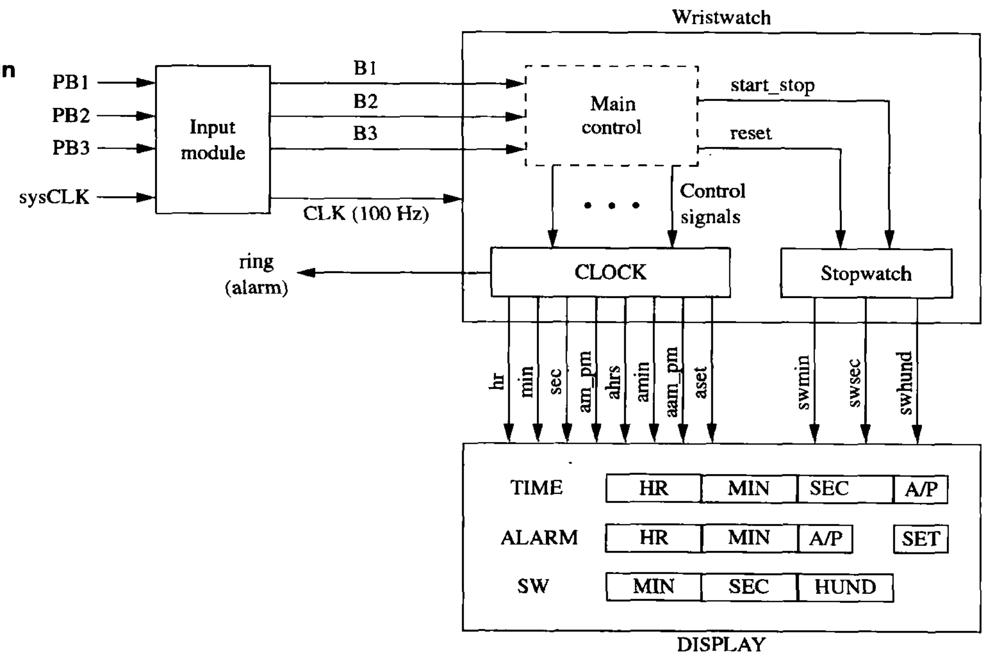
Operation in alarm mode: Display indicates the alarm time and whether it is A.M. or P.M. using the format hh:mm (A or P). Pushing B2 changes the state to Set Alarm Hours or Set Alarm Minutes and then back to Alarm. When in the Set Alarm Hours or Set Alarm Minutes state, each press of B3 advances the alarm hours or minutes by 1. When in the Alarm state, pressing B3 sets or resets the alarm. Once the alarm starts ringing, it will ring for 50 seconds and then shut itself off. It can also be shut off manually by pressing B3 in time mode.

Operation in the stopwatch mode: Display indicates stopwatch time in the format mm:ss.cc (where cc is hundredths of a second). Pressing B2 starts the time counter, pressing B2 again stops it, and then pressing B2 restarts it, and so on. Pressing B3 resets the time. Once the stopwatch is started, it will keep running even when the wristwatch is in time or alarm mode.

8.1.2 Design Implementation

Figure 8-1 shows a block diagram for the design. The **input module** divides the system clock down to a 100-Hz clock, CLK. It debounces the input buttons (PB1, PB2, and PB3) and synchronizes them with CLK. Each time PB1, PB2, or PB3 is pressed, the corresponding signal, B1, B2, or B3, will be 1 for exactly one clock time. The single pulser circuitry that we designed in Section 4.7 can be used to build this module.

FIGURE 8-1: Block Diagram of Wristwatch Design



The wristwatch module contains the **main control** for the wristwatch; the **clock module**, which implements the timekeeping and alarm functions; and the **stopwatch module**, which implements the stopwatch functions. The 100-Hz clock (CLK) synchronizes operation of the control unit and time registers. Figure 8-2 shows the state graph for the controller. This state machine generates the following control signals in response to pressing the buttons:

inch	increments hours in the set_hours state
incm	increments minutes in the set_minutes state
alarm_off	turns off the alarm when it is ringing
incha	increments hours for the alarm
incma	increments minutes for the alarm
set_alarm	toggles the alarm set on and off
start_stop	starts or stops the stopwatch counter
reset	resets the stopwatch counter

FIGURE 8-2: State Graph of Wristwatch Module

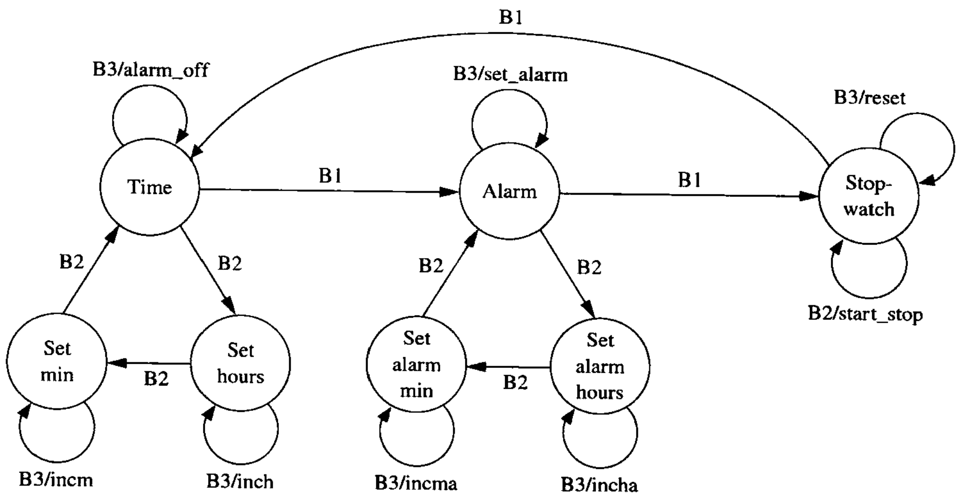


Figure 8-3 shows the VHDL code for the wristwatch module. This module instantiates the clock and stopwatch modules, and it implements the state machine. This state machine tests the B1, B2, and B3 button signals and generates the control signals. Following are some of the signal names used in the VHDL code:

am_pm	A.M. or P.M. in time mode
aam_pm	A.M. or P.M. in alarm mode
alarm_set	indicates that alarm is set
ring	indicates that alarm setting matches time counters, if alarm is set
hours	hours in time mode
ahours	hours in alarm mode
minutes	minutes in the time mode
aminutes	minutes in the alarm mode
seconds	seconds in the time mode
swhundredths	hundredths of a second during stopwatch mode
swseconds	seconds in stopwatch mode
swminutes	minutes in stopwatch mode

FIGURE 8-3: VHDL Code for the Wristwatch Module

```
library IEEE;
use IEEE.numeric_bit.all;

entity wristwatch is
  port(B1, B2, B3, clk: in bit;
       am_pm, aam_pm, ring, alarm_set: inout bit;
       hours, ahours, minutes, aminutes, seconds: inout unsigned(7 downto 0);
       swhundreths, swseconds, swminutes: out unsigned(7 downto 0));
end wristwatch;
```

```
architecture wristwatch1 of wristwatch is
  component clock is
    port(clk, inch, incm, incha, incma, set_alarm, alarm_off: in bit;
         hours, ahours, minutes, aminutes, seconds: inout unsigned(7 downto 0);
         am_pm, aam_pm, ring, alarm_set: inout bit);
  end component;
  component stopwatch is
    port(clk, reset, start_stop: in bit;
         swhundreths, swseconds, swminutes: out unsigned(7 downto 0));
  end component;
  type st_type is (time1, set_min, set_hours, alarm, set_alarm_hrs,
                   set_alarm_min, stop_watch);
  signal state, nextstate: st_type;
  signal inch, incm, alarm_off, set_alarm, incha, incma,
         start_stop, reset: bit;
begin
  clock1: clock port map(clk, inch, incm, incha, incma, set_alarm, alarm_off,
                         hours, ahours, minutes, aminutes, seconds, am_pm,
                         aam_pm, ring, alarm_set);
  stopwatch1: stopwatch port map(clk, reset, start_stop, swhundreths,
                                 swseconds, swminutes);
process(state, B1, B2, B3)
begin
  alarm_off <= '0'; inch <= '0'; incm <= '0'; set_alarm <= '0'; incha <= '0';
  incma <= '0'; start_stop <= '0'; reset <= '0';
  case state is
    when time1 =>
      if B1 = '1' then nextstate <= alarm;
      elsif B2 = '1' then nextstate <= set_hours;
      else nextstate <= time1;
      end if;
      if B3 = '1' then alarm_off <= '1';
      end if;
    when set_hours =>
      if B3 = '1' then inch <= '1'; nextstate <= set_hours;
      else nextstate <= set_hours;
      end if;
      if B2 = '1' then nextstate <= set_min;
      end if;
    when set_min =>
      if B3 = '1' then incm <= '1'; nextstate <= set_min;
      else nextstate <= set_min;
      end if;
      if B2 = '1' then nextstate <= time1;
      end if;
    when alarm =>
      if B1 = '1' then nextstate <= stop_watch;
      elsif B2 = '1' then nextstate <= set_alarm_hrs;
```

```
      else nextstate <= alarm;
      end if;
      if B3 = '1' then set_alarm <= '1'; nextstate <= alarm;
      end if;
    when set_alarm_hrs =>
      if B2 = '1' then nextstate <= set_alarm_min;
      else nextstate <= set_alarm_hrs;
      end if;
      if B3 = '1' then incha <= '1';
      end if;
    when set_alarm_min =>
      if B2 = '1' then nextstate <= alarm;
      else nextstate <= set_alarm_min;
      end if;
      if B3 = '1' then incma <= '1';
      end if;
    when stop_watch =>
      if B1 = '1' then nextstate <= time1;
      else nextstate <= stop_watch;
      end if;
      if B2 = '1' then start_stop <= '1';
      end if;
      if B3 = '1' then reset <= '1';
      end if;
    end case;
  end process;
  process(clk)
  begin
    if clk'event and clk = '1' then
      state <= nextstate;
    end if;
  end process;
end wristwatch1;
```

The clock module contains the counters that keep track of time (*hours*, *minutes*, and *seconds*) as well as the counters that are used to store the hour and minute settings for the alarm (*ahours* and *aminutes*). Each of these counters stores a two-digit BCD (binary-coded-decimal) number that is incremented at the appropriate time. The module also contains a counter that divides the 100-Hz clock by 100 and provides a signal to increment the seconds counter.

The VHDL code in Figure 8-4 instantiates three counters labeled *sec*1, *min*1, and *hrs*1. When the divide-by-100 counter is in state 99, it outputs a signal *c*99, which is used as an increment signal to the *sec*1 counter. *Sec*1 counts the seconds, and when the divide-by-100 counter rolls over, the seconds are incremented by 1 because *c*99 = 1. *Sec*1 is a divide-by-60 counter, and when it reaches 59, it outputs a signal *s*59. *Min*1 counts the minutes. It is incremented when *s*59 and *c*99 are both 1, and

also when *incm* is 1 in the *set_minutes* state. A signal *incmin* is used to denote the condition when minutes has to be incremented, whether due to pressing of a button or due to a control signal while counting. When *min*1 reaches 59, it outputs a signal *m*59. *Hrs*1 counts the hours and also toggles the *am_pm* flip-flop when time changes from 11:59:59:99 to 12:00:00:00. *Hrs*1 is incremented when $m59 = s59 = c99 = 1$, and also when *inch* is 1 in the *set_hours* state. A signal *inchr* denotes the condition when the hour has to be incremented, whether due to pressing of a button or due to a control signal while counting.

FIGURE 8-4: VHDL Code of Clock Module

```
library IEEE;
use IEEE.numeric_bit.all;

entity clock is
  port(clk, inch, incm, incha, incma, set_alarm, alarm_off: in bit;
       hours, ahours, minutes, aminutes, seconds: inout unsigned(7 downto 0);
       am_pm, aam_pm, ring, alarm_set: inout bit);
end clock;

architecture clock1 of clock is
  component CTR_59 is
    port(clk, inc, reset: in bit; dout: out unsigned(7 downto 0);
         t59: out bit);
  end component;
  component CTR_12 is
    port(clk, inc: in bit; dout: out unsigned(7 downto 0); am_pm: inout bit);
  end component;
  signal s59, m59, inchr, incmin, c99: bit;
  signal alarm_ring_time: integer range 0 to 50;
  signal div100: integer range 0 to 99;
  begin
    sec1: ctr_59 port map(clk, c99, '0', seconds, s59);
    min1: ctr_59 port map(clk, incmin, '0', minutes, m59);
    hrs1: ctr_12 port map(clk, inchr, hours, am_pm);
    incmin <= (s59 and c99) or incm;
    inchr <= (m59 and s59 and c99) or inch;
    alarm_min: ctr_59 port map(clk, incma, '0', aminutes, open);
    alarm_hr: ctr_12 port map(clk, incha, ahours, aam_pm);
    c99 <= '1' when div100 = 99 else '0';
    process(clk)
    begin
      if clk'event and clk = '1' then
        if c99 = '1' then div100 <= 0;    -- divide by 100 counter
        else div100 <= div100 + 1;
        end if;
        if set_alarm = '1' then
        alarm_set <= not alarm_set;
      end if;
```

```
      if ((minutes = aminutes) and (hours = ahours) and (am_pm = aam_pm)) and
         seconds = 0 and alarm_set = '1' then
        ring <= '1';
      end if;
      if ring = '1' and c99 = '1' then
        alarm_ring_time <= alarm_ring_time + 1;
      end if;
      if alarm_ring_time = 50 or alarm_off = '1' then
        ring <= '0'; alarm_ring_time <= 0;
      end if;
    end if;
  end process;
end clock1;
```

The clock VHDL code also implements the alarm functions. It instantiates counters for setting the alarm minutes and hours. The *alarm_set* flip-flop is toggled when *alarm_set* is 1. The *ring* flip-flop is set to 1 when the alarm setting matches the time counters and the alarm is set. *Alarm_ring_time* is a counter that counts seconds when the alarm is ringing. The ring flip-flop is cleared after 50 seconds or when the *alarm_off* signal is received.

The VHDL code in Figure 8-5 implements the stopwatch functions. It instantiates counters for hundredths of a second, seconds, and minutes. When a *start_stop* signal is received, the counting flip-flop is toggled. *Ctr*2 is a divide-by-100 BCD counter that is incremented every clock when counting = 1. It generates a signal *swc*99 when it is in state 99. VHDL code for the divide-by-100 counter is shown in Figure 8-6. *Sec*2 is the seconds counter that is incremented when *swc*99 = 1. *Sec*2 generates a signal *s*59 when it is in state 59. The minutes counter, *min*2, is incremented when both *s*59 and *swc*99 are 1.

FIGURE 8-5: VHDL Code of the Stopwatch Module

```
library IEEE;
use IEEE.numeric_bit.all;

entity stopwatch is
  port(clk, reset, start_stop: in bit;
       swhundreths, swseconds, swminutes: out unsigned(7 downto 0));
end stopwatch;

architecture stopwatch1 of stopwatch is
  component CTR_59 is
    port(clk, inc, reset: in bit; dout: out unsigned(7 downto 0); t59: out bit);
  end component;
  component CTR_99 is
    port(clk, inc, reset: in bit; dout: out unsigned(7 downto 0); t59: out bit);
  end component;
  signal swc99, s59, counting, swincmin: bit;
begin
  ctr2: ctr_99 port map(clk, counting, reset, swhundreths, swc99);
```

```
    --counts hundreths of seconds
  sec2: ctr_59 port map(clk, swc99, reset, swseconds, s59);
    --counts seconds
  min2: ctr_59 port map(clk, swincmin, reset, swminutes, open);
    --counts minutes
  swincmin <= s59 and swc99;
  process(clk)
  begin
    if clk'event and clk = '1' then
      if start_stop = '1' then
        counting <= not counting;
      end if;
    end if;
  end process;
end stopwatch1;
```

FIGURE 8-6: VHDL Code for Divide-by-100 Counter

```
library IEEE;
use IEEE.numeric_bit.all;
--divide by 100 BCD counter
entity CTR_99 is
  port(clk, inc, reset: in bit; dout: out unsigned(7 downto 0); t59: out bit);
end CTR_99;

architecture count99 of CTR_99 is
  signal dig1, dig0: unsigned(3 downto 0);
begin
  process(clk)
  begin
    if clk'event and clk = '1' then
      if reset = '1' then dig0 <= "0000"; dig1 <= "0000";
      else
        if inc = '1' then
          if dig0 = 9 then dig0 <= "0000";
            if dig1 = 9 then dig1 <= "0000";
            else dig1 <= dig1 + 1;
            end if;
          else dig0 <= dig0 + 1;
          end if;
        end if;
      end if;
    end if;
  end process;
  t59 <= '1' when (dig1 = 9 and dig0 = 9) else '0';
  dout <= dig1 & dig0;
end count99;
```

VHDL code for the divide-by-60 counter (Figure 8-7) is straightforward. The counter counts to 59 and then resets.

FIGURE 8-7: VHDL Code of Divide-by-60 Counter

```
library IEEE;
use IEEE.numeric_bit.all;
--this counter counts seconds or minutes 0 to 59
entity CTR_59 is
  port(clk, inc, reset: in bit; dout: out unsigned(7 downto 0); t59: out bit);
end CTR_59;

architecture count59 of CTR_59 is
  signal dig1, dig0: unsigned(3 downto 0);
begin
  process(clk)
  begin
    if clk'event and clk = '1' then
      if reset = '1' then dig0 <= "0000"; dig1 <= "0000";
      else
        if inc = '1' then
          if dig0 = 9 then dig0 <= "0000";
             if dig1 = 5 then dig1 <= "0000";
             else dig1 <= dig1 + 1;
             end if;
          else dig0 <= dig0 + 1;
          end if;
        end if;
      end if;
    end if;
  end process;
  t59 <= '1' when (dig1 = 5 and dig0 = 9) else '0';
  dout <= dig1 & dig0;
end count59;
```

The hours counter (Figure 8-8) counts to 12 and then changes to 1 the next time the increment signal is 1. It toggles the *am_pm* signal when the count changes from 11 to 12.

FIGURE 8-8: VHDL Code for Hours Counter

```
library IEEE;
use IEEE.numeric_bit.all;
--this counter counts hours 1 to 12 and toggles am_pm
entity CTR_12 is
  port(clk, inc: in bit; dout: out unsigned(7 downto 0); am_pm: inout bit);
end CTR_12;

architecture count12 of CTR_12 is
  signal dig0: unsigned(3 downto 0);
  signal dig1: bit;
```

```
begin
  process(clk)
  begin
    if clk'event and clk = '1' then
      if inc = '1' then
        if dig1 = '1' and dig0 = 2 then
          dig1 <= '0'; dig0 <= "0001";
        else
          if dig0 = 9 then dig0 <= "0000"; dig1 <= '1';
          else dig0 <= dig0 + 1;
          end if;
          if dig1 = '1' and dig0 = 1 then am_pm <= not am_pm;
          end if;
        end if;
      end if;
    end if;
  end process;
  dout <= "000" & dig1 & dig0;
end count12;
```

8.1.3 Testing the Wristwatch

Next we will write a test bench for the wristwatch module (Figure 8-9). The test bench must generate a series of button pushes as well as the 100-Hz clock, and it must display the time, alarm settings, and stopwatch counters. In effect, the test bench takes the place of the input and display modules in the overall design. To simplify writing the test bench code, we have written two procedures. Procedure `wait1(N1)` waits for $N1$ clocks each time it is called. Procedure `push(button, N)` simulates pushing a button N times each time it is called. Thus `push(B2, 23)` simulates pushing $B2$ 23 times. The push procedure simulates the output from the Input Module. Therefore, each button signal is on for exactly one clock time, and it is synchronized with CLK. The procedure waits 1.2 seconds after each button push. Testing should also be done with longer and shorter wait times between button pushes. Because we are using unsigned numbers and the numeric_bit package, all registers will be clear when the test bench is started. If we used numeric_std instead, we would have to reset all registers before running the simulation.

The test sequence we used is as follows:

1. Set the time to 11:58 P.M.
2. Set the alarm time to 12:00 A.M.
3. Set the alarm and change to time mode, and wait until the time rolls over at midnight.
4. Turn off the alarm 5 seconds later.
5. Change to stopwatch mode and start the stopwatch
6. Switch to time mode and wait for 10 seconds (stopwatch keeps running)
7. Switch back to stopwatch mode and wait until it reads 1 minute and 2 seconds
8. Stop the stopwatch, reset it, and return to time mode.

FIGURE 8-9: Test Bench for Wristwatch

```
library IEEE;
use IEEE.numeric_bit.all;

entity testww is   -- test bench for wristwatch
  port(hours, ahours, minutes, aminutes, seconds,
       swhundreths, swseconds, swminutes: inout unsigned(7 downto 0);
       am_pm, aam_pm, ring, alarm_set: inout bit);
end testww;

architecture testww1 of testww is
  component wristwatch is
    port(B1, B2, B3, clk: in bit;
         am_pm, aam_pm, ring, alarm_set: inout bit;
         hours, ahours, minutes, aminutes, seconds: inout unsigned(7 downto 0);
         swhundreths, swseconds, swminutes: out unsigned(7 downto 0));
  end component;
  signal B1, B2, B3, clk: bit;
begin
  wristwatch1: wristwatch port map(B1, B2, B3, clk, am_pm, aam_pm, ring,
                                   alarm_set, hours, ahours, minutes, aminutes,
                                   seconds, swhundreths, swseconds, swminutes);
  clk <= not clk after 5 ms;   -- generate 100 hz clock
  process
  procedure wait1   -- waits for N1 clocks
    (N1: in integer)
    variable count: integer;
  begin
    count := N1;
    while count /= 0 loop
      wait until clk'event and clk = '1';
      count := count - 1;
      wait until clk'event and clk = '0';
    end loop;
  end procedure wait1;
  procedure push   -- simulates pushing a button N times
    (signal button: out bit; N: in integer) is
  begin
    for i in 1 to N loop
      button <= '1';
      wait1(1);
      button <= '0';
      wait1(120);   -- wait 1200 ms between pushes
    end loop;
  end procedure push;
  begin
    wait1(10);   -- set time to 11:58 pm
    push(b2, 1); push(b3, 23); push(b2, 1); push(b3, 57); push(b2, 1);
```

```
    report "time should be 11:58 P.M.";
    push(b1, 1);    -- set alarm to 12:00 am
    push(b2, 1); push(b3, 24); push(b2, 2); push(b3, 1); push(b1, 2);
    report "alarm should be set to 12:00 A.M.";
    wait until hours = "00010010" and seconds = "00000101";
    push(b3, 1);    -- turn alarm off at 12 hours and 5 seconds
    push(b1, 2);    -- run stopwatch, go to time mode, go back to stopwatch
    push(b2, 1); wait1(120); push(b1, 1); wait1(1000); push(b1, 2);
    wait until swminutes = "00000001" and swseconds = "00000010";
      --stop stopwatch after 1 min. and 2 sec., then reset
    report "stopwatch should read 1 min. 2 sec.";
    push(b2, 1); push(b3, 1); push(b1, 1);
    wait;
  end process;
end testww1;
```

We used the following commands to run the simulation with the preceding test sequence.

```
vsim -t 1ms testww   -- set simulator resolution to 1 ms
add list -hex hours minutes seconds am_pm
              ahours aminutes aam_pm ring
add list b1 b2 b3 wristwatch1/state
add list -hex swminutes swseconds -notrigger swhundredths
run 300000 ms
```

The test results showed that the wristwatch module functions according to the specifications. When the wristwatch module is implemented using the Xillinx Spartan 3 FPGA, it requires 87 slices, 80 flip-flops, and 158 four-input LUTs. To complete the design, we still need to write code for the Input and Display modules.

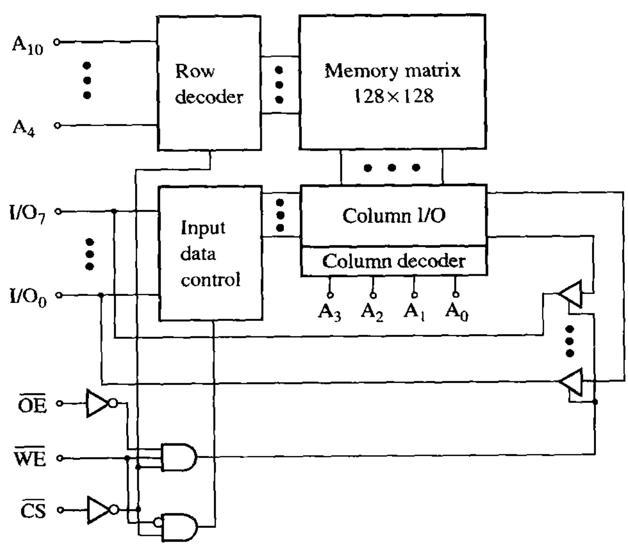
8.2 Memory Timing Models

When we design a complex digital system with several components, many timing constraints must be satisfied in order for the system components to function together properly. For example, if we are interfacing memory components to a microprocessor bus, all bus interface timing specifications must be satisfied. In order to simulate such a system using VHDL, we must develop accurate timing models for each component. In this section we will develop a timing model for a small static RAM. This will illustrate the process of going from manufacturer's specifications to a VHDL model that takes timing parameters into account. These types of timing models are very useful when developing system on a chip (SoC) designs.

Figure 8-10 illustrates the block diagram of a 6116 static RAM, which can store 2048 eight-bit words of data. This memory has 16,384 cells, arranged in a 128 × 128 memory matrix. The RAM contains address decoders and a memory array. The address decoder is typically split into the column decoder and the row decoder. The 11 address lines, which are needed to address the 2^{11} bytes of data, are divided into two groups, one for the column decoder and the other for the row decoder. Lines A0 through A3 select eight columns in the matrix at a time, since there are eight data lines for each address. Lines A4 through A10 select one of the 128 rows in the matrix. The data outputs from the matrix go through tristate buffers before connecting to the data I/O lines. These buffers are disabled except when reading from the memory.

Figure 8-10 also illustrates the connections of the chip select ($\overline{CS}$), output enable ($\overline{OE}$), and write enable ($\overline{WE}$) signals. The functions of these signals were explained in Table 3-7. When $\overline{CS}$ is high (i.e., not asserted), a 0 input reaches the two AND gates; hence, the tristate control voltage is low, resulting in a high-Z output. Similarly, when $\overline{OE}$ is high, even if chip select is asserted, the tristate control is inactive, resulting in high-Z output. When chip select and $\overline{WE}$ are asserted, a write operation happens, and the data on the I/O lines gets written into the RAM. When chip select and $\overline{OE}$ are asserted and $\overline{WE}$ is not asserted, a read operation happens, and the RAM contents appear on the I/O lines.

FIGURE 8-10: Block Diagram of 6116 Static RAM



We presented some static RAM memory models in Section 3.7; however, the models do not take into account timing specifications. Memory timing diagrams and specifications must be considered when designing systems using the memory chips. In this section, we present simulation models of memory chips with particular timing specifications.

Let us consider a CMOS static RAM 6116, whose timing parameters are defined in Table 8-1 for both read and write cycles. Specifications are given for the 6116 SA-15 RAM, which has a 15-ns access time. A dash in the table indicates that either the specification was not relevant or that the manufacturer did not provide the specification.

TABLE 8-1: Timing Specifications for CMOS Static RAM 6116 SA-15

Parameter	Symbol	Timing Specification min(ns)	max(ns)
Read cycle time	t_{RC}	15	—
Address access time	t_{AA}	—	15
Chip select access time	t_{ACS}	—	15
Chip selection to output in low-Z	t_{CLZ}	5	—
Output enable to output valid	t_{OE}	—	10
Output enable to output in low-Z	t_{OLZ}	0	—
Chip deselection to output in high-Z	t_{CHZ}	2*	10
Output disable to output in high-Z	t_{OHZ}	2*	8
Output hold from address change	t_{OH}	5	—
Write cycle time	t_{WC}	15	—
Chip selection to end of write	t_{CW}	13	—
Address valid to end of write	t_{AW}	14	—
Address setup time	t_{AS}	0	—
Write pulse width	t_{WP}	12	—
Write recovery time	t_{WR}	0	—
Write enable to output in high-Z	t_{WHZ}	—	7
Data valid to end of write	t_{DW}	12	—
Data hold from end of write	t_{DH}	0	—
Output active from end of write	t_{OW}	0	—

*Estimated value, not specified by manufacturer.

Figure 8-11(a) shows the read cycle timing for the case where $\overline{CS}$ and $\overline{OE}$ are both low before the address changes. In this case, after the address changes, the old data remains at the memory output for a time t_{OH}; then there is a transition period during which the data may change (as indicated by the cross-hatching). The new data is stable at the memory output after the address access time, t_{AA}. The address must be stable for the read cycle time, t_{RC}.

Figure 8-11(b) shows the timing for the case where $\overline{OE}$ is low and the address is stable before $\overline{CS}$ goes low. When $\overline{CS}$ is high, *Dout* is in the high-Z state, as indicated by a line halfway between '0' and '1'. When $\overline{CS}$ goes low, *Dout* leaves the high-Z state after time t_{CLZ}, there is a transition period during which the data may change, and the new data is stable at time t_{ACS} after $\overline{CS}$ changes. *Dout* returns to high-Z at time t_{CHZ} after $\overline{CS}$ goes high.

FIGURE 8-11: Read Cycle Timing

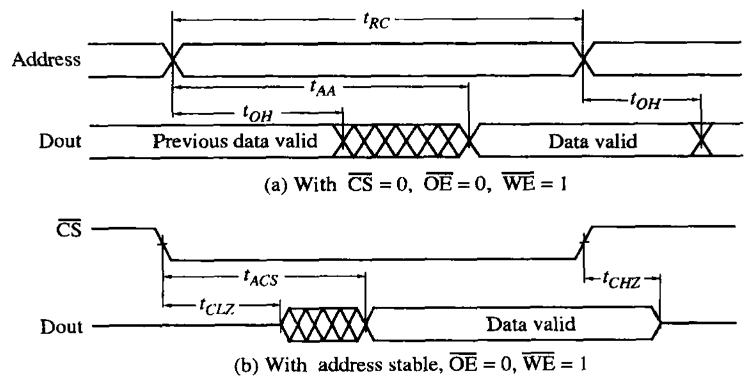


Figure 8-12 shows the write cycle timing for the case where $\overline{OE}$ is low during the entire cycle and where writing to memory is controlled by $\overline{WE}$. In this case, it is assumed that $\overline{CS}$ goes low before or at the same time as $\overline{WE}$ goes low, and goes high before or at the same time as $\overline{CS}$ does. The cross-hatching on $\overline{CS}$ indicates the interval in which it can go from high to low (or from low to high). The address must be stable for the address setup time, t_{AS}, before $\overline{WE}$ goes low. After time t_{WHZ}, the data out from the tristate buffers go to the high-Z state and input data may be placed on the I/O lines. The data into the memory must be stable for the setup time t_{DW} before $\overline{WE}$ goes high, and then it must be kept stable for the hold time t_{DH}. The address must be stable for t_{WR} after $\overline{WE}$ goes high. When $\overline{WE}$ goes high, the memory switches back to the read mode. After t_{OW} (min) and during region (a), *Dout* goes through a transition period and then becomes the same as the data just stored in the memory. Further change in *Dout* may occur if the address changes or if $\overline{CS}$ goes high. To avoid bus conflicts during region (a), *Din* should either be high-Z or the same as *Dout*.

FIGURE 8-12: $\overline{WE}$-Controlled Write Cycle Timing ($\overline{OE} = 0$)

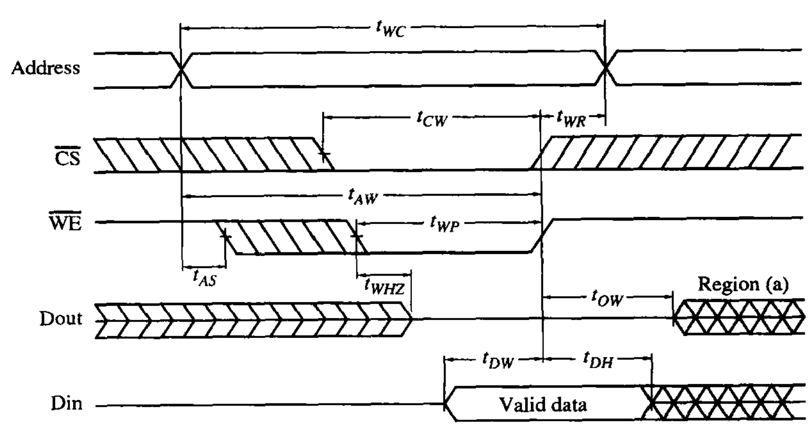
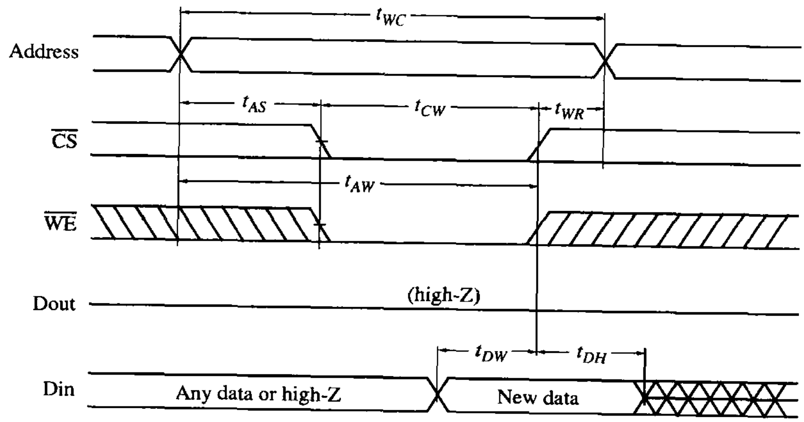


Figure 8-13 shows the write cycle timing for the case where $\overline{OE}$ is low during the entire cycle and where writing to memory is controlled by $\overline{CS}$. In this case, it is assumed that $\overline{WE}$ goes low before or at the same time as $\overline{CS}$ goes low, and $\overline{CS}$ goes high before or at the same time as $\overline{WE}$ does. The address must be stable for the address setup time, t_{AS}, before $\overline{CS}$ goes low. The data into the memory must be stable for the setup time t_{DW} before $\overline{CS}$ goes high, and then it must be kept stable for the hold time t_{DH}. The address must be stable for t_{WR} after $\overline{CS}$ goes high. Note that this write cycle is very similar to the $\overline{WE}$-controlled cycle. In both cases, writing to memory occurs when both $\overline{CS}$ and $\overline{WE}$ are low, and writing is completed when either one goes high.

FIGURE 8-13: CS-Controlled Write Cycle Timing ($\overline{OE}$ = 0)



Next, we revise the RAM model presented in Figure 3-15 to include timing information based on the read and write cycles shown in Figures 8-11, 8-12, and 8-13. We assume that $\overline{OE}$ = '0'. The VHDL RAM timing model in Figure 8-14 uses a generic declaration to define default values for the important timing parameters. Transport delays are used throughout to avoid cancellation problems, which can occur with inertial delays. The RAM process waits for a change in *CS_b*, *WE_b*, or the address. If a rising edge of *WE_b* occurs when *CS_b* is '0', or a rising edge of *CS_b* occurs when *WE_b* is '0', this indicates the end of write, so the data is stored in the RAM, and then the data is read back out after t_{OW}. If a falling edge of *WE_b* occurs when *CS_b* = '0', the RAM switches to write mode and the data output goes to high-Z.

FIGURE 8-14: Timing Simulation Model for 6116 Static CMOS RAM

```
-- memory model with timing (OE_b=0)
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;

entity static_RAM is
  generic(constant tAA:  time := 15 ns;    -- 6116 static CMOS RAM
          constant tACS: time := 15 ns;
```

```
          constant tCLZ: time :=  5 ns;
          constant tCHZ: time :=  2 ns;
          constant tOH:  time :=  5 ns;
          constant tWC:  time := 15 ns;
          constant tAW:  time := 14 ns;
          constant tWP:  time := 12 ns;
          constant tWHZ: time :=  7 ns;
          constant tDW:  time := 12 ns;
          constant tDH:  time :=  0 ns;
          constant tOW:  time :=  0 ns);
  port(CS_b, WE_b, OE_b: in std_logic;
       Address: in unsigned(7 downto 0);
       IO: inout unsigned(7 downto 0) := (others => 'Z'));
end Static_RAM;

architecture SRAM of Static_RAM is
  type RAMtype is array(0 to 255) of unsigned(7 downto 0);
  signal RAM1: RAMtype := (others => (others => '0'));
begin
  RAM: process (CS_b, WE_b, Address)
  begin
    if CS_b='0' and WE_b='1' and Address'event then
    -- read when address changes
      IO <= transport "XXXXXXXX" after tOH,
           Ram1(to_integer(Address)) after tAA; end if;
    if falling_edge(CS_b)and WE_b='1' then
    -- read when CS_b goes low
      IO <= transport "XXXXXXXX" after tCLZ,
           Ram1(to_integer(Address)) after tACS; end if;
    if rising_edge(CS_b) then   -- deselect the chip
      IO <= transport "ZZZZZZZZ" after tCHZ;
      if We_b='0' then -- CS-controlled write
        Ram1(to_integer(Address'delayed)) <= IO; end if;
    end if;
    if falling_edge(WE_b) and CS_b='0' then   -- WE-controlled write
      IO <= transport "ZZZZZZZZ" after tWHZ; end if;
    if rising_edge(WE_b) and CS_b='0' then
      Ram1(to_integer(Address'delayed)) <= IO'delayed;
      IO <= transport IO'delayed after tOW;   -- read back after write
        -- IO'delayed is the value of IO just before the rising edge
    end if;
  end process RAM;

  check: process
  begin
    if NOW /= 0 ns then
      if address'event then
        assert (address'delayed'stable(tWC))   -- tRC = tWC assumed
          report "Address cycle time too short"
          severity WARNING;
```

```
      end if;
    -- The following code only checks for a WE_b controlled write:
      if rising_edge(WE_b) and CS_b'delayed = '0' then
        assert (address'delayed'stable(tAW))
          report "Address not valid long enough to end of write"
          severity WARNING;
        assert (WE_b'delayed'stable(tWP))
          report "Write pulse too short"
          severity WARNING;
        assert (IO'delayed'stable(tDW))
          report "IO setup time too short"
          severity WARNING;
        wait for tDH;
          assert (IO'last_event >= tDH)
          report "IO hold time too short"
          severity WARNING;
      end if;
    end if;
    wait on CS_b, WE_b, Address;
  end process check;
end SRAM;
```

If a rising edge of *CS_b* has occurred, the RAM is deselected, and the data output goes to high-Z after the specified delay. Otherwise, if a falling edge of *CS_b* has occurred and *WE_b* is '1', the RAM is in the read mode. The data bus can leave the high-Z state after time t_{CLZ} (min), but it is not guaranteed to have valid data out until time t_{ACS} (max). The region in between is a transitional region where the bus state is unknown, so we model this region by outputting 'X' on the I/O lines. If an address change has just occurred and the RAM is in the read mode (Figure 8-11(a)), the old data holds its value for time t_{OH}. Then the output is in an unknown transitional state until valid data has been read from the RAM after time t_{AA}.

The check process, which runs concurrently with the RAM process, tests to see if some of the memory timing specifications are satisfied. *NOW* is a predefined variable that equals the current time. (VHDL provides NOW in order to access the current simulation time. It is actually a predefined function. It returns different values when called at different times during the course of a simulation.) To avoid false error messages, checking is not done when *NOW* = 0 or when the chip is not selected. When the address changes, the process checks to see if the address has been stable for the write cycle time (t_{WC}) and outputs a warning message if it is not. Since an address event has just occurred when this test is made, *Address'stable*(t_{WC}) would always return FALSE. Therefore, *Address'delayed* must be used instead of *Address* so that *Address* is delayed one delta and the stability test is made just before *Address* changes. Next, the timing specifications for write are checked. First, we verify that the address has been stable for t_{AW}. Then we check to see that *WE_b* has been low for t_{WP}. Finally, we check the setup and hold times for the data.

VHDL code for a partial test of the RAM timing model is shown in Figure 8-15. This code runs a write cycle followed by two read cycles. The RAM is deselected between cycles. Figure 8-16 shows the test results. We also tested the model for cases where simultaneous input changes occur and cases where timing specifications are violated, but these test results are not included here.

FIGURE 8-15: VHDL Code for Testing the RAM Timing Model

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;

entity RAM_timing_tester is
end RAM_timing_tester;

architecture test1 of RAM_timing_tester is
  component static_RAM is
    port(CS_b, WE_b, OE_b: in std_logic;
         Address: in unsigned(7 downto 0);
         IO: inout unsigned(7 downto 0));
  end component Static_RAM;
  signal Cs_b, We_b: std_logic := '1';    -- active low signals
  signal Data: unsigned(7 downto 0) := "ZZZZZZZZ";
  signal Address: unsigned(7 downto 0):= "00000000";
begin
  SRAM1: Static_RAM port map(Cs_b, We_b, '0', Address, Data);
  process
  begin
    wait for 20 ns;
    Address <= "00001000";                    -- WE-controlled write
    Cs_b <= transport '0', '1' after 50 ns;
    We_b <= transport '0' after 8 ns, '1' after 40 ns;
    Data <= transport "11100011" after 25 ns, "ZZZZZZZZ" after 55 ns;

    wait for 60 ns;
    Address <= "00011000";                    -- RAM deselected
    wait for 40 ns;
    Address <= "00001000";                    -- Read cycles
    Cs_b <= '0';
    wait for 40 ns;
    Address <= "00010000";
    Cs_b <= '1' after 40 ns;
    wait for 40 ns;
    Address <= "00011000";                    -- RAM deselected
    wait for 40 ns;
    report "DONE";
  end process;
end test1;
```

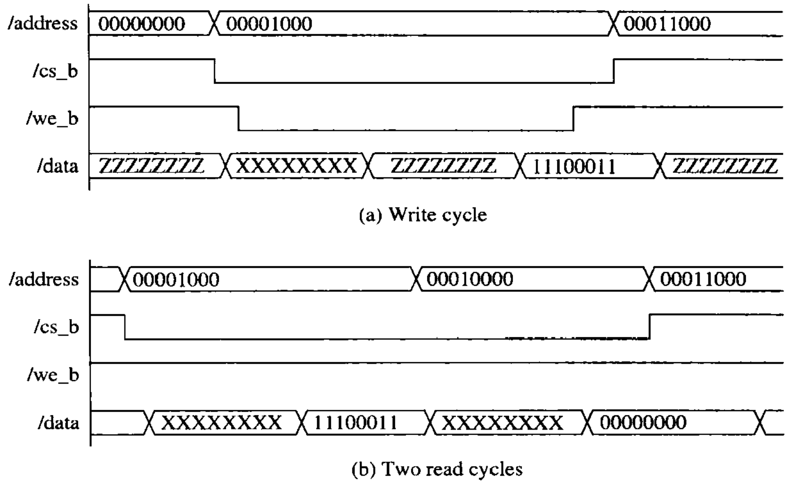


FIGURE 8-16: Test Results for RAM Timing Model

8.3 A Universal Asynchronous Receiver Transmitter

Most computers and microcontrollers have one or more serial data ports used to communicate with serial input/output devices such as keyboards and serial printers. By using a modem (modulator-demodulator) connected to a serial port, serial data can be transmitted to and received from a remote location via telephone lines (see Figure 8-17). The serial communication interface, which receives and transmits serial data, is often called a UART (universal asynchronous receiver-transmitter). In Figure 8-17, *RxD* is the received serial data signal and *TxD* is the transmitted data signal.

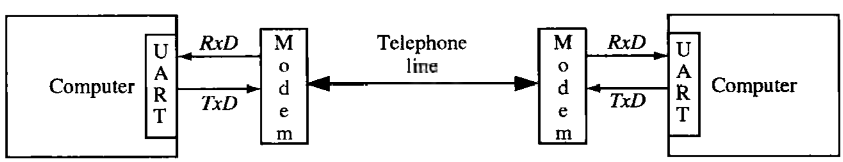


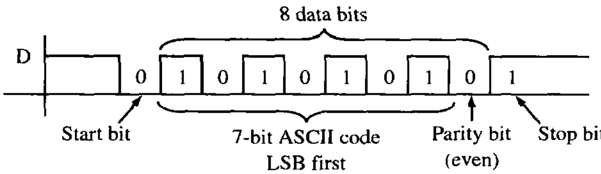
FIGURE 8-17: Serial Data Transmission

Figure 8-18 shows the standard format for serial data transmission. Since there is no clock line, the data (*D*) is transmitted asynchronously, one byte at a time. When no data is being transmitted, *D* remains high. To mark the start of transmission, *D* goes low for one bit time, which is referred to as the start bit. Then eight data bits are transmitted, least significant bit first. When text is being transmitted, ASCII code is usually used. In ASCII code, each alphanumeric character is represented by a 7-bit code. The eighth bit may be used as a parity check bit. In the example, the letter U, coded as 1010101, is transmitted followed by a 0 parity bit, so that the total number of 1's is even (even parity). After 8 bits are transmitted, *D* must go high for at least

one bit time, which is referred to as the stop bit. Then transmission of another character can start at any time.

The number of bits transmitted per second is frequently referred to as the *baud rate.*

FIGURE 8-18: Standard Serial Data Format



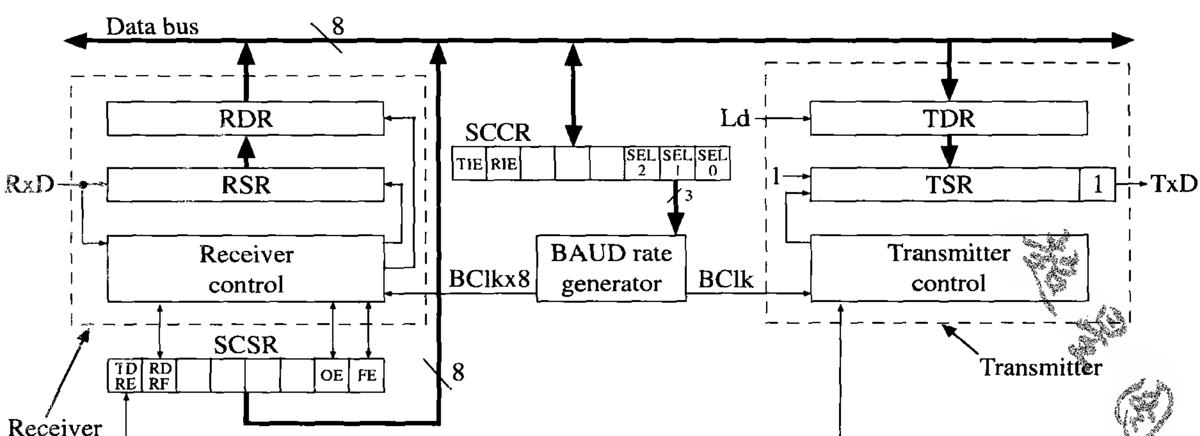
When transmitting, the UART takes 8 bits of parallel data and converts the data to a serial bit stream that consists of a start bit (logic '0'), 8 data bits (least significant bit first), and one or more stop bits (logic '1'). When receiving, the UART detects the start bit, receives the 8 data bits, and converts the data to parallel form when it detects the stop bit. Since no clock is transmitted, the UART must synchronize the incoming bit stream with the local clock.

We now design a simplified version of a UART similar to the one used within the microcontroller MC6805, MC6811, and other microcontrollers. Figure 8-19 shows the UART connected to the 8-bit data bus. The following six 8-bit registers are used:

RSR	Receive shift register
RDR	Receive data register
TDR	Transmit data register
TSR	Transmit shift register
SCCR	Serial communications control register
SCSR	Serial communications status register

The following discussion assumes that the UART is connected to a microcontroller data and address bus so that the CPU can read and write to the registers. *RDR,*

FIGURE 8-19: UART Block Diagram



TDR, *SCCR*, and *SCSR* are memory-mapped; that is, each register is assigned an address in the microcontroller memory space. *RDR*, *SCSR*, and *SCCR* can drive the data bus through tristate buffers; *TDR* and *SCCR* can be loaded from the data bus.

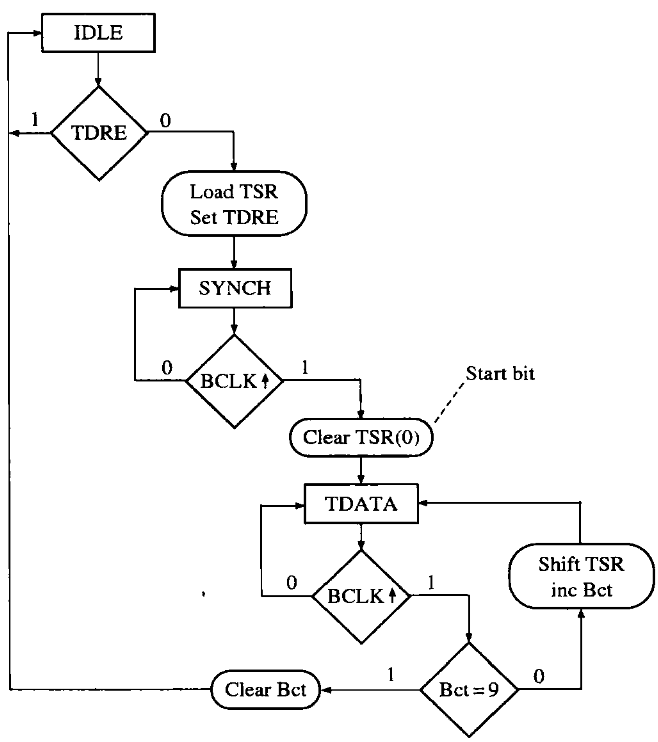
Besides the registers, the three main components of the UART are the baud rate generator, the receiver controller, and the transmitter controller. The baud rate generator divides down the system clock to provide the bit clock (*BClk*) with a period equal to one bit time and also *BClkX8*, which has a frequency eight times the *BClk* frequency.

The *TDRE* (Transmit Data Register Empty) bit in the *SCSR* is set when *TDR* is empty. When the microcontroller is ready to transmit data, the following occurs:

1. The microcontroller waits until *TDRE* = '1' and then loads a byte of data into *TDR* and clears *TDRE*.
2. The UART transfers data from *TDR* to *TSR* and sets *TDRE*.
3. The UART outputs a start bit ('0') for one bit time and then shifts *TSR* right to transmit the eight data bits followed by a stop bit ('1').

Figure 8-20 shows the SM chart for the transmitter. The corresponding sequential machine (SM) is clocked by the microcontroller system clock (*CLK*). In the IDLE state, the SM waits until *TDR* has been loaded and *TDRE* is cleared. In the SYNCH state, the SM waits for the rising edge of the bit clock (*Bclk*↑) and then clears the low-order bit of *TSR* to transmit a '0' for one bit time. In the TDATA state, each time *Bclk*↑ is detected, *TSR* is shifted right to transmit the next data bit,

FIGURE 8-20: SM Chart for UART Transmitter



and the bit counter (*Bct*) is incremented. When *Bct* = 9, 8 data bits and a stop bit have transmitted. *Bct* is then cleared and the SM goes back to IDLE.

The VHDL code for the UART transmitter (Figure 8-21) is based on the SM chart of Figure 8-20. The `use IEEE.numeric_std.all` statement is not necessary if std_logic_vector type is used. We use the unsigned type here and in the other modules of the UART. The transmitter contains the *TDR* and *TSR* registers and the transmit control. It interfaces with *TDRE* and the data bus (DBUS). The first process represents the combinational network, which generates the nextstate and control signals. The second process updates the registers on the rising edge of the clock. The signal *Bclk_rising* is '1' for one system clock time following the rising edge of *Bclk*. To generate *Bclk_rising*, *Bclk* is stored in a flip-flop named *Bclk_Dlayed*. Then *Bclk_rising* is '1' if the current value of *Bclk* is '1' and the previous value (stored in *Bclk_Dlayed*) is '0'. Thus,

```
Bclk_rising <= Bclk and not Bclk_Dlayed;
```

FIGURE 8-21: VHDL Code for UART Transmitter

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all; -- use this if unsigned type is used.

entity UART_Transmitter is
  port(Bclk, sysclk, rst_b, TDRE, loadTDR: in std_logic;
       DBUS: in unsigned(7 downto 0);
       setTDRE, TxD: out std_logic);
end UART_Transmitter;

architecture xmit of UART_Transmitter is
  type stateType is (IDLE, SYNCH, TDATA);
  signal state, nextstate: stateType;
  signal TSR: unsigned(8 downto 0);    -- Transmit Shift Register
  signal TDR: unsigned(7 downto 0);    -- Transmit Data Register
  signal Bct: integer range 0 to 9;    -- counts number of bits sent
  signal inc, clr, loadTSR, shftTSR, start: std_logic;
  signal Bclk_rising, Bclk_Dlayed: std_logic;
begin
  TxD <= TSR(0);
  setTDRE <= loadTSR;
  Bclk_rising <= Bclk and (not Bclk_Dlayed);
  -- indicates the rising edge of bit clock

  Xmit_Control: process(state, TDRE, Bct, Bclk_rising)
  begin
    inc <= '0'; clr <= '0'; loadTSR <= '0'; shftTSR <= '0'; start <= '0';
    -- reset control signals
    case state is
      when IDLE =>
        if (TDRE = '0') then
```

```
          loadTSR <= '1'; nextstate <= SYNCH;
        else nextstate <= IDLE;
        end if;
      when SYNCH =>   -- synchronize with the bit clock
        if (Bclk_rising = '1') then
          start <= '1'; nextstate <= TDATA;
        else nextstate <= SYNCH;
        end if;
      when TDATA =>
        if (Bclk_rising = '0') then nextstate <= TDATA;
        elsif (Bct /= 9) then
          shftTSR <= '1'; inc <= '1'; nextstate <= TDATA;
        else clr <= '1'; nextstate <= IDLE;
        end if;
    end case;
  end process;

  Xmit_update: process(sysclk, rst_b)
  begin
    if (rst_b = '0') then
      TSR <= "111111111"; state <= IDLE; Bct <= 0; Bclk_Dlayed <= '0';
    elsif (sysclk'event and sysclk = '1') then
      state <= nextstate;
      if (clr = '1') then Bct <= 0;
      elsif (inc = '1') then
        Bct <= Bct + 1;
      end if;
      if (loadTDR = '1') then TDR <= DBUS;
      end if;
      if (loadTSR = '1') then TSR <= TDR & '1';
      end if;
      if (start = '1') then TSR(0) <= '0';
      end if;
      if (shftTSR = '1') then TSR <= '1' & TSR(8 downto 1);
      end if;
      -- shift out one bit
      Bclk_Dlayed <= Bclk;   -- Bclk delayed by 1 sysclk
    end if;
  end process;
end xmit;
```

The operation of the UART receiver is as follows:

1. When the UART detects a start bit, it reads in the remaining bits serially and shifts them into the *RSR*.
2. When all the data bits and the stop bit have been received, the *RSR* is loaded into the *RDR*, and the Receive Data Register Full (*RDRF*) flag in the *SCSR* is set.
3. The microcontroller checks the *RDRF* flag, and if it is set, the *RDR* is read and the flag is cleared.

The bit stream coming in on *RxD* is not synchronized with the local bit clock (*Bclk*). If we attempted to read *RxD* at the rising edge of *Bclk*, we would have a problem if *RxD* changed near the clock edge. We could have setup and hold time problems. If the bit rate of the incoming signal differed from *Bclk* by a small amount, we could end up reading some bits at the wrong time. To avoid these problems, we will sample *RxD* eight times during each bit time. (Some systems sample 16 times per bit.) We will sample on the rising edge of *BclkX*8. The arrows in Figure 8-22 indicate the rising edge of *BclkX*8. Ideally, we should read the bit value at the middle of each bit time for maximum reliability. When *RxD* first goes to '0', we will wait for four *BclkX*8 periods, and we should be near the middle of the start bit. Then we will wait eight more *BclkX*8 periods, which should take us near the middle of the first data bit. We continue reading once every eight *BclkX*8 clocks until we have read the stop bit.

FIGURE 8-22: Sampling RxD with BclkX8

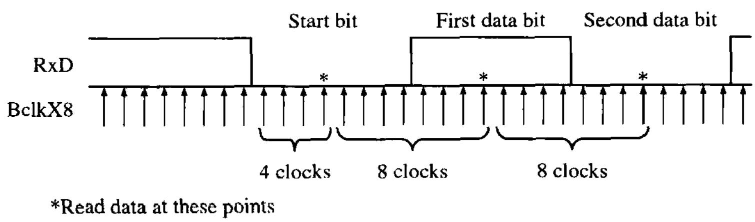
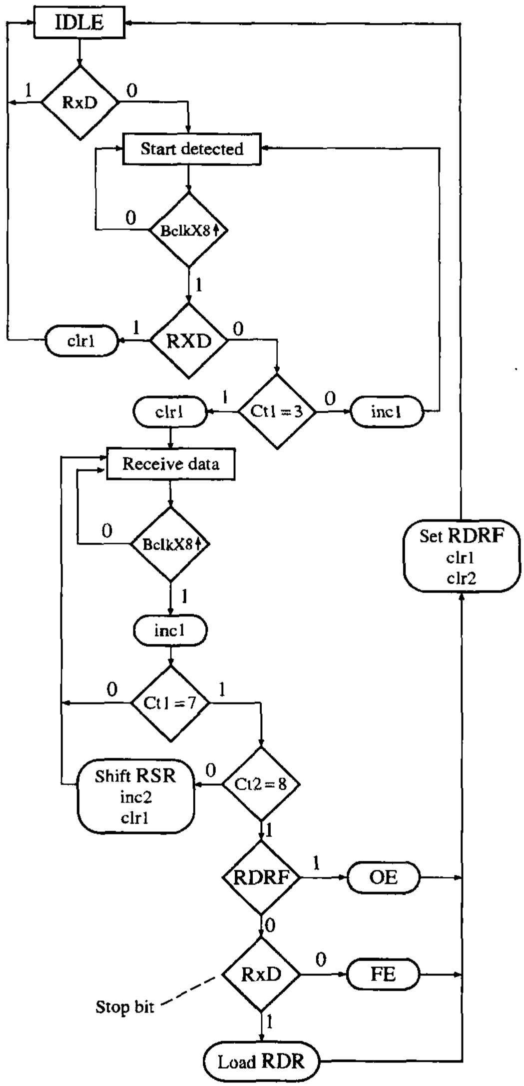


Figure 8-23 shows an SM chart for the UART receiver. Two counters are used. *Ct*1 counts the number of *BclkX*8 clocks. *Ct*2 counts the number of bits received after the start bit. In the IDLE state, the SM waits for the start bit (*RxD* = '0') and then goes to the Start Detected state. The SM waits for the rising edge of *BclkX*8 (*BclkX*8↑) and then samples *RxD* again. Since the start bit should be '0' for eight *BclkX*8 clocks, we should read '0'. *Ct*1 is still 0, so *Ct*1 is incremented and the SM waits for *BclkX*8↑. If *RxD* = '1', this is an error condition and the SM clears *Ct*1 and resets to the IDLE state. Otherwise, the SM keeps looping. When *RxD* is '0' for the fourth time, *Ct*1 = 3, so *Ct*1 is cleared and the state goes to Receive Data. In this state, the SM increments *Ct*1 after every rising edge of *BclkX*8. After the eighth clock, *Ct*1 = 7 and *Ct*2 is checked. If it is not 8, the current value of *RxD* is shifted into *RSR*, *Ct*2 is incremented, and *Ct*1 is cleared. If *Ct*2 = 8, all 8 bits have been read and we should be in the middle of the stop bit. If *RDRF* = '1', the microcontroller has not yet read the previously received data byte, and an overrun error has occurred, in which case the *OE* flag in the status register is set and the new data is ignored. If *RxD* = '0', the stop bit has not been detected properly, and the framing error (*FE*) flag in the status register is set. If no errors have occurred, *RDR* is loaded from *RSR*. In all cases, *RDRF* is set to indicate that the receive operation is completed and the counters are cleared.

FIGURE 8-23: SM Chart for UART Receiver



The VHDL code for the UART receiver (Figure 8-24) is based on the SM chart of Figure 8-23. The receiver contains the *RDR* and *RSR* registers and the receive control. The control interfaces with *SCSR*, and *RDR* can drive data onto the data bus. The first process represents the combinational network, which generates the nextstate and control signals. The second process updates the registers on the rising edge of the clock. The signal *BclkX8_rising* is '1' for one system clock time following the rising edge of *BclkX8*. *BclkX8_rising* is generated the same manner as *Bclk_rising*.

FIGURE 8-24: VHDL Code for UART Receiver

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all; -- to use unsigned type

entity UART_Receiver is
port(RxD, BclkX8, sysclk, rst_b, RDRF: in std_logic;
     RDR: out unsigned(7 downto 0);
     setRDRF, setOE, setFE: out std_logic);
end UART_Receiver;

architecture rcvr of UART_Receiver is
  type stateType is (IDLE, START_DETECTED, RECV_DATA);
  signal state, nextstate: stateType;
  signal RSR: unsigned(7 downto 0);   -- receive shift register
  signal ct1 : integer range 0 to 7;   -- indicates when to read the RxD input
  signal ct2 : integer range 0 to 8;   -- counts number of bits read
  signal inc1, inc2, clr1, clr2, shftRSR, loadRDR: std_logic;
  signal BclkX8_Dlayed, BclkX8_rising: std_logic;
begin
  BclkX8_rising <= BclkX8 and (not BclkX8_Dlayed);
    -- indicates the rising edge of bitX8 clock
  Rcvr_Control: process(state, RxD, RDRF, ct1, ct2, BclkX8_rising)
  begin
    -- reset control signals
    inc1 <= '0'; inc2 <= '0'; clr1 <= '0'; clr2 <= '0';
    shftRSR <= '0'; loadRDR <= '0'; setRDRF <= '0'; setOE <= '0'; setFE <= '0';
    case state is
      when IDLE =>
        if (RxD = '0') then nextstate <= START_DETECTED;
        else nextstate <= IDLE;
        end if;
      when START_DETECTED =>
        if (BclkX8_rising = '0') then nextstate <= START_DETECTED;
        elsif (RxD = '1') then clr1 <= '1'; nextstate <= IDLE;
        elsif (ct1 = 3) then clr1 <= '1'; nextstate <= RECV_DATA;
        else inc1 <= '1'; nextstate <= START_DETECTED;
        end if;
      when RECV_DATA =>
        if (BclkX8_rising = '0') then nextstate <= RECV_DATA;
        else inc1 <= '1';
          if (ct1 /= 7) then nextstate <= RECV_DATA;
            -- wait for 8 clock cycles
          elsif (ct2 /= 8) then
            shftRSR <= '1'; inc2 <= '1'; clr1 <= '1'; -- read next data bit
            nextstate <= RECV_DATA;
          else
            nextstate <= IDLE;
            setRDRF <= '1'; clr1 <= '1'; clr2 <= '1';
```

```
          if (RDRF = '1') then setOE <= '1';        -- overrun error
          elsif (RxD = '0') then setFE <= '1';      -- framing error
          else loadRDR <= '1';                      -- load recv data register
          end if;
        end if;
      end if;
    end case;
  end process;

  Rcvr_update: process(sysclk, rst_b)
  begin
    if (rst_b = '0') then state <= IDLE; BclkX8_Dlayed <= '0';
      ct1 <= 0; ct2 <= 0;
    elsif (sysclk'event and sysclk = '1') then
      state <= nextstate;
      if (clr1 = '1') then ct1 <= 0; elsif (inc1 = '1') then
        ct1 <= ct1 + 1;
      end if;
      if (clr2 = '1') then ct2 <= 0; elsif (inc2 = '1') then
        ct2 <= ct2 + 1;
      end if;
      if (shftRSR = '1') then RSR <= RxD & RSR(7 downto 1);
      end if;
      -- update shift reg.
      if (loadRDR = '1') then RDR <= RSR;
      end if;
      BclkX8_Dlayed <= BclkX8;   -- BclkX8 delayed by 1 sysclk
    end if;
  end process;
end rcvr;
```

Figure 8-25 shows the result of synthesizing the UART receiver using the Xilinx Spartan 3 device FPGA series as a target. The resulting implementation requires 26 flip-flops, 21 slices, and 32 four-input LUTs.

Next we will design a programmable baud rate generator. Three bits in the *SCCR* are used to select any one of eight baud rates. We will assume that the system clock is 8 MHz and we want baud rates 300, 600, 1200, 2400, 4800, 9600, 19,200, and 38,400. The maximum *BclkX8* frequency needed is 38,400 × 8 = 307,200. To get this frequency, we should divide 8 MHz by 26.04. Since we can divide only by an integer, we need to either accept a small error in the baud rate or adjust the system clock frequency downward to 7.9877 MHz to compensate.

Figure 8-26 shows a block diagram for the baud rate generator. The 8-MHz system clock is first divided by 13 using a counter. This counter output goes to an 8-bit binary counter. The outputs of the flip-flops in this counter correspond to divide by 2, divide by 4, . . . , and divide by 256. One of these outputs is selected by a multiplexer. The MUX select inputs come from the lower 3 bits of the *SCCR*. The MUX

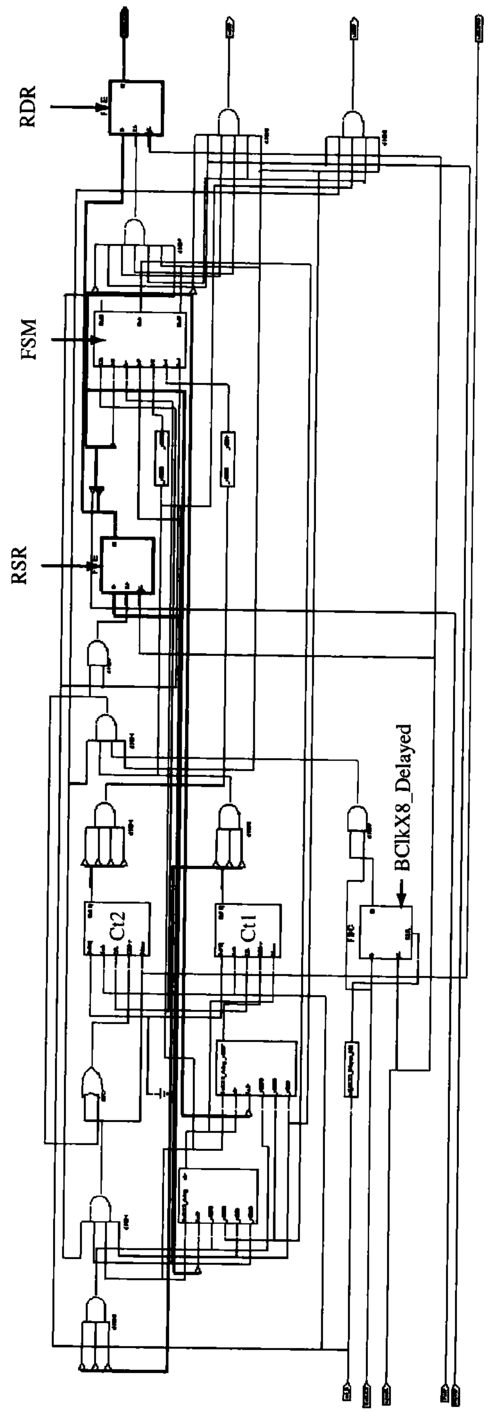
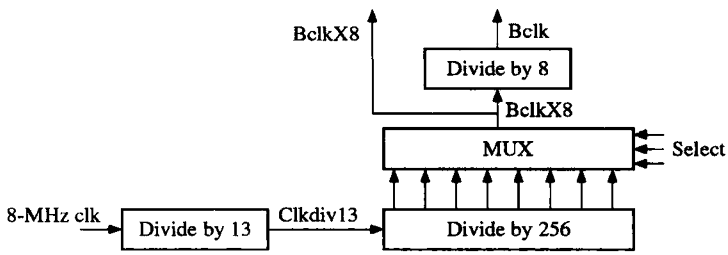


FIGURE 8-25: Synthesized UART Receiver

output corresponds to *BclkX*8, which is further divided by 8 to give *Bclk*. Assuming an 8-MHz clock, the frequencies generated are given by the following table:

Select Bits	BAUD Rate (*Bclk*)
000	38,462
001	19,231
010	9615
011	4808
100	2404
101	1202
110	601
111	300.5

FIGURE 8-26: Baud Rate Generator



The VHDL code for the baud rate generator is given in Figure 8-27. The first process increments the divide-by-13 counter on the rising edge of the system clock. The second process increments the divide-by-256 counter on the rising edge of *Clkdiv*13. A concurrent statement generates the MUX output, *BclkX*8. The third process increments the divide-by-8 counter on the rising edge of *BclkX*8 to generate *Bclk*.

FIGURE 8-27: VHDL Code for Baud Rate Generator

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;   -- for overloaded + operator and conversion functions

entity clk_divider is
  port(Sysclk, rst_b: in std_logic;
       Sel: in unsigned(2 downto 0);
       BclkX8: buffer std_logic;
       Bclk: out std_logic);
end clk_divider;

architecture baudgen of clk_divider is
  signal ctr1: unsigned(3 downto 0) := "0000";    -- divide by 13 counter
  signal ctr2: unsigned(7 downto 0) := "00000000";    -- div by 256 ctr
  signal ctr3: unsigned(2 downto 0) := "000";   -- divide by 8 counter
  signal Clkdiv13: std_logic;
```

```
begin
  process(Sysclk)    -- first divide system clock by 13
  begin
    if (Sysclk'event and Sysclk = '1') then
      if (ctr1 = "1100") then ctr1 <= "0000";
      else ctr1 <= ctr1 + 1;
      end if;
    end if;
  end process;
  Clkdiv13 <= ctr1(3);     -- divide Sysclk by 13

  process(Clkdiv13)    -- ctr2 is an 8-bit counter
  begin
    if (Clkdiv13'event and Clkdiv13 = '1') then
      ctr2 <= ctr2 + 1;
    end if;
  end process;

  BclkX8 <= ctr2(to_integer(sel));    -- select baud rate
  process(BclkX8)
  begin
    if (BclkX8'event and BclkX8 = '1') then
      ctr3 <= ctr3 + 1;
    end if;
  end process;
  Bclk <= ctr3(2);    -- Bclk is BclkX8 divided by 8
end baudgen;
```

To complete the UART design, we need to interconnect the three components we have designed, connect them to the control and status registers, and add the interrupt generation logic and the bus interface. Figure 8-28 gives the VHDL code for the complete UART.

FIGURE 8-28: VHDL Code for Complete UART

```
library IEEE;
use IEEE.std_logic_1164.all;
use IEEE.numeric_std.all;

entity UART is
  port(SCI_sel, R_W, clk, rst_b, RxD: in std_logic;
       ADDR2: in unsigned(1 downto 0);
       DBUS: inout unsigned(7 downto 0);
       SCI_IRQ, TxD: out std_logic);
end UART;

architecture uart1 of UART is
  component UART_Receiver
    port(RxD, BclkX8, sysclk, rst_b, RDRF: in std_logic;
```

```
        RDR: out unsigned(7 downto 0);
        setRDRF, setOE, setFE: out std_logic);
  end component;
  component UART_Transmitter
    port(Bclk, sysclk, rst_b, TDRE, loadTDR: in std_logic;
        DBUS: in unsigned(7 downto 0);
        setTDRE, TxD: out std_logic);
  end component;
  component clk_divider
    port(Sysclk, rst_b: in std_logic;
        Sel: in unsigned(2 downto 0);
        BclkX8: buffer std_logic; Bclk: out std_logic);
  end component;
  signal RDR: unsigned(7 downto 0);   -- Receive Data Register
  signal SCSR: unsigned(7 downto 0);   -- Status Register
  signal SCCR: unsigned(7 downto 0);   -- Control Register
  signal TDRE, RDRF, OE, FE, TIE, RIE: std_logic;
  signal BaudSel: unsigned(2 downto 0);
  signal setTDRE, setRDRF, setOE, setFE, loadTDR, loadSCCR: std_logic;
  signal clrRDRF, Bclk, BclkX8, SCI_Read, SCI_Write: std_logic;
begin
  RCVR: UART_Receiver port map(RxD, BclkX8, clk, rst_b, RDRF, RDR,
                                setRDRF, setOE, setFE);
  XMIT: UART_Transmitter port map(Bclk, clk, rst_b, TDRE, loadTDR,
                                   DBUS, setTDRE, TxD);
  CLKDIV: clk_divider port map(clk, rst_b, BaudSel, BclkX8, Bclk);
  -- This process updates the control and status registers
  process(clk, rst_b)
  begin
    if (rst_b='0') then
      TDRE <= '1'; RDRF <= '0'; OE <= '0'; FE <= '0';
      TIE <= '0'; RIE <= '0';
    elsif (rising_edge(clk)) then
      TDRE <= (setTDRE and not TDRE) or (not loadTDR and TDRE);
      RDRF <= (setRDRF and not RDRF) or (not clrRDRF and RDRF);
      OE <= (setOE and not OE) or (not clrRDRF and OE);
      FE <= (setFE and not FE) or (not clrRDRF and FE);
      if (loadSCCR = '1') then TIE <= DBUS(7); RIE <= DBUS(6);
        BaudSel <= DBUS(2 downto 0);
      end if;
    end if;
  end process;

  -- IRQ generation logic
  SCI_IRQ <= '1' when ((RIE='1' and (RDRF='1' or OE='1')) or
                      (TIE='1' and TDRE='1'))
                 else '0';
  -- Bus Interface
  SCSR <= TDRE & RDRF & "0000" & OE & FE;
  SCCR <= TIE & RIE & "000" & BaudSel;
```

```
  SCI_Read <= '1' when (SCI_sel = '1' and R_W = '0') else '0';
  SCI_Write <= '1' when (SCI_sel = '1' and R_W = '1') else '0';
  clrRDRF <= '1' when (SCI_Read = '1' and ADDR2 = "00") else '0';
  loadTDR <= '1' when (SCI_Write = '1' and ADDR2 = "00") else '0';
  loadSCCR <= '1' when (SCI_Write = '1' and ADDR2 = "10") else '0';
  DBUS <= "ZZZZZZZZ" when (SCI_Read = '0')   -- tristate bus when not reading
          else RDR when (ADDR2 = "00")
  -- write appropriate register to the bus
          else SCSR when (ADDR2 = "01")
          else SCCR;   -- dbus = sccr, if ADDR2 is "10" or "11"
end uart1;
```

SCI_IRQ is an interrupt signal that interrupts the CPU when the UART receiver or transmitter needs attention. When the *RIE* (receive interrupt enable) is set in *SCCR, SCI_IRQ* is generated whenever *RDRF* or *OE* is '1'. When *TIE* (transmit interrupt enable) is set in *SCCR, SCI_IRQ* is generated whenever *TDRE* is '1'.

The UART is interfaced to microcontroller address and data buses so that the CPU can read and write to the UART registers when the UART is selected by *SCIsel* = '1'. The last two bits of the address (*ADDR*2), together with the *R_W* signal, are used for register selection as follows:

ADDR2	*R_W*	Action
00	0	DBUS ← RDR
00	1	TDR ← DBUS
01	0	DBUS ← SCSR
01	1	DBUS ← hi-Z
1–	0	DBUS ← SCCR
1–	1	SCCR ← DBUS

When the UART is not selected for reading, the data bus is driven to high-Z.

The VHDL code in Figure 8-28 was synthesized using the Xilinx SPARTAN 3 series FPGA as a target. The resulting implementation required 62 slices, 109 four-input LUTs, and 74 flip-flops.

This chapter presented three examples for the use of VHDL in design and simulation of digital systems. Two design examples, a wristwatch and a UART, and a simulation example, a memory chip, were presented. In the design examples, we first developed a block diagram for the design and state machine charts representing the controller of the system. Then, we presented behavioral VHDL models for the various blocks in the system. Use of test benches is illustrated. The VHDL code was then synthesized for FPGAs. Designs were downloaded and operation verified.

We also presented a simulation model for a memory chip. This model included timing parameters for the memory chip and built-in checks to verify that setup and hold times and other timing specifications are met. Such models are helpful when third party cores/chips are utilized during system on a chip (SoC) design.

8.4 Problems

8.1 Assume that you are implementing the wristwatch design from Section 8.1 on an FPGA board. Design the input module for the wristwatch for the FPGA board that you have and write the VHDL code.

8.2 Assume that you are implementing the wristwatch design from Section 8.1 on an FPGA board. Design the display module for the wristwatch and write the VHDL code. Use an FPGA board with an LCD display. Display the time, the alarm setting, or the stopwatch time depending on which mode the wristwatch is in.

8.3 **(a)** Add a count-down timer mode to the wristwatch module of Figures 8-2 and 8-3. The timer should count seconds, minutes, and hours. When in the timer state, B2 should change states to allow setting the hours, minutes, and seconds with B3. When setting is complete and the wristwatch is back in the main timer state, B3 should start the count-down. If B3 is pressed again, it should stop the count-down; otherwise, the count-down stops when it reaches 00:00:00, in which case the timer beeps for one second.

(b) Modify the test bench and test your timer.

8.4 The problem concerns the design of a simple calculator for adding unsigned binary numbers. Operation is similar to a simple hand-held calculator, except all inputs and outputs are in binary, and the only operation is +. The calculator displays 8 bits with a binary point. The calculator has only five keys: 0, 1, ., +, and reset. Reset clears all registers and resets the calculator to the starting state. After entering the first number, the + key terminates that entry and allows a second number to be entered. When + is pushed again, the sum is put in the accumulator, and another number can be entered. This continues until the calculator is reset. Note that there is no equals key. You may assume that only normal input sequences occur, that is, a number will always be entered each time before + is pressed. Before addition can be done, the binary points of the numbers to be added must be aligned by shifting. If addition produces an overflow, the overflow should be corrected if possible. If not, set $E = 1$ to indicate an error.

The keys are not encoded. The calculator has six input signals: *zero, one, dot, plus, reset*, and V. Assume that all input signals are debounced, and $V = 1$ for one clock time whenever a key is pressed. Outputs to the display are 8 bits from the A register, *RCTA* (the number of bits to the right of the binary point), and E.

(a) Draw a block diagram for the calculator showing required registers, counters, adders, and so on. Show the necessary control signals and tell what they mean. For example, *RSHA* means right shift A. Specify the size of each register.

(b) Draw an SM chart for the main calculator code. Include inputting the binary numbers, aligning the binary points, adding, and correcting for overflow if possible. Define all control signals used.

(c) Write VHDL code for the main calculator module.
(d) Write a test bench for your VHDL module.

8.5 This question refers to static RAM read and write cycles (refer to Figures 8-11 and 8-12). Answer this question in general, not for any specific set of numeric values.

(a) If $\overline{WE} = 1$, and the address changes at the same time $\overline{CS}$ goes to 0, what is the maximum time before valid data is available at the RAM output? (*Note*: The timing diagrams are not drawn to scale).
(b) What determines the maximum number of bytes per second that can be read from the RAM? State any assumptions which you make.
(c) For a $\overline{WE}$-controlled write cycle, what is the normal sequence of events which occur when writing to RAM?
(d) State clearly what timing conditions must be satisfied in order to correctly write data to the RAM. For example, $\overline{WE}$ must be 0 for at least t_{wp}.

8.6 Answer the following questions for the 6116 SA-15 static CMOS RAM. Refer to the timing specifications in Table 8-1.

(a) What is the maximum clock frequency that can be used?
(b) What is the minimum time after a change in address or $\overline{CS}$ at which valid data can be read?
(c) For a $\overline{WE}$-controlled write cycle, what is the earliest time new data can be driven after $\overline{WE}$ goes low?
(d) For a write cycle, what is the minimum time that valid data must be driven onto the data bus?

8.7 This problem concerns a simplified memory model for a 6116 CMOS RAM. Assume that both $\overline{CS}$ and $\overline{OE}$ are always low, so memory operation depends only on the address and $\overline{WE}$.

(a) Write a simple VHDL model for the memory that ignores all timing information. (Your model should not contain $\overline{CS}$ or $\overline{OE}$.)
(b) Add the following timing specs to your model: t_{AA}, t_{OH}, t_{WHZ}, and t_{OW}. For reads, *Dout* should go to "XXXXXXXX" (unknown) after t_{OH} and then to valid data out after t_{AA}. For writes, *Dout* should go to high-*Z* after t_{WHZ}, and it should go to the value just stored after t_{OW}.
(c) Add another process that gives appropriate error messages if any of the following specs are not satisfied: t_{WP}, t_{DW}, and t_{DH}.

8.8 A VHDL model that describes the operation of the 6116 memory is given in Figure 8-14.

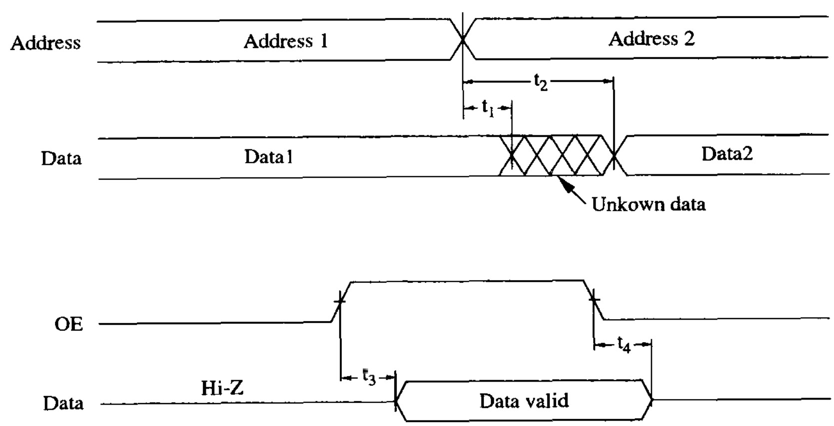
(a) Verify that the code will report a warning if the data setup time for writing to memory is not met, if the data hold time for writing to memory is not met, or if the minimum pulse width spec for *WE_b* is not met.

(b) Indicate the changes and additions to the original VHDL code that are necessary if *OE_b* ($\overline{\text{OE}}$) is taken into account. Note that for reads, if *OE_b* goes low after *CS_b* goes low, the t_{OE} access time must be considered. Also note that when *OE_b* goes high, the data bus will go high-Z after time t_{OHZ}.

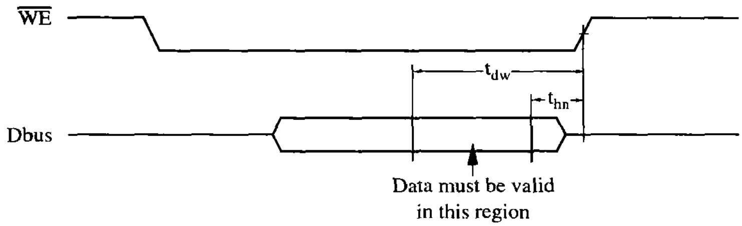
8.9 What modifications must be made in the check process in the VHDL 6116 RAM timing model (Figure 8-14) in order to verify the address setup time (t_{AS}) and the write recovery time (t_{WR}) specifications?

8.10 Consider the *CS*-controlled write cycle for a static CMOS RAM (Figure 8-13). What VHDL code is needed in the check process in the timing model (Figure 8-14) to verify the correct operation of a *CS*-controlled write? You must check timing specifications such as t_{CW}, t_{DW}, and t_{DH}.

8.11 A ROM (read-only memory) has an 8-bit address input, an output enable (*OE*), and an 8-bit data output. When $OE = 0$, *data* = hi-Z; when $OE = 1$, *data* is read from the ROM. Timing diagrams are shown below. Write a VHDL model for the ROM that includes the timing specifications.



8.12 A static RAM memory uses a $\overline{\text{WE}}$ controlled write cycle as shown in the figure. This memory has a negative data hold time with a magnitude t_{hn}. This means that as long as the setup time (t_{dw}) is satisfied, it is okay for the input data to change anytime during the interval t_{hn} before the rising edge of $\overline{\text{WE}}$. Write a process that will report an error if the input data (*Dbus*) changes at any time during the time interval t_{dw} to t_{hn} before $\overline{\text{WE}}$ rises.



8.13 Make necessary changes in the UART receiver VHDL code so that it uses a 16X bit clock instead of an 8X bit clock. Using a faster sampling clock can improve the noise immunity of the receiver.

8.14 **(a)** Write a VHDL test bench for the UART. Include cases to test overrun error, framing error, noise causing a false start, change of BAUD rate, and so on. Simulate the VHDL code.

(b) If suitable hardware is available, write a simpler test bench to allow a loop-back test with *TxD* externally connected to *RxD*. Synthesize the test bench along with the UART, download to the target device, and verify correct operation of the hardware.

8.15 Make necessary changes to the VHDL code to add a parity option to the UART described in Section 8.3. Add 2 bits (P_1P_0) to the SCCR that select the parity mode as follows:

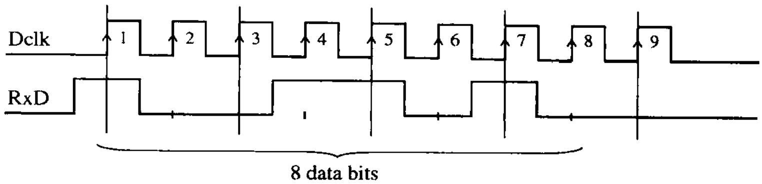
$P_1P_0 = 00$ 8 data bits, no parity bit
$P_1P_0 = 01$ 7 data bits, 8th bit makes parity even
$P_1P_0 = 10$ 7 data bits, 8th bit makes parity odd
$P_1P_0 = 11$ 7 data bits, 8th bit is always '0'

The transmitter should generate the even, odd, or '0' parity bit as specified. The receiver should check the parity bit to verify that it is correct. If not, it should set a PE (parity error) flag in the SCSR.

8.16 The operation of a synchronous receiver is somewhat similar to the UART receiver discussed in Section 8.3, except both data (*RxD*) and a data clock (*Dclk*) are transmitted so there is no need to synchronize data with a local clock, and no start and stop bits are required. As shown below, when 8 bits of data are transmitted, the clock is actually active for nine clock times and then it becomes inactive. On the first eight clocks data is shifted into the receive shift register (RSR), and on the ninth clock, the data is transferred to the receive data register (RDR) and the RDRF flag is set.

(a) Draw a block diagram for the synchronous receiver, including a counter. (*Note*: A state machine is not necessary, but generation of control signals *Load* and *Shift* is required.)

(b) Write synthesizable VHDL code that corresponds to (a). Signals *Load* and *Shift* should appear explicitly in your code.



8.17 Write a test bench for the UART that performs a loop-back test. The test bench connects the *TxD* output of the UART to the *RxD* input so that any data loaded into TDR will automatically be transmitted from *TxD*, received into *RxD*, and loaded into RDR. The test bench should simulate the action of a CPU that writes "01010101" to TDR, reads the status register in a loop until RDRF = '1', and then reads from RDR.

APPENDIX A

VHDL Language Summary

Disclaimer: This VHDL summary is not complete and contains some special cases. Only VHDL statements used in this text are listed. For a complete description of VHDL syntax, refer to References 6, 9, and 23.

Notes:

- VHDL is not case sensitive.
- Signal names and other identifiers may contain letters, numbers, and the underscore (_) character.
- An identifier must start with a letter.
- An identifier cannot end with an underscore.
- Every VHDL statement must be terminated with a semicolon.
- VHDL is a strongly typed language. In general, mixing of data types is not allowed.

LEGEND

bold	reserved word
[ ]	optional items
{ }	repeated zero or more times
|	or

1. Predefined Types

bit	'0' or '1'
boolean	FALSE or TRUE
integer	an integer in the range $-(2^{31} - 1)$ to $+(2^{31} - 1)$ (some implementations support a wider range)
real	floating-point number in the range $-1.0E38$ to $+1.0E38$
character	any legal VHDL character including upper- and lowercase letters, digits, and special characters (each printable character must be enclosed in single quotes; e.g., 'd', '7', '+')
time	an integer with units fs, ps, ns, us, ms, sec, min, or hr
natural	integers ≥ 0
positive	integers > 0

bit_vector	array of bits
string	array of characters
delay_length	time ≥ 0

2. Operators By Increasing Precedence

1. Binary logical operators: **and or nand nor xor xnor**
2. Relational operators: = /= < <= > >=
3. Shift operators: **sll srl sla sra rol ror**
4. Adding operators: + − & (concatenation)
5. Unary sign operators: + −
6. Multiplying operators: * / **mod rem**
7. Miscellaneous operators: **not abs** **

3. Predefined Attributes

Signal attributes that return a value:

Attribute	Returns
S'ACTIVE	true if a transaction occurred during the current delta, else false
S'EVENT	true if an event occurred during the current delta, else false
S'LAST_EVENT	time elapsed since the previous event on S
S'LAST_VALUE	value of S before the previous event on S
S'LAST_ACTIVE	time elapsed since previous transaction on S

Signal attributes that create a signal:

Attribute	Creates
S'DELAYED [(time)]*	signal same as S delayed by specified time
S'STABLE [(time)]*	boolean signal that is true if S had no events for the specified time
S'QUIET [(time)]*	boolean signal that is true if S had no transactions for the specified time
S'TRANSACTION	signal of type bit that changes for every transaction on S

*Delta is used if no time is specified.

Array attributes:

```
type ROM is array (0 to 15, 7 downto 0) of bit;
signal ROM1 : ROM;
```

Attribute	Returns	Examples
A'LEFT(N)	left bound of Nth index range	ROM1'LEFT(1) = 0 ROM1'LEFT(2) = 7
A'RIGHT(N)	right bound of Nth index range	ROM1'RIGHT(1) = 15 ROM1'RIGHT(2) = 0
A'HIGH(N)	largest bound of Nth index range	ROM1'HIGH(1) = 15 ROM1'HIGH(2) = 7
A'LOW(N)	smallest bound of Nth index range	ROM1'LOW(1) = 0 ROM1'LOW(2) = 0
A'RANGE(N)	Nth index range	ROM1'RANGE(1) = 0 to 15 ROM1'RANGE(2) = 7 downto 0
A'REVERSE_RANGE(N)	Nth index range reversed	ROM1'REVERSE_RANGE(1) = 15 downto 0 ROM1'REVERSE_RANGE(2) = 0 to 7
A'LENGTH(N)	size of Nth index range	ROM1'LENGTH(1) = 16 ROM1'LENGTH(2) = 8

4. Predefined Functions

NOW	returns current simulation time
FILE_OPEN([status], FileID, string, mode)	open file
FILE_CLOSE(FileID)	close file

5. Declarations

entity declaration:

```
entity entity-name is
        [generic (list-of-generics-and-their-types);]
        [port (interface-signal-declaration);]
        [declarations]
end [entity] [entity-name];
```

interface-signal declaration:

```
list-of-interface-signals: mode type [:= initial-value]
{; list-of-interface-signals: mode type [:= initial-value]}
```

Note: An interface signal can be of mode in, out, inout, or buffer.

architecture declaration:

```
architecture architecture-name of entity-name is
        [declarations]      -- variable declarations not allowed
begin
        architecture-body
end [architecture] [architecture-name];
```

Note: The architecture body may contain component-instantiation statements, processes, blocks, assignment statements, procedure calls, etc.

integer type declaration:
```
type type_name is range integer_range;
```

enumeration type declaration:
```
type type_name is (list-of-names-or-characters);
```

subtype declaration:
```
subtype subtype_name is type_name [index-or-range-constraint];
```

variable declaration:
```
variable list-of-variable-names: type_name [:= initial_value];
```

signal declaration:
```
signal list-of-signal-names: type_name [:= initial_value];
```

constant declaration:
```
constant constant_name: type_name := constant_value;
```

alias declaration:
```
alias identifier[:identifier-type] is item-name;
```

Note: Item-name can be a constant, signal, variable, file, function name, type name, etc.

array type and object declaration:
```
type array_type_name is array index_range of element_type;
signal|variable|constant array_name: array_type_name
[:= initial_values];
```

procedure declaration:
```
procedure procedure-name (parameter list) is
      [declarations]
begin
      sequential statements
end procedure-name;
```

Note: Parameters may be signals, variables, or constants.

function declaration:
```
function function-name (parameter-list) return return-type is
       [declarations]
begin
       sequential statements          -- must include return
                                         return-value;
end function-name;
```

Note: Parameters may be signals or constants.

library declaration:

```
library list-of-library-names;
```

use statement:

```
use library_name.package_name.item;        (.item may be .all)
```

package declaration:

```
package package-name is
        package declarations
end [package][package-name];
```

package body:

```
package body package-name is
        package body declarations
end [package body][package name];
```

component declaration:

```
component component-name
      [generic (list-of-generics-and-their-types);]
      port (list-of-interface-signals-and-their-types);
end component;
```

file type declaration:

```
type file_name is file of type_name;
```

file declaration:

```
file file_name: file_type [open mode] is "file_pathname";
```

Note: Mode may be read_mode, write_mode, or append_mode.

6. Concurent Statements

signal assignment statement:

```
signal <= [reject pulse-width | transport] expression [after
delay_time];
```

Note: If signal assignment done as concurrent statement, signal value is recomputed every time a change occurs on the right-hand side. If `[after delay_time]` is omitted, signal is updated after delta time.

conditional assignment statement:

```
signal <= expression1 when condition1
          else expression2 when condition2
          . . .
          [else expression];
```

selected signal assignment statement:

```
with expression select
      signal <= expression1 [after delay_time] when choice1,
                expression2 [after delay_time] when choice2,
                . . .
                [expression [after delay_time] when others];
```

assert statement:

```
assert boolean-expression
      [report string-expression]
      [severity severity-level];
```

component instantiation:

```
label: component-name
      [generic map (generic-association-list);]
      port map (list-of-actual-signals);
```

Note: Use **open** if a component output has no connection

generate statements:

```
generate_label: for identifier in range generate
[begin]
      concurrent statement(s)
end generate [generate_label];

generate_label: if condition generate
[begin]
      concurrent statement(s)
end generate [generate_label];
```

process statement (with sensitivity list):

```
[process-label:] process (sensitivity-list)
      [declarations]      -- signal declarations not allowed
begin
      sequential statements
end process [process-label];
```

Note: This form of process is executed initially and thereafter only when an item on the sensitivity list changes value. The sensitivity list is a list of signals. No wait statements are allowed.

process statement (without sensitivity list):

```
[process-label:] process
      [declarations]      -- signal declarations not allowed
begin
      sequential statements
end process [process-label];
```

Note: This form of process must contain one or more wait statements. It starts execution immediately and continues until a wait statement is encountered.

procedure call:
```
procedure-name (actual-parameter-list);
```

Note: An expression may be used for an actual parameter of mode in; types of the actual parameters must match the types of the formal parameters; open cannot be used.

function call:
```
function-name (actual-parameter list)
```

Note: A function call is used within (or in place of) an expression. Function call is not a statement by itself, it is part of a statement.

7. Sequential Statements

signal assignment statement:
```
signal <= [reject pulse-width | transport] expression [after
delay_time];
```

Note: If `[after delay_time]` is omitted, signal is updated after delta time.

variable assignment statement:
```
variable := expression;
```

Note: This can be used only within a process, function, or procedure. The variable is always updated immediately.

wait statements can be of the form:
```
wait on sensitivity-list;
wait until boolean-expression;
wait for time-expression;
```

if statement:
```
if condition then
        sequential statements
{elsif condition then
        sequential statements}  -- 0 or more elsif clauses may
                                   be included
[else sequential statements]
end if;
```

case statement:
```
case expression is
      when choice1 => sequential statements
      when choice2 => sequential statements
      . . .
      [when others => sequential statements]
end case;
```

for loop statement:

```
[loop-label:] for identifier in range loop
        sequential statements
end loop [loop-label];
```

Note: You may use **exit** to exit the current loop.

while loop statement:

```
[loop-label:] while boolean-expression loop
        sequential statements
end loop [loop-label];
```

exit statement:

```
exit [loop-label] [when condition];
```

assert statement:

```
assert boolean-expression
        [report string-expression]
        [severity severity-level];
```

report statement:

```
report string-expression
        [severity severity-level];
```

procedure call:

```
procedure-name (actual-parameter-list);
```

Note: An expression may be used for an actual parameter of mode in; types of the actual parameters must match the types of the formal parameters; open cannot be used.

function call:

```
function-name (actual-parameter list)
```

Note: A function call is used within (or in place of) an expression. Function call is not a statement by itself, it is part of a statement.

APPENDIX B

IEEE Standard Libraries

The two packages from the IEEE libraries that we have used in the book are NUMERIC_BIT and NUMERIC_STD. The headers of these packages read as follows:

Standard VHDL Synthesis Package (1076.3, NUMERIC_BIT)

```
-- Developers: IEEE DASC Synthesis Working Group, PAR 1076.3
-- Purpose: This package defines numeric types and arithmetic functions
--        :for use with synthesis tools. Two numeric types are defined:
--        :--> UNSIGNED: represents an UNSIGNED number in vector form
--        :--> SIGNED: represents a SIGNED number in vector form
--        :The base element type is type BIT.
--        :The leftmost bit is treated as the most significant bit.
--        :Signed vectors are represented in two's complement form.
--        :This package contains overloaded arithmetic operators on
--        :the SIGNED and UNSIGNED types. The package also contains
--        :useful type conversions functions, clock detection
--        :functions, and other utility functions.
```

Standard VHDL Synthesis Package (1076.3, NUMERIC_STD)

```
-- Developers: IEEE DASC Synthesis Working Group, PAR 1076.3
-- Purpose: This package defines numeric types and arithmetic functions
--        :for use with synthesis tools. Two numeric types are defined:
--        :--> UNSIGNED: represents UNSIGNED number in vector form
--        :--> SIGNED: represents a SIGNED number in vector form
--        :The base element type is type STD_LOGIC.
--        :The leftmost bit is treated as the most significant bit.
--        :Signed vectors are represented in two's complement form.
--        :This package contains overloaded arithmetic operators on
--        :the SIGNED and UNSIGNED types. The package also contains
--        :useful type conversions functions.
```

The entire package listings can be viewed at

http://www.eda.org/rassp/vhdl/models/standards/numeric_bit.vhd
http://www.eda.org/rassp/vhdl/models/standards/numeric_std.vhd

Useful conversion functions in the numeric_bit package:

`TO_INTEGER(A)`: converts an unsigned (or signed) vector *A* to an integer
`TO_UNSIGNED(B,N)`: converts an integer to an unsigned vector of length *N*
`TO_SIGNED(B,N)`: converts an integer to an signed vector of length *N*
`UNSIGNED(A)`: causes the compiler to treat a bit_vector *A* as an unsigned vector
`SIGNED(A)`: causes the compiler to treat a bit_vector *A* as a signed vector
`BIT_VECTOR(B)`: causes the compiler to treat an unsigned (or signed) vector *B* as a bit_vector

The same conversion functions are available in the numeric_std package, except replace bit_vector with std_logic_vector.

Notes:

1. The numeric_bit package provides an overloaded operator to add an integer to an unsigned, but not to add a bit to an unsigned type. Thus, if *A* and *B* are unsigned, `A+B+1` is allowed, but a statement of the form

```
Sum <= A + B + carry;
```

is not allowed when carry is of type bit. The carry must be converted to unsigned before it can be added to the unsigned vector A+B. The notation **`unsigned'(0=>carry)`** will accomplish the necessary conversion. Use the statement

```
Sum <= A + B + unsigned'(0=>carry);
```

2. If we want more bits in the sum than there are in the numbers being added, we must extend the numbers by concatenating '0'. For example, if *X* and *Y* are 4 bits, and a 5-bit sum including the carry out is desired, extend *X* to 5 bits by concatenating '0' and *X*. (*Y* will automatically be extended to match.) Hence:

```
Sum5 <= '0' & X + Y;
```

accomplishes the addition of two 4-bit numbers and provides a 5-bit sum.

APPENDIX C

TEXTIO Package

```
package TEXTIO is
  -- Type definitions for text I/O:
  type LINE is access STRING; -- A LINE is a pointer to a STRING value.
  -- The predefined operators for this type are as follows:
  -- function "=" (anonymous, anonymous: LINE) return BOOLEAN;
  -- function "/=" (anonymous, anonymous: LINE) return BOOLEAN;
  type TEXT is file of STRING; -- A file of variable-length ASCII records.
  -- The predefined operators for this type are as follows:
  -- procedure FILE_OPEN (file F: TEXT; External_Name; in STRING;
  -- Open_Kind: in FILE_OPEN_KIND := READ_MODE);
  -- procedure FILE_OPEN (Status: out FILE_OPEN_STATUS; file F: TEXT;
  -- External_Name: in STRING;
  -- Open_Kind: in FILE_OPEN_KIND := READ_MODE);
  -- procedure FILE_CLOSE (file F: TEXT);
  -- procedure READ (file F: TEXT; VALUE: out STRING);
  -- procedure WRITE (file F: TEXT; VALUE: in STRING);
  -- function ENDFILE (file F: TEXT) return BOOLEAN;
  type SIDE is (RIGHT, LEFT); -- For justifying output data within fields.
  -- The predefined operators for this type are as follows:
  -- function "=" (anonymous, anonymous: SIDE) return BOOLEAN;
  -- function "/=" (anonymous, anonymous: SIDE) return BOOLEAN;
  -- function "<" (anonymous, anonymous: SIDE) return BOOLEAN;
  -- function "<=" (anonymous, anonymous: SIDE) return BOOLEAN;
  -- function ">" (anonymous, anonymous: SIDE) return BOOLEAN;
  -- function ">=" (anonymous, anonymous: SIDE) return BOOLEAN;
  subtype WIDTH is NATURAL; -- For specifying widths of output fields.

  -- Standard text files:
  file INPUT: TEXT open READ_MODE is "STD_INPUT";
  file OUTPUT: TEXT open WRITE_MODE is "STD_OUTPUT";
  -- Input routines for standard types:
  procedure READLINE (file F: TEXT; L: inout LINE);
  procedure READ (L: inout LINE; VALUE: out BIT; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out BIT);
  procedure READ (L: inout LINE; VALUE: out BIT_VECTOR; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out BIT_VECTOR);
```

```
  procedure READ (L: inout LINE; VALUE: out BOOLEAN; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out CHARACTER; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out CHARACTER);
  procedure READ (L: inout LINE; VALUE: out INTEGER; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out INTEGER);
  procedure READ (L: inout LINE; VALUE: out REAL; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out REAL);
  procedure READ (L: inout LINE; VALUE: out STRING; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out STRING);
  procedure READ (L: inout LINE; VALUE: out TIME; GOOD: out BOOLEAN);
  procedure READ (L: inout LINE; VALUE: out TIME);
  -- Output routines for standard types:
  procedure WRITELINE (file F: TEXT; L: inout LINE);
  procedure WRITE (L: inout LINE; VALUE: in BIT;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0);
  procedure WRITE (L: inout LINE; VALUE: in BIT_VECTOR;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0);
  procedure WRITE (L: inout LINE; VALUE: in BOOLEAN;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0);
  procedure WRITE (L: inout LINE; VALUE: in CHARACTER;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0);
  procedure WRITE (L: inout LINE; VALUE: in INTEGER;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0);
  procedure WRITE (L: inout LINE; VALUE: in REAL;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0;
                   DIGITS: in NATURAL:= 0);
  procedure WRITE (L: inout LINE; VALUE: in STRING;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0);
  procedure WRITE (L: inout LINE; VALUE: in TIME;
                   JUSTIFIED: in SIDE:= RIGHT; FIELD: in WIDTH := 0;
                   UNIT: in TIME:= ns);
  -- File position predicate:
  -- function ENDFILE (file F: TEXT) return BOOLEAN;
end TEXTIO;
```

APPENDIX D

Projects

For each of these projects, choose an appropriate FPGA or CPLD as a target device and carry out the following steps:

1. Work out an overall design strategy for the system and draw block diagrams. Divide the system into modules if appropriate. Develop an algorithm, SM charts, or state graphs as appropriate for each module. Unless otherwise specified, your design should be a synchronous system with appropriate circuits added to synchronize the inputs with the clock.
2. Write synthesizable VHDL code for each module, simulate it, and debug it. To avoid timing problems in the hardware, use signals instead of variables and make sure the code synthesizes without latches. Use test benches when appropriate to verify correct operation of each module.
3. Integrate the VHDL code for the modules, simulate, and test the overall system.
4. Make any needed changes and synthesize the VHDL code for the target device. Simulate the system after synthesis.
5. Generate a bit file for the target device and download it. Verify that the hardware works correctly.

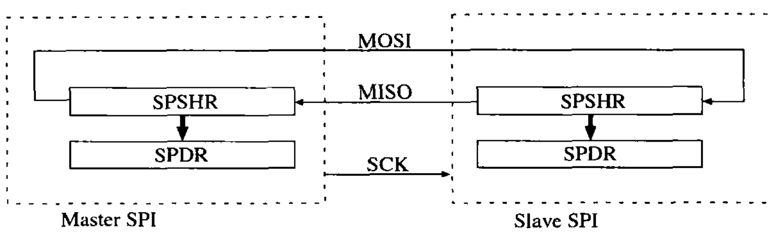
P1. Push-Button Door Lock

Design a push-button door lock that uses a standard telephone keypad as input. Use the keypad scanner designed in Chapter 4 as a module. The length of the combination is 4 to 7 digits. To unlock the door, enter the combination followed by the # key. As long as # is held down, the door will remain unlocked and can be opened. When # is released, the door is relocked. To change the combination, first enter the correct combination followed by the * key. The lock is then in the "store" mode. The "store" indicator light comes on and remains on until the combination has been successfully changed. Next enter the new combination (4 to 7 digits) followed by #. Then enter the new combination a second time followed by #. If the second time does not match the first time, the new combination must be entered two times again. Store the combination in an array of eight 4-bit registers or in a small RAM. Store the 4-bit key codes followed by the code for the # key. Also provide a reset button that is not part of the keypad. When the reset button

is pushed, the system enters the "store" state and a new combination may be entered. Use a separate counter for counting the inputs as they come in. A 4-bit code, a key-down signal (*Kd*), and a valid data signal (*V*) are available from the keypad module.

P2. Synchronous Serial Peripheral Interface

Design an SPI (synchronous serial peripheral interface) module suitable for use with a microcontroller. The SPI allows synchronous serial communication with peripheral devices or with other microcontrollers. The SPI contains four registers—*SPCR* (SPI control), *SPSR* (SPI status), *SPDR* (SPI data), and *SPSHR* (SPI shift register). The following diagram shows how two SPIs can be connected for serial communications. One SPI operates as a master and one as a slave. The master provides the clock for synchronizing transmit and receive operations. When a byte of data is loaded into the master *SPSHR*, it initiates serial transmission and supplies a serial clock (*SCK*). Data is exchanged between the master and slave shift registers in eight clocks. As soon as transmission is complete, data from each *SPSHR* is transferred to the corresponding *SPDR*, and the SPI flag (*SPIF*) in the *SPSR* is set.



The function of the pins depends on whether the device is in master or slave mode:

MOSI—output for master, input for slave
MISO—input for master, output for slave
SCK—output for master, input for slave

The *SPDR* and *SPSHR* are mapped to the same address. Reading from this address reads the *SPDR*, but writing loads the *SPSHR*. *SPSR* bit 7 is the SPI flag (*SPIF*). *SPSR* may also contain error flags, but we will omit them from this design. The following sequence will clear *SPIF*:

Read *SPSR* when *SPIF* is set.
Read or write to the *SPDR* address.

The *SPCR* register contains the following bits:

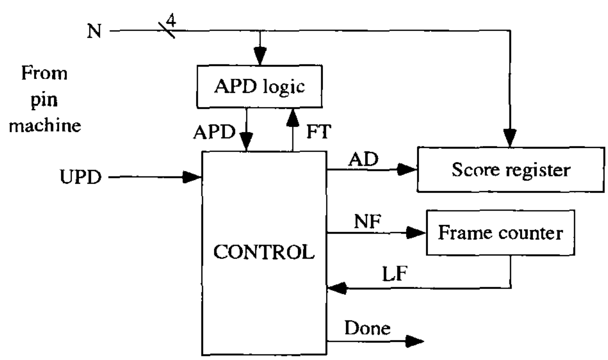
SPIE—enable SPI interrupt
SPE—enable the SPI
MSTR—set to '1' for master mode, '0' for slave mode

SPR1 and *SPR0*—set *SCLK* rate as follows:

$SPR1\&SPR0 = 00$	SCK rate = Sysclk rate/2
$SPR1\&SPR0 = 01$	SCK rate = Sysclk rate/4
$SPR1\&SPR0 = 10$	SCK rate = Sysclk rate/16
$SPR1\&SPR0 = 11$	SCK rate = Sysclk rate/32

P3. Bowling Score Keeper

The digital system shown below will be used to keep score for a bowling game. The score-keeping system will score the game according to the following (regular) rules of bowling: A game of bowling is divided into ten frames. During each frame, the player gets two tries to knock down all of the bowling pins. At the beginning of a frame, ten pins are set up. If the bowler knocks all ten pins down on his or her first throw, then the frame is scored as a *strike*. If some (or all) of the pins remain standing after the first throw, the bowler gets a second try. If the bowler knocks down all of the pins on the second try, the frame is scored as a *spare*. Otherwise, the frame is scored as the total number of pins knocked down during that frame.



The total score for a game is the sum of the number of pins knocked down plus bonuses for scoring strikes and spares. A strike is worth 10 points (for knocking down all ten pins) plus the number of pins knocked down on the next two *throws* (not frames). A spare is worth 10 points (for knocking down ten pins) plus the number of pins knocked down on the next throw. If the bowler gets a spare on the tenth frame, then he or she gets one more throw. The number of pins knocked down from this extra throw are added to the current score to get the final score. If the bowler gets a strike on the last frame, then he or she gets two more throws, and the number of pins knocked down are added to the score. If the bowler gets a strike in frame 9 and 10, then he or she also gets two more throws, but the score from the first bonus throw is added into the total *twice* (once for the strike in frame 9, once for the strike in frame 10), and the second bonus throw is added in once. The maximum score for a perfect game (all strikes) is 300. An example of bowling game scoring follows:

Frame	First Throw	Second Throw	Result	Score
1	3	4	7	7
2	5	5	spare	7 + 10 = 17
3	7	1	8	17 + 7 (bonus for spare in 2) + 8 = 32
...	...	...	...	87
9	10	—	strike	87 + 10 = 97
10	10	—	strike	97 + 10 (for this throw) + 10 (bonus for strike in 9)
—	6	3	—	117 + 6 (bonus for strike in 9) + 6 (bonus for strike in 10) + 3 (bonus for strike in 10) = 132

The score-keeping system has the form shown in the preceding table. The control network has three inputs: *APD* (All Pins Down), *LF* (Last Frame), and *UPD* (update). *APD* is 1 if the bowler has knocked all ten pins down (in either one or two throws). *LF* is 1 if the frame counter is in state 9 (frame 10). *UPD* is a signal to the network that causes it to update the score. *UPD* is 1 for exactly one clock cycle after every throw the bowler makes. There are many clock cycles between updates.

The control network has four outputs: *AD, NF, FT*, and *Done*. *N* represents the number of pins knocked down on the current throw. If *AD* is 1, *N* will be added to the score register on the rising edge of the next clock. If *NF* is 1, the frame counter will increment on the rising edge of the next clock. *FT* is 1 when the first throw in a frame is made. *Done* should be set to 1 when all ten frames and bonus throws, if applicable, are complete.

Use a 10-bit score register and keep the score in BCD form rather than in binary. That is, a score of 197 would be represented as 01 1001 0111. The lower two decimal digits of the register should be displayed using two 7-segment LED indicators, and the upper 2 bits can be connected to two single LEDs. When *ADD* = 1 and the register is clocked, *N* should be added to the register. *N* is a 4-bit binary number in the range 0 through 10. Use a 4-bit BCD counter module for the middle BCD digit. Note that in the lower 4 bits, you will add a binary number to a BCD digit to give a BCD digit and a carry.

P4. Simple Microcomputer

Design a simple microcomputer for 8-bit signed binary numbers. Use a keypad for data entry and a 256 × 8 static RAM memory. The microcomputer should have the following 8-bit registers: *A* (accumulator), *B* (multiplier), *MDR* (memory data register), *PC* (program counter), and *MAR* (memory address register). The *IR* (instruction register) may be 5 to 8 bits, depending on how the instructions are encoded. The *B* register is connected to the *A* register so that *A* and *B* can be shifted together during the multiply. Only one 8-bit adder and one complementer is allowed. The microcomputer should have a 256-word-by-8-bit memory for storing instructions and data. It should have two modes: (a) memory load and (b) execute program. Use a DIP switch to select the mode.

Memory load mode operates as follows: Select mode = 0 and reset the system. Then press two keys on the keypad followed by pushing a button to load each word in memory. The first word is loaded at address 0, the second word at address 1, and so on. Data should be loaded immediately following the program. Execution mode operates as follows: Select mode = 1 and press reset. Execution begins with the instruction at address 0.

Each instruction will be one or two words long. The first word will be the opcode, and the second word (if any) will be an 8-bit memory address or immediate operand. One bit in the opcode should distinguish between memory address or immediate operand mode. Represent negative numbers in 2's complement. Implement the following instructions:

LDA <memadd>	load *A* from the specified memory address
LDA <imm>	load *A* with immediate data
STA <memadd>	store *A* at the specified memory address
ADD <memadd>	add data from memory address to *A*, set carry flag if carry, set *V* if 2's complement overflow
ADD <imm>	add immediate data to *A*, set carry flag if carry, set *V* if overflow
SUB <memadd>	subtract data from memory address from *A*, set carry flag if borrow, set *V* if 2's complement overflow
SUB <imm>	subtract immediate data from *A*, set carry flag if borrow, set *V* if overflow
MUL <memadd>	multiply data from memory address by *B*, result in *A* & *B*
MUL <imm>	multiply immediate data by *B*
SWAP	swap *A* and *B*
PAUSE	pause until a button is pressed and released (*Note*: A register should always be displayed on LEDs.)
JZ <target addr>	jump to target address if $A = 0$
JC <target addr>	jump to target address if carry flag (*CF*) is set
JV <target addr>	jump to target address if overflow flag (*V*) is set

The control module should be implemented as a linked state machine, with a separate state machine for the multiplier control. Try to keep the number of states small. (A good solution should have about ten states for the main control.) The multiplier control should use a separate counter to count the number of shifts. Assume that the clock speed is slow enough so that memory can be accessed in one clock period.

P5. Stack-Based Calculator

Design a stack-based calculator for 8-bit signed binary numbers. Input data to the calculator can come from a keypad or from DIP switches with a separate push-button to enter the data. The calculator should have the following operations:

enter	push the 8-bit input data onto the stack
0 – clear	clear the top of the stack, reset the stack counter, reset overflow, and so on.
1 – add	replace the top two data entries on the stack with their sum

2 – sub	replace the top two data entries on the stack with their difference (stack top—next entry)
3 – mul	replace the top two data entries on the stack with their product (8 bits × 8 bits to give 8-bit product)
4 – div	replace the top two data entries on the stack with their quotient (stack top / next entry) (8 bits divided by 8 bits to give 8-bit quotient)
5 – xchg	exchange the top two data entries on the stack
6 – neg	replace the top of the stack with its 2's complement

Negative numbers should be represented in 2's complement. Provide an overflow indicator for 2's complement overflow. This indicator should also be set if the product requires more than 8 bits including sign or if divide by 0 is attempted.

Implement a stack module that has four 8-bit words. The stack should have the following operations: push, pop, and exchange the top two words on the stack. The top of the stack should always be displayed on eight LEDs. Include an indicator for stack overflow (attempt to push a fifth word) and stack underflow (attempt to pop an empty stack or to exchange the top of stack with an empty location).

Design the control unit for the calculator using linked state machines. Draw a main SM chart with separate SM charts for the multiplier and divider control. When you design the arithmetic unit, try to avoid adding unnecessary registers. You should be able to implement the arithmetic unit with three registers (8 or 9 bits each), an adder, two complementers, and so on.

P6. Floating-Point Arithmetic Unit

Design a floating-point arithmetic unit. Each floating-point number should have a 4-bit fraction and a 4-bit exponent, with negative numbers represented in 2's complement. (This is the notation used in the examples in Chapter 6.) The unit should accept the following floating-point instructions:

001	FPL—load floating-point accumulator (fraction and exponent)
010	FPA—add floating-point operand to accumulator
011	FBS—subtract floating-point operand from accumulator
100	FPM—multiply accumulator by floating-point operand
101	FPD—(optional) divide floating-point accumulator by floating-point operand

The result of each operation (4-bit fraction and 4-bit exponent) should be in the floating-point accumulator. All output should be properly normalized. The accumulator should always be displayed as hex digits on 7 segment LEDs. Use an LED to indicate an overflow.

The input to the floating-point unit will come from a 4 × 4 hexadecimal keypad, using a scanner similar to the one designed in Chapter 4. Each instruction will be represented by three hex digits from the keypad—the opcode, the fraction, and the exponent. For example, FPA $1.011 \times 2^{\ 3}$ is coded as 2 B D = 0010 1011 1101. Assume that all inputs are properly normalized or zero. Your design should include the following modules: fraction unit, exponent unit, control module, and 4-bit binary to seven-segment display conversion logic.

P7. Tic-Tac-Toe Game

Design a machine to play the defensive game of tic-tac-toe using an FPGA. Input will be a 3 × 3 keypad, a reset button, and a switch SW1. If SW1 is off, the machine should always win if possible, or draw (nobody wins) if winning is not possible. If SW1 is on, part of the machine's logic should be bypassed so that the player can win occasionally. Output will be a 3 × 3 array of LEDs with a red and a green LED in each square. Use two LEDs to indicate player wins or machine wins. If the game is a draw, light both LEDs. Since the machine is playing a defensive game, the human player will always move first. Each time the player moves, the machine should wait two seconds before making its move. Your VHDL code should represent a synchronous digital system that makes efficient use of available hardware resources.

Here is one strategy for playing the game: (player = X, machine = O)

1. Player moves first.
2. Machine makes an appropriate initial move. If player starts in center, machine plays corner; otherwise, machine plays center.
3. After each subsequent move by the player, the machine checks the following in sequence:
   (a) Two O's in a row: machine plays in the third square and wins.
   (b) Two X's in a row: machine plays in the third square to block player.
   (c) If it is the machine's second move, a special move may be required: If player's first two moves are opposite corners, the machine's second move must be side. If player's first move is center, the machine's *second* move should be corner if rule (b) does not apply.
   (d) Two intersecting rows each contain only one X: Machine plays in the square at the intersection of the two rows (this blocks the player from forcing a win).
   (e) If there is no better move, play anywhere.

The preceding rules obviously apply only when the appropriate squares are empty.

P8. CORDIC Computing Unit

CORDIC (coordinate rotation digital computer) is a computing technique that uses two-dimensional planar rotation to compute trigonometric functions. This algorithm has a wide variety of applications, ranging from your calculator to global positioning systems. The algorithm is perfect for digital systems since computation is merely a set of repeated adds and shifts. For details of this algorithm, review the paper[1]. "A Survey of CORDIC Algorithms for FPGA-Based Computers," located at http://www.andraka.com/files/crdcsrvy.pdf.

Implement the CORDIC algorithm using an FPGA. Your implementation must correctly produce the sine or cosine of an input angle ranging from −179 to +180 degrees, inclusive. You will only be required to satisfy 8-bit precision. Input will be received in decimal format via a keypad. Three decimal digits will be input (most

[1]R. Andraka, "A Survey of CORDIC Algorithms for FPGA-Based Computers," in *Proceedings of the 1998 ACM/SIGDA Sixth International Symposium on Field Programmable Gate Arrays*, pp. 191–200, February 22–24, 1998.

significant digit first) followed by a sign. The angle should be initially represented in BCD and then converted to binary (negative angles represented 2's complement). Designate two special keys for sine and cosine. Output will be displayed on a set of four 7-segment LEDs.

The following pseudocode demonstrates the basics of the CORDIC algorithm. Read the document referenced above and then iterate through this process by hand to help you understand this algorithm.

```
for i = 0 to n              // n-bit precision
    dx = x/(2^i)            // x is 16-bit register representing
                               fractional values. It should be
                            // initialized to .607 (1001_1011_0111_
                               0001). After the algorithm
                            // completes, x holds cos(a). dx is also
                               16 bits.
    dy = y/(2^i)            // y is a 16-bit register representing
                               fractional values. It should
                            // be initialized to 0 (0000_0000_0000_
                               0000). After the
                            // algorithm completes, y holds sin(a). dy
                               is also 16 bits.
    da = arctan(2^-i)       // pre-calculated values in a lookup table
                            // these values should be represented as
                               follows: upper
                            // 8 bits whole number part, lower 8 bits
                               fractional part
                            // a is the input angle represented with
                               at least 10 bits.
                            // All input angles are whole numbers.
    if (a >= 0) then
        x = x - dy; a = a - da; y = y + dx;
    else
        x = x + dy; a = a + da; y = y - dx;
    end if
end loop
```

When you work through this algorithm, notice that it does not produce the negative and positive values associated with sine and cosine. Create separate logic to determine the sign. The algorithm shown above only works for −90 to +90 input angles. You can simplify your design if you do all calculations in the first quadrant (e.g., sin(105) is the same as sin(75)).

P9. Calculator for Average and Standard Deviation

Design a special-purpose calculator to calculate the average and standard deviation of a set of test scores. Input will be from a decimal keypad and output will be an LCD display. Each test score will be an integer in the range 0 to 100. The number of scores will be in the range 1 to 31.

Entry sequence: For each score, enter one, two, or three digits followed by E (enter). After all scores have been entered, press A to calculate the average and then press D to compute the standard deviation. The average and standard deviation should be displayed with one digit after the decimal point.

The formula for the standard deviation is

$$\text{s.d} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - A)^2}{N}} = \sqrt{\frac{\sum_{i=1}^{N}x_i^2}{N} - A^2}$$

where A is the average. Use the latter form because it is not necessary to store the N scores.

Your design should have three main modules: input, computation, and display. The computation module computes the average and standard deviation of the input data. All computation should be done with binary integers. The input data will be scaled up by a factor of 10 and converted to binary by the input module. The outputs will be converted to decimal and scaled down by a factor of 10 by the display module.

The input module should include a keypad scanner similar to the one designed in Chapter 4. Every time a key is pressed, the scanner will debounce and decode the key. It will then output a 4-bit binary code for the key that was pressed, along with a valid signal (V). This input module will process the digits from the keypad scanner and convert the input number to binary. This module should perform the following tasks:

1. If the input is a digit in the range 0 through 9, store it in a register. Ignore invalid inputs.
2. After one, two, or three digits have been entered followed by E, check to see that the number is within range ($\leq$ 100). If not, turn on an error signal.
3. If the input number is in range, append BCD 0, which in effective multiplies by 10. Example: If the entry sequence is 7, 9, E, the BCD register should contain 0000 0111 1001 0000 (790).
4. Convert the BCD to binary and signal the computation unit when conversion is complete.
5. When the A or D key is pressed, generate a signal for the computation module.

The computation module should have one register to accumulate the sum of the inputs and another to accumulate the sum of the squares of the inputs. The data input should be a binary integer with the decimal range 0 to 1000 (score $\times$ 10). Assume three input control signals: V1 (valid data), A (compute and output the average), S (compute and output the standard deviation). Ignore S unless computation of the average has been completed. The computation module should include a square root circuit which will find the square root of a 18-bit binary integer to give a 9-bit integer result. Refer to Reference 35 for a binary square root algorithm. When testing the computation module, be sure to include the worst cases: largest average with 31 inputs (s.d. should be 0), largest standard deviation with 30 inputs (average should be 500).

The display module should drive a two-line LCD display. This module serves two functions: First it displays each number as it is being input, and second it displays the average and standard deviation. During input, each valid decimal digit should be shifted into the display. When E is pushed, the input number will remain displayed until another key is pushed. After the average has been computed, the display module should convert it to BCD and output it to the first line of the LCD display. After the standard deviation has been computed, it should be converted to BCD and displayed on the second line.

P10. Four-Function Decimal Calculator

Design a four-function hand-held calculator for decimal numbers and implement it using an FPGA. The input will be a keypad and the output will be an LCD display. When you implement your design on the FPGA, optimize for area since speed is unimportant for a hand calculator. General operation of the calculator should be similar to a standard four-function calculator.

The main calculator input keypad has 16 keys to be labeled as follows:

7 8 9 ÷
4 5 6 *
1 2 3 –
0 . = +

Use one additional key for the clear function. The input and output will be a maximum of eight decimal digits and a decimal point with an optional minus sign. Assume that at any time, any key may be pressed. Either take appropriate action or ignore the key press. If more than eight digits are entered, extra digits are ignored.

If the answer requires more than eight digits, some digits to the right of the decimal point are truncated.

Example: 123.45678 + 12345.678 = 12469.134

If more than eight digits are required to the left of the decimal point, display the letter E to indicate an error. For numbers less than 1, display a 0 before the decimal point.

Your calculator should have three modules. The input module scans, debounces, and decodes the keypad. The main module accepts digits and commands from the input module and processes them. The display module displays the input numbers and results on an LCD display.

The main module should have two 8-digit BCD registers, A and B. Register A should have an associated counter that counts the number of digits (ctrA), another counter that counts the number of digits to the right of the decimal point (rctA), and a sign flip-flop (signA). Register B should have similar associated hardware. As each decimal digit is entered, its BCD code should be shifted into A. The result of each computation should be placed in A. The display module should always display the contents of A, along with the associated decimal point and sign. When the first digit of a new number is entered into A, the previous contents of A should be transferred to B. Although input and output is sign and magnitude BCD, internal computations

should be done using 2's complement binary arithmetic. A typical sequence of calculations to add A and B is

1. Adjust A and B to align the decimal points.
2. Convert A and B to binary (Abin and Bbin).
3. Add Abin and Bbin
4. Convert the result to BCD, store in A.
5. If an overflow occurs, correct it if possible, else set the E (error) flag.

The display module should output signals to the LCD to properly display the contents of the A register. After initializing and clearing the LCD, it should display "E" if the error flip-flop is set. Otherwise it should output a minus sign if signA = '1', followed by up to eight digits with the decimal point in the correct place. Leading zeros should be replaced by blanks.

References

References 14, 19, 21, 27, 28, 39, 41, 46, and 48 are general references on digital logic and digital system design. References 2, 3, 4, 15, 16, 20, 24, 30, 40, 42, 44, 47, 49, and 50 provide information on PLDs, FPGAs, and CPLDs. References 10, 21, 31, 38, 43, 45, and 52 provide a basic introduction to VHDL. References 5, 6, 8, 9, 17, 18, 22, 23, 33, 34 and 51 cover more advanced VHDL topics. References 1, 7, 11, 29, 32, and 36 relate to hardware testing and design for testability. The MIPS ISA and architectures of several MIPS processors are described in references 13, 25, 26 and 37.

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